LuGre-Model-Based Friction Compensation

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Abstract—A tracking problem for a mechanical system is considered. We start with a feedback controller that is designed without attention to disturbances, which are assumed to be adequately described by a dynamic LuGre friction model. We are interested in deriving a superimposed observer-based compensator to annihilate or reduce the influence of such a disturbance. We exploit a recently suggested approach for observer design for LuGre-friction-model-based compensation. In order to apply this technique, it is necessary to know the Lyapunov function for the unperturbed system, as well as the parameters of the dynamic friction model, and to verify that a certain structural property satisfied. The case when the system is passive with respect to the matching disturbance related to the given Lyapunov function is illustrated in this brief with a DC-motor example. The main contribution is some new insights into the numerical real-time implementation of a compensator for disturbances describable by one of various LuGre-type models. The other contribution, which is built upon the main one, is experimental verification of the suggested model-based observer design procedure.

Index Terms—Friction compensation, LuGre model, observer design, real-time implementation.

I. INTRODUCTION

W E CONSIDER here the problem of overcoming the influence of friction forces, which is a very common obstacle for successful feedback control design in mechanical systems. Typically, a control law solving the tracking problem for a mechanical system is derived first under the assumption that friction forces are negligible. However, in every mechanical system friction forces of different physical nature, as well as other disturbances, are always present and quite often significant in comparison with the admissible levels of control signals. In practice, compensation of such disturbances is done using various ad hoc procedures. The most common way is to derive one or several friction maps and use them for an approximate cancellation of disturbances mainly defined by friction. The major difficulty is taking into consideration various nonlinear effects like hysteresis in the approximate friction-velocity dependence, difference between static and Coulomb friction, Strubeck effect, variations of model parameters due to environment and time, etc., [3], [8], [10], [11], [15], [22]. Another difficulty is due to the variety of places where there is interaction between surfaces of a particular hardware setup. As for the details of the commonly used friction models—e.g., the Dahl model, the LuGre model [11], the Leuven model [19], [25], the elastoplastic model [13], and the generalized Maxwell-slip model [1], [25]—and their applications, we refer to the survey paper [3] and, for robotic application, to [2], [7], and [12].

Often various disturbances mainly defined by friction, acting in the system at different places, can be approximately described by one of lumped dynamic models such as LuGre [4] attractive from the point of view of observer-based design of a compensator. However, there is a lack of such rigorously motivated design techniques. Two recently proposed theoretical strategies, based on the LuGre model, rely on perfect knowledge of all the parameters and of a Lyapunov function for a friction-free design [23], [24]. In the approach of [24], matching and passivity with respect to the control input are exploited; while in [23], the key is a certain property of the derivative of the Lyapunov function, which is case of a linear-quadratic-regulator-type design. In both cases, the observer structure is classical: a copy of the model with an additional term, which is a function of either the passive output [24] (this is a direct generalization of the observer suggested in [11] for a particular benchmark example) or of the control input [23].

It is worth to underline the practical usefulness of designing an appropriate compensator to complement a fixed controller, which is preliminarily designed for a disturbance-free plant.

In this brief, we do not discuss important practical issues related to uncertainty of the disturbance model. We assume that the LuGre-like model structure with nominal values of parameters is adequate, and we concentrate on the fundamental issues of implementation of a model-based compensator. Therefore, we leave adaptive modifications, such as in [10] and others, as well as robustification, such as the design of an additional disturbance/unknown input observer (see [14] and others), out of consideration.

In Section II, we proceed with the standard pole-placement feedback control design, followed by a passivity-based compensation of friction for a particular simple mechanical system: a DC motor with an attached rigid arm (see the lower right corner of Fig. 1). Since the control design itself is not based on passivity, the theorem from [24] is not applicable. However, the design of friction observer is to be done conceptually along the lines proposed in [24] and can be similarly generalized to a class of fully actuated mechanical systems. Note that the convergence proof for the tracking problem is simpler than for the case of stabilization of a set as in [23], [24].
The proposed observer design procedure is verified experimentally in Section III. During the verification stage, we have to face an important intrinsic difficulty of code generation and for each LuGre-type-model-based friction compensation scheme that contains that is linear with respect to the internal state. We demonstrate that, typically, the Euler discretization of observer dynamics is not appropriate and should not be used. Instead, we propose a new observer implementation technique that is built on an assumption that all the input signals for the observer are constant during the sampling period. This is a natural approach since, between sampling instances, these signals are unavailable anyway. In the main part of this brief (Section II-C), we present some preliminary results of the performance analysis for the new scheme and compare it to the Euler scheme. This brief is concluded with some observations and a discussion on the proposed discretization technique in Section IV and with a summary in Section V.

II. DC MOTOR WITH DYNAMIC FRICTION

Let us consider the simplified model for a DC motor with a rigid arm attached

$$J\ddot{\phi} = k_c u - F$$  \hspace{1cm} (1)

where $J$ is the total moment of inertia of the rotor and the arm, $k_c$ is the motor constant, $u$ is the applied control signal, and $F$ is an unknown friction torque. Following [11], we assume that friction can be modeled by the dynamic LuGre model

$$F = z + \sigma_1\dot{z} + \sigma_2\dot{\phi}, \quad \dot{\phi} = \phi - \frac{|\dot{\phi}|}{g(\dot{\phi})} z$$ \hspace{1cm} (2)

where $z$ is the average deflection of the bristles modeling the contact between the interacting surfaces [11], $\varepsilon_0 = 1/\sigma_0$ is the reciprocal of the averaged stiffness of the bristles $\sigma_0$, $\sigma_1$ is the damping coefficient of the bristles, $\sigma_2$ is the viscous friction coefficient, and the Stribeck curve is defined by

$$g(v) = \begin{cases} F_{c+} + (F_{s+} - F_{c+})e^{-(v/v_s)^2}, & v > 0 \\ F_{c-} + (F_{s-} - F_{c-})e^{-(v/v_s)^2}/2, & v < 0 \\ (g(0^+) + g(0^-))/2, & v = 0 \end{cases}$$

with $F_{c+}$ and $F_{c-}$ being the Coulomb and static friction values and $v_s$ being the Stribeck velocity. It is assumed that $|z(0)| \leq \max\{F_{s-}, F_{s+}\}$.

A. Control Design for Tracking

Suppose that our goal is to design a dynamic output feedback controller to ensure $L_2$-stability [17, p. 198] and a certain transient behavior of the tracking error

$$e_1(t) = \varphi(t) - r(t)$$

where $\varphi(t)$ is the only measured output of the system and $r(t)$ is the desired trajectory, assumed to be smooth and known with a required number of continuous bounded derivatives. For simplicity, let us assume that it is enough to achieve

$$\dot{e}_2 + 2\eta e_2 + \omega_0^2 e_1 \approx 0, \quad e_2 = \dot{e}_1$$ \hspace{1cm} (3)

where $\eta > 0.25$ is the desired damping ratio and $\omega_0 > 0$ is the desired frequency of the error dynamics.
Assuming that all parameters of the system are known and neglecting the friction force, i.e., assuming that the system can be described by (1) with \( F \equiv 0 \), we can take
\[
u = \text{sat}(\hat{J} (\dot{\varphi} - k_p (\dot{x}_1 - r) - k_d (\dot{x}_2 - \dot{r}))) / \hat{k}_c
\]
where \( \hat{J} = J \), \( \hat{k}_c = k_c \), \( \text{sat}(\cdot) \) is the standard saturation function with a certain level, \( k_p = \omega_0^2 \), \( k_d = 2 \eta \omega_0 \), \( \dot{x}_1 \) and \( \dot{x}_2 \) are the states of the linear high-gain observer
\[
\begin{align*}
\dot{x}_1 &= \dot{x}_2 + 2(\varphi - \dot{x}_1) / \varepsilon \\
\dot{x}_2 &= (\varphi - \dot{x}_1) / \varepsilon^2
\end{align*}
\]
and \( \varepsilon > 0 \) is a sufficiently small parameter.\(^1\) Note that the level of saturation function should be designed either analytically or through extensive simulations to ensure that the control signal is not saturated whenever the states are perfectly estimated and the initial conditions of the closed-loop system are taken from a reasonable compact set. The saturation function is introduced to protect the closed-loop system from the destabilizing peaking effect (see [17, Sec. 14.5] for details).

It follows from the results of [5] and [17] that if the initial conditions belong to a known compact set and the level of saturation is chosen appropriately, the trajectories of the closed-loop system (1), (4), (5) with \( F \equiv 0 \) are such that
\[
\bar{e}_1 + 2 \eta \omega_0 \bar{e}_1 + \omega_0^2 e_1 = O(\varepsilon)
\]
i.e., we have ideal dynamic (3) up to small terms. Moreover, the closed-loop system is exponentially stable, provided that the initial conditions belong to a given compact set and \( \varepsilon > 0 \) is chosen sufficiently small.

However, if friction is present \( F \neq 0 \) and, for example, approximately satisfies (2)], there is no guarantee that (6) is valid; moreover, stability may be lost.

To check that neglecting friction could indeed be a problem, we test it on our experimental setup: a brush-commutated DC-motor Penta M1 with an attached rigid arm, which is equipped with a tacho generator servo amplifier SCA-SS-70-10 and an incremental encoder Elitra EH53A (the resolution is 1024 imp/turn). We use MATLAB/Simulink in combination with a dSPACE\(^2\) board (DS1104) to obtain discrete-time sampled versions for our controllers and to implement them in real time with the sampling period of \( h = 0.0001 \) s. We take \( \hat{k}_c = 2.5, \hat{J} = 0.005, \varepsilon = 0.01, r(t) = a_r \sin(\omega t), a_r = 1, \omega = 2, \omega_0 = 1 \), and \( \eta = 0.7 \).

The result is shown in Fig. 1. We see that there is a significant tracking error, which could be due to parametric uncertainty (i.e., \( k_c / (J = \hat{k}_c / \hat{J}) \)), uncompensated friction torque, neglected hidden dynamics, and measurement noise. It is worth noting that the largest tracking error is on the intervals around the points where the velocity is crossing zero, which we attribute to a manifestation of friction forces.

\(^1\)The transfer function \( s / (s^2 + 1)^2 \) defined by the linear system (5) approaches pure derivative as \( \varepsilon \to 0 \). It is also a low-pass filter with the cut-off frequency around \( 1 / \varepsilon \); this is the source of the trade off between better differentiating ability and noise attenuation.

\(^2\)MATLAB and Simulink are trademarks of MathWorks Inc., while dSPACE is a trademark of dSPACE Inc.

### B. Model-Based Friction Compensation

We believe that friction is the major issue in the system at hand (and so we are going to ignore unavoidable parametric uncertainty). One way to improve performance is to supplement (4) with an appropriate adaptive or robust addition [17]. For an approach, based on the extended high-gain observer, which is experimentally tested on our setup, see [14]. Here, however, we would like to concentrate on model-based friction compensation.

For the purpose of analysis aiming at exploiting the trajectory recovery property of high-gain observers, we start with the case when one can apply the following unsaturated state-feedback controller:
\[
u = (J (\dot{r} - k_p e_1 - k_d e_2) + \tilde{F}) / k_c
\]
where \( e_1 = \varphi - r(t), e_2 = \dot{\varphi} - \dot{r}(t) \), and \( \tilde{F} \) is a compensating term for \( F \).

We take
\[
\tilde{F} = \dot{z} + \sigma_1 z_0 \dot{z} + \sigma_2 \dot{z}, \quad e_0 \dot{z} = \dot{\varphi} - \frac{1}{g(\varphi)} \dot{z} + K(\cdot)
\]
where the friction parameters are assumed to be identified perfectly and \( K(e_1, e_2, \dot{\varphi}) \) is a function to be defined. The results of identification for our setup are \( \sigma_2 = 0.001, F_{c_k} = 0.02 \cdot k_c, F_{c_0} = 0.021 \cdot k_c, F_{c_s} = 0.058 \cdot k_c \), and \( F_{c_{ss}} = 0.052 \cdot k_c \).

We took \( \sigma_1 = 1.5 \) and \( e_0 = 0.01 \) since we failed to identify reliable values for these parameters due to insufficient resolution of our encoders; we have checked that varying them in certain ranges does not influence performance.

Exploiting the strictly positive real property of the transfer function \( (\sigma_1 s + \sigma_2) / (J s^2 + k_d s + k_p) \) defined by the closed-loop system (1), (2), (7), (8) and for the special case when \( r(t) \) is constant, taking \( K(e_1, e_2, \dot{\varphi}) = k e_1 \), with \( k > 0 \), has been proposed in [11]. For a more general case, various formulas for \( K(e_1, e_2, \dot{\varphi}) \) have been recently derived in [23] and [24]. Passivity of the friction-free closed-loop system was used in [24], an approach that is straightforward to apply for our system. In order to follow [23], a special, although pretty general, property of the derivative of the Lyapunov function has to be shown. Despite the fact that the synthesized observers are different, in all these three papers, the first step of observer design is the same, and we are going to follow it.

Let us analyze the closed-loop system formed by (1), (2), (7), and (8)
\[
\begin{align*}
\dot{e}_2 + 2 \eta \omega_0 e_2 + \omega_0^2 e_1 &= -\frac{\dot{\varphi} + \sigma_1 e_0 \dot{z}}{J} \\
\dot{e}_1 &= e_2 \\
e_0 \dot{z} &= -\frac{1}{g(\varphi)} \dot{z} - K(e_1, e_2, v)
\end{align*}
\]
where \( \dot{z} = z - \dot{z} \) and \( v = \dot{\varphi} = e_2 + \dot{r} \), exploiting the Lyapunov function candidate in the form
\[
W(e_1, e_2, \dot{z}) = V(e_1, e_2) + \frac{\rho e_0^2}{2} \dot{z}^2
\]
with \( \rho > 0 \) and \( V(e_1, e_2) \) being a Lyapunov function for (3).
Let us take
\[ V(e_1, e_2) = \omega_0 (1 + \eta) e_1^2 + e_1 e_2 + \frac{e_2^2}{\omega_0} \]
so that
\[ \dot{W} = -\omega_0^2 e_1^2 - (4\eta - 1) e_2^2 - \left( 1 + \frac{2e_2}{\omega_0} \right) \frac{\dot{z}}{J} + \rho \varepsilon_0 \dot{z}^2 \]
and after eliminating \( \dot{z} \)
\[ \dot{W} = -\omega_0^2 e_1^2 - (4\eta - 1) e_2^2 - w \dot{z}^2 + (\sigma_1 w - \rho \dot{z}) \left( \frac{|v|}{g(v)} \dot{z} + K(\cdot) \right) \]
where
\[ w = \frac{e_1 + 2e_2}{J} \]
is the passive output for the first equation in (9) with respect to its right-hand side as the input and \( V(e_1, e_2) \) as a storage function.

It is not hard to see that
\[ \dot{W} = -\omega_0^2 e_1^2 - (4\eta - 1) e_2^2 - \sigma_1 w^2 + (\sigma_1 w - \rho \dot{z}) \left( \frac{|v|}{g(v)} \dot{z} + K(\cdot) + \frac{w}{\rho} \right) \]
Therefore, we take
\[ K(e_1, e_2, v) = -\frac{w}{\rho} - \frac{|v|}{g(v)} \frac{\sigma_1 w}{\rho} \]
so that
\[ \dot{W} = -\omega_0^2 e_1^2 - (4\eta - 1) e_2^2 - \sigma_1 w^2 - \left( \sigma_1 w - \rho \dot{z} \right)^2 \frac{|v|}{g(v)} \]
\[ \dot{W} \leq -\omega_0^2 e_1^2 - (4\eta - 1) e_2^2 - \sigma_1 w^2 - \left( \sigma_1 w - \rho \dot{z} \right)^2 \frac{|\hat{p}(t)| - |e_2|}{\max\{F_{sw}, F_{sw}\}} \]

It follows that \( e_1(t), e_2(t) \equiv \hat{e}_1(t), \) and \( \hat{z}(t) \) are bounded. It is worth noting also that \( \hat{z}(t) \) is bounded as well [11]. It is not hard to show now (see [17, Th. 8.4] or [24]) that, along any solution of the state-feedback closed-loop system, we have \( e_1(t) \to 0 \) and \( e_2(t) \to 0 \).

Now, complementing the previous analysis with the separation principle ideas from [5, we take, instead of (8)
\[ \dot{\hat{F}} = \dot{z} + \sigma_1 \dot{\hat{z}} + \sigma_2 \dot{v}, \quad \sigma_0 \dot{z} = \hat{v} \quad \frac{|\hat{v}|}{g(v)} \dot{z} + K(\cdot) \] (11)
with
\[ K(\cdot) = \left( 1 + \frac{\sigma_1}{g(v)} \right) \frac{e_1 + 2e_2}{\omega_0 \rho J} \]
\[ \hat{v} = \text{sat}(\dot{\hat{z}}), \quad \dot{\hat{v}} = \hat{v} - \hat{r}(t) \] (12)

where \( \rho > 0 \), combined with the linear high-gain observer (5), and let
\[ u = \text{sat} \left( \hat{J} \left( \hat{z} - k_p e_1 - k_d \dot{e}_2 + \hat{F} \right) / k_c \right) \] (13)
with the level of saturation function being appropriately tuned. Asymptotic elimination of the tracking errors in the presence of a friction torque satisfying (2)—or recovery thereof—does not follow directly from [5] since asymptotic stability has not been shown for the system with the controller using velocity measurements. However, for the case of known parameters, closely following the arguments presented in [5] to complement the previous Lyapunov-function-based analysis and exploiting the singular perturbation technique, convergence to a set where the tracking error is small can be shown.

Proposition 1: There exists \( \varepsilon > 0 \) such that, for all \( \varepsilon \in (0, \varepsilon_0] \), all the trajectories of the closed-loop system (1), (2), (5), (11), (12), (13) initiated inside a given compact set are bounded, converge to a set where \( |e_1(t)| \) is ultimately bounded by a bound which approaches zero as \( \varepsilon \) vanishes, and stay thereafter, provided that \( \hat{J} = J \) and \( \hat{k}_c = k_c \).

C. Discretization of the LuGre Model

In order to implement (8) with any choice of \( K(\cdot) \) in real time with a sampling time \( h > 0 \), we need to obtain an appropriate sampled discrete-time version. For the purpose of qualitative analysis, we rewrite the differential part of (8) as
\[ \dot{z} = a(t) \dot{z} + b(t) \] (14)

where the coefficients are
\[ a(t) = \frac{|\dot{z}(t)|}{\varepsilon_0 g(\dot{z}(t))} \geq 0 \]
\[ b(t) = \frac{\dot{z}(t) + K(e_1(t), e_2(t), \dot{v}(t))}{\varepsilon_0} \]

Under sufficiently fast sampling, due to continuity, one might assume the coefficients of this equation to be approximately constant during the \( k \)th sampling period \( (k \in \mathbb{Z}) \), i.e., for \( t \in (kh, kh + h) \). This assumption is, of course, unavoidable if we would like to implement the controller in real time with zero-order hold. Hence, it is useful to consider the patched continuous solution \( \{ \bar{x}_k(t) \}_{k=1}^{\infty} \) of the infinite family of differential equations
\[ \bar{\ddot{x}}_{k+1} = -\bar{a}_k \bar{z}_{k+1} + \bar{b}_k, \quad \bar{z}_{k+1}(kh) = \bar{z}_k(kh) \] (15)

each of which is defined for \( k: h < t \leq kh + h \), with
\[ \bar{a}_k = \frac{|\dot{z}(kh)|}{\varepsilon_0 g(\dot{z}(kh))} \geq 0 \]
\[ \bar{b}_k = \frac{\dot{z}(kh) + K(e_1(kh), e_2(kh), \dot{v}(kh))}{\varepsilon_0} \]

If we leave the discretization of (14) to real-time implementation—e.g., dSPACE or Real-Time Workshop—a fixed-step-
based approximation will be performed. To be specific, let us assume that the Euler method is used so that

\[ \dot{z}_{k+1} = (1 - \alpha_k h) \dot{z}_k + h \tilde{r}_k. \]  

(16)

It is not hard to see that if \( |\dot{z}(t)| \neq 0 \), then \( \tilde{a}_k \neq 0 \) and \( \alpha(t) \neq 0 \). As a result, the homogeneous parts of (14) and (15) are exponentially stable, while the homogeneous part of (16) is unstable if speed is restricted so that

\[ \alpha_k > \frac{2}{h}. \]

From this remark, the following statement follows.

**Lemma 1:** Suppose that \( \dot{z}(0) = z_k(0) \) and, for all \( t \geq 0 \)

\[ |\dot{z}(t)| \geq \tilde{a} = \frac{\max \{ F_{\text{inc}}, F_{\text{exc}} \}}{h} \]

and \( |b(t)| \) is globally bounded. Then, the solutions of (14) and (15) are bounded and are close to each other for sufficiently small values of \( h > 0 \), while for almost all \( \tilde{a}_k \), the solutions of (16) are unbounded.

It is intuitively clear from this result that Euler discretization (16) is not reliable whenever there are infinitely many intervals of time where \( |\dot{z}(t)| \geq \tilde{a} \).

In order to suggest a better discretization approach, it is crucial to notice that (15) can be solved analytically in the case when the functions \( \alpha(t) \) and \( b(t) \) are given and simple. In particular, one can integrate (15) exactly and obtain

\[ \dot{z}_{k+1} = e^{-\alpha_k h} \dot{z}_k + \frac{e^{-\alpha_k h} - 1}{\tilde{a}_k} b_k \]

(17)

where \( \dot{z}_k = z_k(kh - h) \) and the singularity at \( \tilde{a}_k = 0 \) is removed (substituted with the limiting value). For closeness of the solutions of (17) and (15), the sampling period \( h \) must be sufficiently small but not necessarily as small as needed for a reliable implementation of (16). It is important to notice that \( e^{-\alpha_k h} < 1 \) over any time interval where \( |\dot{z}(t)| \) is separated from zero. As a result, we have the following statement.

**Lemma 2:** For any \( \tilde{a} > 0 \) and \( h > 0 \), the difference equation (17) with \( |\dot{z}(t)| \geq \tilde{a} \) is stable and satisfies the bounded-input bounded-state property with the sequence \( \{\tilde{u}_k\} \) considered as an input. Moreover, the solutions of (14) and (17) are close to each other for sufficiently small values of \( h > 0 \).

We conclude that (17) is a better approximate sampled realization of (14), correspondingly for (8), than (16). It should be noticed, however, that, in order to use the new scheme, we need a procedure for the evaluation of \( (e^{\alpha} - 1)/\alpha \), which is a difficult function for numerical implementation when \( \alpha \) is small. It is recommended to use the mathematically equivalent formula\(^3\):

\( (e^{\alpha} - 1)/\ln(e^{\alpha}) \), which is reliable from a numerical point of view.

We believe that, although the aforementioned computations are simple, our suggestion to use (17) instead of (16) is crucial. It is worth mentioning that if any of the other more involved friction models [12], [13], [19] with similar dynamics is used for compensation, it also has to be carefully discretized following the same approach.

\(^3\)The idea is due to W. Kahan, unpublished course notes, according to materials of MathWorks, Inc.
III. EXPERIMENTAL VALIDATION

We show in the following the results of our experiments with the controller (13), (5) for the two cases:
1) Fig. 2: using (11) and (12) with \( \rho = 10 \), discretized using (17);
2) Fig. 3: using (8) with \( K \varphi \equiv 0 \), discretized by (17), and \( \hat{\varphi} \) substituted with \( \dot{\varphi} \).

The parameters of the controllers are the same as that in the previous.

We can see that the reduction of the magnitude of the tracking error is not significant. Note, however, that if we are concerned about the distances between the trajectory and the reference, the improvement is noticeable. One can see in Fig. 1 that the system gets stuck due to stiction for the time interval where the desired velocity is small. This problem is eliminated when the observer is acting (see Fig. 2) at the expense of generating a peaklike compensation signal. As seen in Fig. 3, the open-loop friction compensator works, as well as the passivity-based one; however, it would be useless for the case when a periodic orbit (or a certain set) is stabilized orbitally [17, Sec. 8.4] as in [23] and [24].

IV. DISCUSSION

We would like to notice that the value of \( \rho \) in (12) that we have tuned to obtain an acceptable performance is very high, and this choice reduces the influence of the injection term. An intuitive choice \( K(\cdot) \varphi \equiv 0 \) can be partially informally argued as follows. Obviously, the error in the estimation of nonviscous friction is proportional to the error in the estimation of the interior friction state \( \hat{z} = z - \hat{z} \), which can be computed as

\[
\hat{z}(t) = \hat{z}(0) \exp \left\{ -\frac{1}{\varepsilon_0} \int_0^t \frac{|u(\tau)|}{g(x(\tau))} d\tau \right\}
\]

and vanishes exponentially on every time interval where \( u(t) = \dot{\varphi}(t) \) is separated from zero. The closed-loop system with such an open-loop observer is a singular perturbation of the system with a perfectly compensated friction. However, the rigorous analysis, based on the assumption that \( \varepsilon_0 > 0 \) is sufficiently small, is not standard since the fast subsystem has a discontinuity in the right-hand side and the assumption that it is exponentially stable uniformly in \( v \) is violated (vanishes at \( v = 0 \)). It should be noticed that taking \( K(\cdot) \equiv 0 \) and substituting \( \hat{\varphi} \) in (8) with \( \dot{\varphi} \) (or with a weighted sum of \( \dot{\varphi} \) and an estimate from the measurements \( \dot{\varphi} \)) may work well in practice (cf. Fig. 3). It is intuitively clear that it should be a reasonable approximation in the case when the tracking error and its derivative are small. In such a way, robustness with respect to measurement noise is improved as well. However, to the best of our knowledge, there is no rigorous justification for such an approach.

It is useful to compute the critical bound on speed between the interacting surfaces, which is given in Lemma 1. For our values of the parameters, we need the following: \( |\varphi| > \bar{\tau} \approx 0.0013 / h \). Therefore, if we WOULD use the sampling time \( h = 0.001 \) s, we COULD NOT allow the approximation (16) with speeds greater than 1.3 rad/s. From the practical point of view, the situation is even worse; since we do not measure velocity and use a high-gain observer (5) to estimate it, the transient during the peaking period [17, Sec. 14.5] is potentially dangerous for the stability of the overall system with (16), followed by zero-order hold, in the loop. It is worth noticing that if the value of \( \varepsilon_0 \) is less than \( 10^{-4} \) (which is often the case, see, e.g., [8], [11], and ), the range of “safe velocities” is practically unacceptable. This problem is a well-known obstacle for a real-time implementation of any kind of observer for a LuGre-model-based or similar

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Fig. 3. Experimental results. Performance under the pole-placement-based controller with the open-loop friction observer.
friction compensation. A possible ad hoc solution, taken from [8, slide 41], is as follows: “For larger velocities, it is advisable to stop the integration of $\dot{z}$ and to use its steady-state value. Another alternative is to change the method for discretization.” Note that higher-order discretization techniques would probably be limited as well; we have observed no benefits from using higher order fixed-step integration methods. We believe that the solution proposed in Section II-C is useful; it resolved our problems with the implementation of the model-based observer designed as here or as in [23] and [24].

Discussion on the validity of the LuGre model from the physical point of view is outside the scope of this brief. It should be noticed, however, that, in a series of previously published papers, various features of this model have been criticized from both theoretical and practical points of view, and even some modifications have been suggested (see, e.g., [6], [12], [13], [16], [19], and [21]).

Other approaches dealing with equilibrium sets (stick-slip motion), stability, and limit-cycle behavior in friction compensation include variable-structure control for uncertain nonsmooth friction [18] and friction compensation based on reduced-order observers [20].

V. CONCLUSION

We have considered the problem of observer-based design for friction compensation. Our choice for the description of the friction phenomenon has been the dynamic LuGre model.

The LuGre model (or a similar alternative) is an attractive choice for a rigorously justified observer-based design of approximate compensators. It is worth to be tested experimentally for a particular hardware setup and might work better than an ad hoc compensation procedure. Nevertheless, despite the fact that the model is known for a long time, there is a lack of reported compensator design techniques, which are successfully verified in experiments. Due to our belief that the main reason for this is an intrinsic problem with discretization, which is known to the authors of the model, we have concentrated here on resolving this issue.

We have recalled that the standard Euler-scheme-based discretization of any LuGre-type-model-based observer is typically not reliable, and we have suggested an alternative solution. The new discretization approach, combined with a simplified version of the passivity-based compensator from [23], has been applied to treat the tracking problem for a DC motor. We have presented experimental results showing that the problem with real-time implementation is removed. At this point, to make the compensation scheme more practical, some efforts should be directed to incorporation of adaptation to the uncertainty of the parameters of the friction model.

REFERENCES