KINEMATIC ANALYSIS AND PROTOTYPE OF A METAMORPHIC ANTHROPOMORPHIC HAND WITH A RECONFIGURABLE PALM

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A novel metamorphic anthropomorphic hand is for the first time introduced in this paper. This robotic hand has a reconfigurable palm that generates changeable topology and augments dexterity and versatility of the hand. Structure design of the robotic hand is presented and based on mechanism decomposition kinematics of the metamorphic anthropomorphic hand is characterized with closed-form solutions leading to the workspace investigation of the robotic hand. With characteristic matrix equation, twisting motion of the metamorphic robotic hand is investigated to reveal both dexterity and manipulability of the metamorphic hand. Through a prototype, grasping and prehension of the robotic hand are tested to illustrate characteristics of the new metamorphic anthropomorphic hand.

Keywords: Metamorphic mechanisms; metamorphic hand; reconfigurable palm; Kinematics.

1. Introduction
Since the first industrial robot being invented by Engelberger in the early 1960s, there is a growing interest in robotic hand research and in the past three decades a great variety of robotic hands have been developed. The mechanical palm of a robotic hand is usually designed as a non-foldable block incorporated with fingers. This includes the Stanford/JPL hand, Utah/MIT hand, GIFU hand, Belgrade/USC hand, DLR hand, NAIST-Hand, MANUS hand, and UBH3 hand. Corresponding author.
The non-foldable palms of the above hands have limited dexterity of the multi-fingered hands and mechanisms with more functionality are needed for secure grasping and manipulation of different and complex objects. This then leads to the DLR II hand,\textsuperscript{11} the HIT/DLR hand,\textsuperscript{12} Shadow Hand,\textsuperscript{13} DLR/HIT Hand II,\textsuperscript{14} Fluidic Hand,\textsuperscript{15} and the more recently development of two anthropomorphic hands by IIT (Italian Institute of Technology)\textsuperscript{16} and Bi-on-ics\textsuperscript{17} by breaking a palm into two to three movable sections in order to increase dexterity and manipulability of the hands.

However, in contrast to a human hand with a foldable and flexile palm, the above robotic and anthropomorphic hands do not have more versatility. In particular, conventional design of a finger has three parallel joints and the finger usually operates on a plane, this is the main limitation of robotic hands resulting in limitation for manipulation and difficulty of adapted the hands to the geometric shape of a manipulated object, and realizing fine manipulation. This also limits the fine-tuning ability\textsuperscript{18} and subsequently the use of robotic hand in prosthesis. Therefore, with the advancement of the robotics field, it is important to re-examine the mechanism of a multi-fingered robotic hand. The mechanism needs to provide additional functionality to robotic hands that the hands can only not grasp as well as manipulate but also have more dexterity and versatility. The introduction of using the metamorphic mechanism as the palm for the novel metamorphic robotic three-fingered hand\textsuperscript{19–21} marked a turning point and shed light on using the reconfigurable mechanism as a palm of robotic hands to improve their dexterity and versatility. Metamorphic mechanisms\textsuperscript{22} are a class of mechanisms that are capable of altering topological configurations from one to another with a resultant change in the mobility of mechanisms. Originated from artworks, this class of mechanisms can be extracted from origami folds that change topological structure\textsuperscript{23} during motion.

The initial concept of metamorphic mechanisms\textsuperscript{24} is originated by metamorphosis, which depicts the change in form, topology, and configuration. Metamorphic mechanism is capable of adapting its topology and functions to meet the various environments and demands. The mechanism is not just designed to perform a monotonous task, but an integrated system that can be reconfigured into various types of submechanisms to execute a variety of tasks. The reconfigurable topology and variable mobility characteristics of the metamorphic mechanisms offer versatility, adaptability, and low cost for diverse and changing applications.\textsuperscript{25}

Based on metamorphic mechanism, this paper for the first time presents a novel metamorphic anthropomorphic hand with a reconfigurable palm. Structure design of the metamorphic hand is introduced and kinematics of the hand is investigated. Augmented workspace of the hand is subsequently explored and twisting motion is studied based on characteristic equation matrix. A prototype of metamorphic hand is presented illustrating the characteristics of the hand.
2. Structure Design of the Metamorphic Anthropomorphic Hand

Based on the previous development of the metamorphic three-fingered hand stemming from an origami fold as having been presented in the patent by Dai, a dexterous metamorphic anthropomorphic hand is developed in this paper as illustrated in Fig. 1. The hand consists of a reconfigurable palm and five fingers, i.e. a four-DOF thumb and three-DOF index finger, middle finger, ring finger, and little finger. The reconfigurable palm is a spherical five-bar linkage containing five links \( l_1 \) to \( l_5 \) with the base link \( l_5 \) connected to the wrist. Two actuated joints A and E are introduced for adjusting position and orientation of the spherical reconfigurable palm. Joint A is in particular used to change the structure of the reconfigurable palm by rotating the crank link \( l_1 \), forming a four-bar linkage at a metamorphic phase. A three-phalanxed thumb of the hand is amounted at link \( l_2 \) with joint T providing the fourth DOF, a three-phalanx index finger is mounted at link \( l_3 \), and the three-planlax middle, ring, and little fingers are amounted at link \( l_4 \). In accordance with the size of an adult’s hand, the maximum radius of the spherical linkage palm is assigned as 50mm. The angles corresponding to links \( l_1 \) to \( l_5 \) (see Fig. 2) are \( \alpha_1 \) to \( \alpha_5 \) complying with \( \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = 2\pi \). In order to increase the dexterity of the palm, both human hand arrangement and rotatability criterion of spherical linkage are considered, so that the angles of links are assigned as \( \alpha_1 = 25^\circ, \alpha_2 = 40^\circ, \alpha_3 = 70^\circ, \alpha_4 = 112^\circ, \) and \( \alpha_5 = 113^\circ \). In this case, the fundamental representation of the link angles \( \{\bar{\alpha}\}_{i=1}^{5} = \{25^\circ, 40^\circ, 67^\circ, 68^\circ, \) and \( 70^\circ \} \) satisfies

\[
\alpha_1 + \alpha_2 + \alpha_5 = \alpha_3 + \alpha_4. \tag{1}
\]

Thus, excluding the indeterminate position where all links fall on a great circle, all joints have full rotatability except for the joint between links 4 and 5.

![Fig. 1. A metamorphic anthropomorphic hand with a reconfigurable palm.](image-url)
The structure designed of this palm is indicated in Fig. 3, where finger bases are mounted at the corresponding links. In order to reduce the size of the palm, tendon drive method is considered, so that three pulleys are embedded in links $l_5$ and $l_2$ at joints A, E, and T (see Fig. 1) to actuate the palm and the fourth degree of the thumb. The pulleys are driven by steel tendons connected to three DC motors amounted in the forearm. The bases of the index, middle, ring, and little fingers are arranged in such a manner that, when all the palm links lie on a same plane (i.e. all links fall on a same great circle), the MCP joints of these fingers are perpendicular to plane $\Sigma$ (see. Fig. 3) passing through the center of the palm and perpendicular
to twist joint. While, the finger base of the thumb is connected to \( l_2 \) through joint T whose axis passing through the center of the palm, leading to an additional DOF for the thumb.

3. Kinematics of the Metamorphic Anthropomorphic Hand

The beneficial effect of the reconfigurable palm can be indicated by its ability of changing palm size and changing finger positions and postures to suit various tasks. The motion of \( l_1 \) and \( l_4 \) results in the change of the palm topological structure. When joint E is fixed at a certain value, the palm operates as a spherical four-bar linkage by evolving into a one-DOF phase. This results in an instant metamorphic phase. When \( l_2 \) overlaps the base link \( l_1 \), and two links are locked, the palm evolves into a four-bar phase and becomes another one-DOF phase in Fig. 2. This results in an innate metamorphic phase. While an instant metamorphic phase can always be achieved, the innate metamorphic phase needs to be considered in the mechanical design stage.

Thus, with the reconfigurable palm, the metamorphic hand indicates more dexterity, adaptability, and manipulability. In order to reveal the kinematic characteristics of the metamorphic hand, in this section, the geometry and kinematics of the metamorphic hand is investigated based on mechanism decomposition. From mechanism viewpoint, the metamorphic hand is a hybrid mechanism. Therefore, the whole hand can be decomposed and the kinematics of the metamorphic palm and fingers can be separately studied and then integrated leading to the investigation of the hand kinematics. Closed-form solutions are obtained leading to the workspace study of the metamorphic anthropomorphic hand.

3.1. Geometric constraint of the reconfigurable palm

Figure 4 gives the schematic diagram of the reconfigurable palm. This is a spherical five-bar linkage with the base link \( l_5 \) connected to the wrist. The right-hand-side

Fig. 4. Parameters of the reconfigurable palm.
of the base link connects the first input link $l_1$ at joint A and the left-hand-side connects the second input link $l_4$ at joint E. The five fingers are mounted at points $F_1$ to $F_5$, respectively. The angle between joint B and OF$_2$, OF$_3$, OF$_4$, and OF$_5$ are $\delta_2$, $\delta_3$, $\delta_4$, and $\delta_5$. For various configurations of the palm, points $F_1$, $F_2$, $F_3$, $F_4$, and $F_5$ form various pentagons as illustrated in Fig. 4.

In the spherical five-bar linkage, joints A and E are active joints and joints B, C, and D are passive joints. In order to derive the geometric constraints of this reconfigurable palm, coordinate frames are set up in Fig. 4 in such a way that, for all the local coordinate frames of links $l_1$ to $l_5$, they are all centered at point O with $z_i$-axis aligned with proximal joint of the link $l_i$, $y_i$-axis directed along $z_i \times z_{i+1}$, and $x_i$-axis determined by $y_i$ and $z_i$ with the right-hand rule. A global coordinate frame is set up at point O and has its $z$-axis aligned with joint E and its $y$-axis directed along $z_5 \times z_1$, coinciding with $y_5$ in Fig. 4. Based on this, given the values of angles $\theta_1$ and $\theta_5$, coordinates of points B, C, and D can be obtained in the global coordinate frame as

$$p_B = \begin{bmatrix} x_B \\ y_B \\ z_B \end{bmatrix} = \mathbf{R}(y, \alpha_5)\mathbf{R}(z_1, \theta_1)\mathbf{R}(y_1, \alpha_1)k = \begin{bmatrix} c\alpha_1 s\alpha_5 + s\alpha_1 c\alpha_5 c\theta_1 \\ s\alpha_1 s\theta_1 c\alpha_5 - c\alpha_1 c\alpha_5 c\theta_1 \end{bmatrix},$$

$$p_C = \begin{bmatrix} x_C \\ y_C \\ z_C \end{bmatrix} = \mathbf{R}(z_5, \theta_5)\mathbf{R}(y_4, \alpha_4)\mathbf{R}(z_4, \theta_4)\mathbf{R}(y_3, \alpha_3)k$$

$$= \begin{bmatrix} c\alpha_3 s\alpha_4 c\theta_5 - s\alpha_3 (s\theta_4 s_5 - c\alpha_4 c\theta_4 c\theta_5) \\ c\alpha_3 s\alpha_4 c\theta_5 + s\alpha_3 (s\theta_4 s_5 + c\alpha_4 c\theta_4 c\theta_5) \\ c\alpha_3 c\alpha_4 - s\alpha_3 s\alpha_4 c\theta_4 \end{bmatrix}$$

and

$$p_D = \begin{bmatrix} x_D \\ y_D \\ z_D \end{bmatrix} = \mathbf{R}(z_5, \theta_5)\mathbf{R}(y_4, \alpha_4)k = \begin{bmatrix} c\alpha_4 c\theta_5 \\ s\alpha_4 s\theta_5 \\ c\alpha_4 \end{bmatrix},$$

where $s$ and $c$ denote the sine and cosine functions and $k$ is a unit vector as $k = [0, 0, 1]^T$.

From Fig. 4, the geometric constraints of the spherical five-bar linkage yield,

$$p_C^T p_B = \cos \alpha_2,$$

$$p_C^T p_D = \cos \alpha_3,$$

$$p_C^T p_C = 1.$$
Combination of Eqs. (5) and (6) gives the coordinates of $x_C$ and $y_C$ in terms of $z_C$ as

$$x_C = U + Vz_C, \quad (8)$$

$$y_C = P + Qz_C, \quad (9)$$

where $U = (y_D c_\alpha_2 - y_B c_\alpha_3)/(x_B y_D - y_B x_D)$, $V = (y_B z_D - z_B y_D)/(x_B y_D - y_B x_D)$, $P = (x_B c_\alpha_3 - x_D c_\alpha_2)/(x_B y_D - y_B x_D)$, and $Q = (z_B x_D - x_B z_D)/(x_B y_D - y_B x_D)$. Substituting Eqs. (8) and (9) into Eq. (7) leads to a quadratic equation in terms of $z_C$ as

$$Az_C^2 + Bz_C + C = 0, \quad (10)$$

where $A = V^2 + Q^2 + 1$, $B = 2(UV + PQ)$, and $C = U^2 + P^2 - 1$.

Thus, the coordinate of $z_C$ can be obtained as

$$z_C = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}. \quad (11)$$

Equating Eq. (11) to the element $z_C$ in Eq. (3) the joint angle $\theta_4$ can be obtained as

$$\theta_4 = \arccos \left( \cot \alpha_5 \cot \alpha_4 - \frac{z_C}{s_\alpha_3 s_\alpha_4} \right). \quad (12)$$

Above two angles of $\theta_4$ result from the fact that the spherical triangle $\triangle BCD$ can be assembled with $C$ on either side of the diagonal $BD$. Where “+” corresponds to the case that $C$ locates above the diagonal and “−” corresponds to the case that $C$ locates below the diagonal.

Further, from Fig. 4, it is found that coordinate of point $C$ can also be expressed as

$$p_C = R(y, \alpha_5)R(z_1, \theta_1)R(y_1, \alpha_1)R(z_2, \theta_2)R(y_2, \alpha_2)k. \quad (13)$$

Substituting parameters into Eq. (13) yields

$$z_C = c_\alpha_2 (c_\alpha_1 c_\alpha_5 - s_\alpha_1 s_\alpha_5 c_\theta_1) - s_\alpha_2 ((s_\alpha_1 c_\alpha_5 + c_\alpha_1 s_\alpha_5 c_\theta_1) c_\theta_2 - s_\alpha_5 s_\theta_1 s_\theta_2). \quad (14)$$

Equating Eq. (14) to Eq. (11) and rearranging the equation lead to

$$E \cos \theta_2 + F \sin \theta - 2 = G, \quad (15)$$

where $E = s_\alpha_2 (s_\alpha_1 c_\alpha_5 + c_\alpha_1 s_\alpha_5 c_\theta_1)$, $F = -s_\alpha_2 s_\alpha_5 s_\theta_1$, and $G = c_\alpha_5 (c_\alpha_1 c_\alpha_5 - s_\alpha_1 s_\alpha_5 c_\theta_1) + (B \mp \sqrt{B^2 - 4AC})/2A$.

Solving Eq. (15) gives the joint angle $\theta_2$ is given as

$$\theta_2 = \arctan \frac{F}{E} \pm \arccos \left( \frac{G}{\sqrt{E^2 + F^2}} \right). \quad (16)$$
Two joint angles $\theta_2$ are obtained in spherical triangle $\triangle ABC$ with $\lambda = \arctan(E/F)$ locating the diagonal $AC$, and $\mu = \arctan(G/\sqrt{E^2 + F^2})$ denoting the angle above and below this diagonal.

Further, the joint angle $\theta_3$ can similarly be obtained by using the geometric constraints of link length. Joint velocities can be obtained by differentiating Eqs. (12) and (16). The above gives the motion characteristics of the articulated palm.

### 3.2. Transformation from palm to finger base

Fingers of the robotic hand are connected to the above reconfigurable palm through finger bases attached at points $F_1$, $F_2$, $F_3$, $F_4$, and $F_5$ as in Fig. 5. In order to relate palm motion to finger motion, local coordinate frames $F_i-x_{F_i}y_{F_i}z_{F_i}$ are set up at points $F_1$ to $F_5$ with $z_{F_1}$-axis directed along $OF_1$, $y_{F_2}$ directed along $z_{F_1} \times z_3$, and $y_{F_i}$ ($i = 3, 4, 5$) directed along $z_{F_i} \times z_5$. Local coordinate frames $M_i-x_{Mi}y_{Mi}z_{Mi}$ of the MCP joints of the fingers are set up with $x_{Mi}$-axis aligned with the $i$th MCP joint and $z_{Mi}$-axis directed along $F_iM_i$. The angle between $z_{F_i}$ and $z_{Mi}$ is $\gamma_i$ and the distance between $F_i$ and $M_i$ is $a_{i0}$ as in Fig. 5. It should be pointed out herein that $\gamma_1$ equals 0.

From the above analysis, the coordinate transformations from the finger base coordinate frames to the global coordinate frames can be obtained as

$$
R_{Fi} = \begin{cases} 
R(y, \alpha_5)R(z_1, \theta_1)R(y_1, \alpha_1)R(z_2, \theta_2)R(y_2, \delta_1) & \text{if } i = 1 \\
R(z, \theta_5)R(y_4, \alpha_4)R(z_4, \theta_4)R(y_3, \delta_2) & \text{if } i = 2 \\
R(z, \theta_5)R(y_4, \alpha_4 - \delta_3) & \text{if } i = 3, 4, 5
\end{cases}
$$

(17)

Thus, the homogeneous transformation matrix from the finger base coordinate frame to the global coordinate frame can be derived as

$$
D_{Fi} = \begin{bmatrix} R_{Fi} & R_{Fi}k' \\
0 & 1 \end{bmatrix} (i = 1, 2, \ldots, 5),
$$

(18)

Fig. 5. Parameters of finger base.
where \( k = [0, 0, R]^T \) and \( R_{F_i}k' \) gives the position vector of point \( F_i \) in the global coordinate frame. \( R \) is the radius of the sphere on which all the links move.

Then, the homogeneous transformation from coordinate frames of the MCP joints to the global coordinate frame can be given according to Fig. 5 as

\[
D_{Mi} = \begin{cases} 
D_{F_i} D_{FMi} D_{10} & (i = 1) \\
D_{F_i} D_{FMi} & (i = 2, 3, \ldots, 5) 
\end{cases}
\]  

(19)

where

\[
D_{FMi} = \begin{bmatrix} 
  c\gamma_i & 0 & -s\gamma_i & -a_{i0}s\gamma_i \\
  s\gamma_i & 0 & c\gamma_i & a_{i0}c\gamma_i \\
  0 & 0 & 0 & 1 
\end{bmatrix}
\]

denotes the transformation from coordinate frame \( M_i-x_iy_iz_i \) to coordinate frame \( F_i-x_{Fi}y_{Fi}z_{Fi} \). It should be noted that for the thumb finger base, it has \( \gamma_1 \) equals 0. Moreover, \( D_{10} \) presents the additional DOF for the thumb as

\[
D_{10} = \begin{bmatrix} 
  R(z_{F1}, \theta_{10}) & 0 \\
  0 & 0 & 0 & 1 
\end{bmatrix}
\]

with \( R(z_{F1}, \theta_{10}) \) denoting the rotation matrix about \( z_{F1} \) of \( \theta_{10} \).

### 3.3. The finger operation-planes and its relationship with the palm motion

In the metamorphic hand, the configuration of the palm changes orientation and position of robotic fingers attached to the palm. The change can now be represented by finger operation-planes. A finger operation-plane presents a two-dimensional (2D) workspace of the finger when the attached link is stationary. When the attached link moves, the finger operation-plane moves in 3D space. The trajectory of normal of the operation-plane presents movement of the plane.

As shown in Fig. 6, operation-planes corresponding to the five fingers are given as \( \sum_1 \) to \( \sum_5 \). These operation-planes are placed in such a way that the planes pass through the finger tips and are perpendicular to MCP joints of the finger bases. Let \( r_{10} \) to \( r_{50} \) denote the position vectors from origin \( O \) to original points of the finger bases \( M_1 \) to \( M_5 \), and let \( r_1 \) to \( r_5 \) denote the point vectors of the finger tips. Further, let normals of the finger operation-planes indicated as \( n_1 \) to \( n_5 \), the operation-planes can be given as

\[
\sum_i : (r_i - r_{10}) \cdot n_i = 0 \quad (i = 1, 2, \ldots, 5).
\]

(20)

From the above analysis, position vectors \( r_{10} \) can be obtained as

\[
\begin{bmatrix} 
  r_{10} \\
  1 
\end{bmatrix} = D_{Mi} \begin{bmatrix} 
  \theta \\
  1 
\end{bmatrix},
\]

(21)

where \( \theta = [0 \ 0 \ 0]^T \) is a null vector.
Further, normals of the operation-planes can be derived from Eq. (19) as

$$n_i = ED_M E^T i,$$

where unit vector $i = [1 \ 0 \ 0]^T$ directs along $x_1$-axis and $E^T$ is the transpose of $E$ with $E$ being an elementary matrix as

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

4. Workspace Analysis of the Metamorphic Anthropomorphic Hand

In the metamorphic hand, five fingers are mounted on three spherical links. Each finger consisting of three revolute joints operates on an operation-plane perpendicular to the corresponding finger base. Thus, the finger workspace in this metamorphic hand is affected by the palm motion. This leads to decomposing the workspace into a workspace of fingers and that of the hand. The former is planar workspace of three serial links connected by three revolute joints when the palm is fixed, which have been studied widely. The latter enlarges the former by sweeping the former to form an augmented workspace.
4.1. **Workspace of the reconfigurable palm**

The palm workspace can be presented by a pentagon determined by finger mounting points $F_1$, $F_2$, $F_3$, $F_4$, and $F_5$ in Fig. 4. As aforementioned, the palm workspace pentagon varies with the palm motion. From Eq. (18), coordinates of the finger mounting points can be obtained as

$$p_{Fi} = R_{Fi}k',$$

where $k' = [0 \ 0 \ R]^T$, without loss of generality, herein it is assumed that $R = 1$, and $i = 1, 2, \ldots, 5$. Using Gauss map, the palm workspace pentagon can be mapped onto a 4D space with input angle $\theta_5$ as an additional dimension along the $x$-axis in Fig. 7. This gives a helical sweep of the workspace pentagons, forming the workspace tube in the figure. Form the workspace helical tube, the variation of input angle $\theta_5$ at the palm gives the configuration variation of the palm and subsequently results in the change of finger orientation and hand pose.

4.2. **Augmented workspace of the metamorphic hand**

Let the kinematic of a single three-revolute-joint finger expressed by $D_i$.\textsuperscript{27,28} Integrate the above kinematic analysis of the reconfigurable palm and single robotic finger, the kinematics of the whole metamorphic hand can be readily obtained by combining the above derivations. The positions and orientations of the finger tips in the hand can be expressed by multiplying Eqs. (19) and $D_i$ as

$$F_i = D_{Mi}D_i \quad (i = 1, 2, \ldots, 5).$$

The velocities of the finger tips can then be obtained by differentiating Eq. (24).

Further, Eq. (24) implies that workspace of single robotic finger is augmented benefiting from the introduction of the reconfigurable palm indicated by coefficient matrix $D_{Mi}$ in Eq. (24). Take the index finger as an example and substitute structure parameters into Eq. (24) with two inputs of the articulated palm as $\theta_1 \in [0, 2\pi]$ and $\theta_5 \in [\pi/4, \pi]$, workspace of the index finger in the metamorphic

![Fig. 7. Palm workspace helical tube.](image-url)
Fig. 8. Workspace of index finger in metamorphic hand.

hand is obtained in Fig. 8. Figure 8 indicates that the workspace of the index finger is greatly enlarged. Similarly, one can find that with the motion of the reconfigurable palm, workspace of each individual finger in the hand is augmented leading to the augmentation of the workspace of the whole metamorphic hand. Thus, reconfigurable palm provides additional dexterity and gives extra control to this novel robotic hand. Continuously change the palm configuration with input variables $\theta_1$ and $\theta_5$, workspace of the metamorphic hand can be written as

$$W = \bigcup_{\theta_1} \bigcup_{\theta_5} \left( \sum_{i=1}^{5} F_i \right),$$

(25)

where $\sum_{i=1}^{5} F_i$ denotes the union of the workspace form by the five fingers. Equation (25) reveals that the reconfigurable palm enhances and enlarges the hand workspace that generates various hand configurations and poses.

5. Foldability and Index-Finger-Involved Grasping without Abducted Motion

Reconfigurability of the palm not only enlarges workspace of robotic hand, but also greatly increases the adaptability and grasps the ability of the metamorphic hand for producing various grasping poses. The metamorphic hand can be entirely folded as illustrated in Fig. 9, which gives the hand more flexibility and saves the space of storage. This fully foldability owns to the structure of the reconfigurable palm that the sum of link angles of $l_3$ and $l_4$, i.e., $\alpha_3 + \alpha_4 = 182^\circ$ approximately equals the sum of angles of links $l_1$, $l_2$, and $l_5$, i.e., $\alpha_1 + \alpha_2 + \alpha_5 = 178^\circ$. 
Further, as aforementioned, the index finger and the middle finger individually just have three DOFs rather than the human hand having four DOFs and they have no adducted and abducted motions. However, the metamorphic hand can execute the adduction prehensile actions such as griping a chalk (Fig. 10) benefited from the flexibility of the articulated palm changing the positions and orientations of the fingers. Figure 10 shows that two inputs of the palm lead to the change of
configurations of links $l_3$ and $l_4$ resulting in the position and orientation changes of the index finger and the middle finger, so that their radial sides move toward each other to implement the gripping of a chalk. However, to realize this prehension, path planning of the hand is requested in the future work. If the positions of contact of the object with the radial sides of the index finger and the middle finger are specified, the rotation angles of the input links can be obtained from the inverse geometric analysis of the metamorphic hand. From Eqs. (4) and (13), input angles $\theta_1$ and $\theta_5$ can be obtained as

$$\theta_1 = \arctan \left( \frac{M}{L} \right) \pm \arccos \left( \frac{N}{\sqrt{L^2 + M^2}} \right), \quad (26)$$

and

$$\theta_5 = \arccos \left( \frac{x_D}{\sin \theta_4} \right), \quad (27)$$

where $L = \sin \alpha_2 \sin \theta_2$, $M = \sin \alpha_2 \cos \theta_2 + \cos \alpha_2 \sin \theta_1$ and $N = y_C$. The values of $y_C$, $x_D$ and $\theta_2$ can be determined from the inverse kinematics of the index finder with respect to the palm.

6. Twisting Motion of the Metamorphic Anthropomorphic Hand

A further feature of the metamorphic hand is its twisting motion since it involves both lateral and radial movements of fingers being substantiated by the reconfigurable palm. Shown in Fig. 11, twist motion is demonstrated by rotating link $l_1$ by $\theta_1$ of 20° with respect to the base link $l_5$ and by rotating link $l_4$ by $\theta_5$ of 12° with respect to the base link. These inputs cause the thumb to move downward from the intersecting point resulting in clockwise rotation, and the index finger and the middle finger to move upward from the intersecting points in the tangential direction of the ball contributing to the clockwise rotation. All these lead to the twist motion of the ball. This can also be explained by characteristic matrix equation of the anthropomorphic hand as follows leading to the kinematic characteristics of the twist motion. Assumption are made that there is no relative motion at the contact points between the three fingertips (see Fig. 11) of the robotic hand and the ball and

![Fig. 11. Twist motion of metamorphic hand.](image-url)
that the contact points of the fingers are within the workspace of the metamorphic hand. Then, hypothetical joints, which assume that point contact is a spherical joint, are employed to express the freedom at the contact point of the object with respect to each fingertip.

Let screws $S_1$ to $S_5$ denote the joint axes A to E of the palm, $S_{i1}$ to $S_{i3}$ denote the joint axes of the $i$th finger and $S_{10}$ denotes axis of joint T (see Fig. 11). Angles $\theta_1$ to $\theta_5$ are associated with screws $S_1$ to $S_5$, angles $\theta_{i1}$ to $\theta_{i3}$ are associated with screws $S_{i1}$ to $S_{i3}$, and angle $\theta_{10}$ associated with screw $S_{10}$. Then based on Ref. 29, let twist $S$ represents the instantaneous motion of the ball. This twist can be expressed as a linear combination of the N joint twists and the three-point contact twist in any of the three involved fingers, i.e., the thumb, the index finger, and the middle finger.

In terms of the thumb, twist $S$ can be expressed as

$$S = \dot{\theta}_1 S_1 + \dot{\theta}_2 S_2 + \dot{\theta}_{10} S_{10} + \dot{\theta}_{i1} S_{i1} + \dot{\theta}_{i2} S_{i2} + \dot{\theta}_{i3} S_{i3} + (\dot{\theta}_{14} S_{14} + \dot{\theta}_{15} S_{15} + \dot{\theta}_{16} S_{16}).$$

(28)

where $S_{14}$ to $S_{16}$ are the screws that represent the hypothetical spherical joint at the contact point between the thumb and the ball and $\dot{\theta}_{14}$ to $\dot{\theta}_{16}$ are the joint rates of the hypothetical spherical joint as illustrated in Fig. 11.

Similarly, with respect to the index finger and the middle finger, twist $S$ can be given as

$$S = \dot{\theta}_5 S_5 + \dot{\theta}_4 S_4 + \dot{\theta}_{21} S_{21} + \dot{\theta}_{22} S_{22} + \dot{\theta}_{23} S_{23} + (\dot{\theta}_{24} S_{24} + \dot{\theta}_{25} S_{25} + \dot{\theta}_{26} S_{26}),$$

(29)

and

$$S = \dot{\theta}_3 S_3 + \dot{\theta}_{31} S_{31} + \dot{\theta}_{32} S_{32} + \dot{\theta}_{33} S_{33} + (\dot{\theta}_{34} S_{34} + \dot{\theta}_{35} S_{35} + \dot{\theta}_{36} S_{36}),$$

(30)

where $S_{34}$ to $S_{36}$ are motion screws representing the hypothetical spherical joint at the contact point between the index finger/middle finger and the ball and $\dot{\theta}_{34}$ to $\dot{\theta}_{36}$ are the joint rates of the hypothetical spherical joint.

Let screws $S'_{i1}$ to $S'_{i3}$ denote a three-system of screws that is reciprocal to the three-system of screws corresponding to the point contact $S_{i4}$ to $S_{i6}$ of the $i$th finger. Taking the reciprocal product of both sides of Eqs. (25) to (27) with each reciprocal screw $S'_{i1}$ to $S'_{i3}$ gives nine linear equations and these nine equations can be written in matrix form as

$$J^T_q \Delta S = J_\theta \dot{\theta},$$

(31)

where

$$J^T_q = \begin{bmatrix} S_{11}^T & S_{12}^T & \cdots & S_{i1}^T & S_{i2}^T & \cdots & S_{33}^T \end{bmatrix}, \quad J_\theta = \begin{bmatrix} J_\theta^1 \\ J_\theta^2 \\ J_\theta^3 \end{bmatrix}$$
with

$$J^i_\theta = \begin{bmatrix} S^r_1 T S_k & \cdots & S^r_1 T S_{33} \\ S^r_2 T S_k & \cdots & S^r_2 T S_{33} \\ S^r_3 T S_k & \cdots & S^r_3 T S_{33} \end{bmatrix}, \quad \Delta = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}$$

and $\dot{\theta} = [\dot{\theta}_1 \ \dot{\theta}_2 \ \ldots \ \dot{\theta}_{33}]^T$. Equation (28) is characteristic matrix equation of the metamorphic hand involving the thumb, the index finger, and the middle finger. This equation describes the instantaneous motion of the ball grasped by the three aforementioned fingers. The matrix $J^T_q$ is referred to as forward Jacobian matrix and $J_\theta$ as inverse Jacobian matrix. $J^T_q$ is a $9 \times 6$ matrix, $J_\theta$ is a $9 \times 15$ matrix, $I$ is a $3 \times 3$ identity matrix, $\dot{\theta}$ is a $15 \times 1$ vector, and $S$ is a $6 \times 1$ vector. Thus, from Eq. (28), given the joint rates of the palm and fingers, twist of the ball $S$ can be obtained, which characterizes the twist motion. However, it should be pointed out that Eq. (28) is an overdetermined linear system of equations where there are more equations than variables. The solution of this equation can only be found approximately and usually the residue is minimized in the least squares sense.

7. Prototype of the Metamorphic Hand and Its Initial Tests

Based on the kinematic analysis and computational simulation, structure design of the robotic hand is refined and then the manufacturing of an aluminum prototype of the metamorphic anthropomorphic hand is accomplished (Fig. 12). The robotic hand is subsequently assembled and tendon drive method is used for the actuation of the reconfigurable palm. In order to simplify the control of the hand, in this prototype underactuation strategy\textsuperscript{31–33} is employed and tendon-actuated method

![Fig. 12. Prototype and control system of the metamorphic hand.](image)
is also used for the five fingers and the finger extension is achieved by torsion springs. Carl Stahl® stainless steel cables of diameter 0.5 mm are used as tendons. Further, control system is established and the actuation of the whole hand system is implemented by eight Maxon® DC motors with encoders and gearboxes. For this current test rig, motors and control boards are installed in the control box instead of embedded in the forearm. NI® labview is chosen as user interface and the communication BUS for transferring the required setpoints between the PC and the control board has been chosen to maximize the robustness of the communication. Between the possible serial communication networks (RS-232, USB, and CAN) CAN has been chosen, as it is the most robust. The protocol for the communication is CANopen and communication speed is 1 Mbps.

Various postures have been translated into different setpoints and precaution steps have been taken to avoid collision among the links during the transitions between the setpoints. These steps include performing the motion in sequence of motions and defining a set of temporary ‘safe’ steps between the sets. Based on this, tests of the hand are carried out and Fig. 13 indicates prehension of the metamorphic hand to our daily life objects of different geometric features. These prehensile actions are completed by changing the configurations of the reconfigurable palm to change the positions and orientations of the fingers for various objects and environments. However, it should be pointed out that, although tendon-actuated method

![Operate pliers](image1)
![Operate scissors](image2)
![Hold a bowl](image3)
![Grasp a volleyball](image4)

![Operate a remote control](image5)
![Hold a mug](image6)
![Grasp a cylinder](image7)
![Hold a bottle](image8)

![Grip a card](image9)
![Turn a key](image10)
![Pinch a coin](image11)
![Use a comb](image12)

Fig. 13. Prehensile tests of the metamorphic anthropomorphic hand.
can reduce the size of the hand, friction between the tendons and guide tube greatly reduces the torques applied to the pulleys, so that in some configurations the palm cannot move smoothly and the hand could not have enough force to grasp heavy objects. Thus, static and dynamic analyses of this metamorphic hand need to be carefully studied in the future work.

8. Conclusion

This paper for the first time presented a novel metamorphic anthropomorphic hand with a reconfigurable palm. The introduction of the reconfigurable palm added dexterity and versatility to the anthropomorphic hand. With the structure design of the hand and mechanism decomposition, this paper developed the closed-form kinematic equations for modeling the metamorphic hand and investigated kinematics and workspace of the hand. Foldability of the hand was then discussed and twisting motion was explored based on the characteristic equation matrix. A prototype of the metamorphic anthropomorphic hand was produced accompany with a control system and grasping and manipulation tests of the hand were conducted to investigate dexterity, adaptability, and manipulability of the metamorphic anthropomorphic hand.

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References


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