Robotic Ball Catching
with an Eye-in-Hand Single-Camera System

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Abstract—In this paper, a unified control framework is pro-
posed to realize a robotic ball catching task with only a moving
single-camera (eye-in-hand) system able to catch flying, rolling
and bouncing balls in the same formalism. The thrown ball is
visually tracked through a circle detection algorithm. Once the
ball is recognized, the camera is forced to follow a baseline in the
space so as to acquire an initial data-set of visual measurements.
A first estimate of the catching point is initially provided through
a linear algorithm. Then, additional visual measurements are
acquired to constantly refine the current estimate by exploiting
a nonlinear optimization algorithm and a more accurate ballistic
model. A classic partitioned visual servoing approach is employed
to differently control the translational and rotational components
of the camera. Experimental results performed on an industrial
robotic system prove the effectiveness of the presented solution.
A motion-capture system is employed to validate the proposed
estimation process via ground truth.

Index Terms—Robotic Ball catching, Ball detection, Ball track-
ing tracking, Real-time trajectory estimation, Partitioned visual
servoing, Bouncing and rolling balls

I. INTRODUCTION

CATCHING a thrown ball is often a good exercise to check
human reflexes. The same task could be applied to an
advanced robotic platform to test new control methodologies.
Furthermore, such a challenging scenario covers several areas
like fast visual detection and tracking, motion estimation,
prediction and coordination, on-line trajectory planning, and
so on. Hence, the ball catching task might contribute in paving
the way towards the next generation of robots which should
be provided with the above mentioned skills.

The task of catching a thrown ball can be generally split
into three sub-problems: ball detection, trajectory estimation,
and robot motion control. Several works are present in the
literature covering each of the aforementioned sub-problems.
Usually, high-speed stereo vision system is required to solve
the first sub-problem. Through the approach proposed in this
paper, partially introduced in [1], [3], it is shown both from
a theoretical and practical point of view that the problem of
catching a thrown ball can be solved by using only one camera.
Moreover, in this way, the cost of the overall equipment can
be reduced. Even if recently many low cost stereo systems
are available off-the-shelf, the computational cost to elaborate
at high frame rate (e.g., about 140 Hz) is still demanding in
the ball catching task and this can be lowered by using a
monocular system, saving computational resources. Moreover,
with respect to previous works, the estimate of the trajectory is
improved in this paper considering also bouncing and rolling
balls within the same framework.

Each year an intensive progress in ball tracking and catching
tasks is achieved by the RoboCup competition; many table
tennis robots are challenging humans with great outcomes;
plenty of videos can be found on the net showing differ-
ent robotic ball catching techniques. Even mobile humanoid
robots, with a stereo-camera system, have been used in such
a task [4], [5]. Nevertheless, authors do believe that the first
case of a robotic ball catcher employing only a single moving
camera and coping with rolling, bouncing and flying balls in
the same framework, i.e., without changing either the estimator
or the control law, is presented in this paper. In more detail, a
standard industrial robot manipulator is equipped with a CCD
camera mounted directly on the manipulator end-effector (eye-in-hand
configuration). Differently from [1]–[3], the ball is
recognized through a circle detection algorithm based on the
method developed in [6]. One of the novelties of this paper
is the improvement of the approach proposed in [6] along
the lines of the work introduced in [7], so as to provide a sub-
pixel accuracy to have a measure of the ball centroid that
ameliorates the trajectory estimation process. Therefore, the
proposed estimator is composed of a continuous refinement
of the ball interception point through a nonlinear algorithm,
whose initial starting condition is provided by a fast linear
estimation process. The initial camera motion is thus com-
mmanded along a suitable baseline so as to collect a sufficient
initial number of visual data from different points of view and
provide such initial estimate. As another novelty, the proposed
ball trajectory estimator extends what already presented by
the authors [1], [3] since now the process is able to cope
with rolling and bouncing balls too. A statistical reliability
test of the measures has been also introduced to discard
possible outliers in the measurements set. Last, but not least,
the employed control law is more detailed than in [1], [3], as
well as the policy employed for the intersection point selection.
Related stability proofs are now provided. Experimental results
demonstrate the effectiveness of the presented solution. The
performance of the proposed trajectory estimator is shown
through a comparison with the ground truth provided by a motion-capture system.

The outline of the paper is as follows. The related work is presented in the next section. A general overview of the proposed algorithm is given in Section III. The algorithms for the ball detection and tracking are analysed in Section IV. The ball trajectory estimator and interception policies are described in Section V. Section VI is devoted to describe the replanning of the robot path. The partitioned visual servoing is revised in Section VII. The employed set-up is introduced in Section VIII and the experiments are critically discussed. Section IX gives concluding remarks.

II. RELATED WORK

A brief literature review is now presented. It is clear from the following that very few papers address the monocular case in ball detection and trajectory estimation operations. Moreover, in this paper, the camera is also moving during the tracking/estimation since it is mounted in an eye-in-hand configuration. Modifying and/or employing some methods already presented in the literature concerning ball detection, trajectory estimation and visual servoing, authors have achieved the goal of catch a flying/bouncing/rolling ball through a single moving camera system within the same framework.

A. Ball detection

Working in unstructured (or at least partially structured) environments requires recognizing some objects and key features in the scene [3]. Detecting a ball, or in general a thrown object, is thus crucial in robotic catching applications.

A first distinction can be made on the basis of the number of the employed cameras in the visual system. On the one hand, a stereo visual system benefits by using the triangulation method to reconstruct the 3D position of the ball in the scene [6], [9]–[11], but requires a more accurate calibration procedure and sophisticated elaboration hardware. On the other hand, a monocular visual system has an easier calibration procedure, but more effort has to be put in the 3D reconstruction of the scene [12]–[16]. Besides this, in a monocular visual system, a classification about the position of the camera with respect to the robot can be taken into account. An eye-to-hand configuration is considered in [13], while the camera is mounted in an eye-in-hand configuration in [1], [2].

Another distinction can be made about the employed techniques to detect the thrown object. By using a threshold method, the difference between the actual image and some reference images is employed in [17]. An equalized color-based clustering in the HSL color space is considered in [1], [2], while a circular gradient method to detect the ball in the images is utilized in [6].

Other objects rather than balls can be thrown and detected in the image. The work in [18] can be taken as a starting point to investigate further aspects and details.

B. Trajectory estimation

The problem about motion estimation has been addressed in several ways. This subsection takes into account the estimation performed by using a visual system in the particular context of the ball catching. Hence, in order to predict the correct catching point where the robot should intercept the thrown object, the motion trajectory of this last has to be estimated.

A 2D task is defined in [19] on the image plane of each available camera. The robotic arm may catch the ball if such 2D tasks are achieved simultaneously. The 3D position of the ball can be instead estimated by resorting to an extended Kalman filter (EKF) based on a Newtonian system which also considers the effect of the air drag [17]. Six nonlinear regression methods are compared in [20] to estimate translational and rotational velocities of free-flying objects.

By starting from a set of images taken from a monocular system, the estimate of the motion trajectory of a thrown object can be performed by using a least squares method [16] (without modeling the air resistance). A least squares solution is considered in [2], [13], [21], too.

Many estimators are based on the so-called Chapman’s strategy—the fielder should run at a proper speed to maintain a constant increasing rate of the tangent of the ball’s elevation angle [22]—for the ball catching [13], [14], [23].

When the ball hits the ground, or other objects, the estimate becomes more complicated due to the rebound effects. As an example, this condition always appears during table tennis games. In order to take into account not only the energy loss of the ball after the collision with the ground, but also the air resistance, the visual measurements errors and the friction between the ball and the ground, a first-order polynomial describing the bouncing model is employed in [24]. Another model relating flying and self-rotational velocities just before and after the rebound is considered in [25]. The lift and drag aerodynamic effects of the rebound are examined in [26], while the bouncing phenomenon between a ping-pong ball and both the rigid table and the racket rubber are exploited in [27], [28]. A 3D model for rigid body impact with tangential compliance, represented through springs, is presented in [29].

Finally, other estimators either make use of proper neural networks [30], or follow a programming-by-demonstration approach to find a feasible catching configuration in a probabilistic manner [31].

C. Robot motion control

On the basis of the configuration of the visual system, the robot controller has to take into account either the ball tracking, or the ball interception or both the tasks. The most common visual servoing approaches are the position-based and image-based methods [32]. A combination of these lasts is used in [12] to catch a ball moving on a table, and in [13] to solve the complete 3D task. A dynamic version of the aforementioned approaches is proposed in [33]. Decoupling translational and angular components in a visual servoing task is employed in partitioned visual approaches [34]–[36].

The importance in regulating the impedance of the hand, or the arm, during a ball catching action is highlighted in [37]: a human could miss the ball when the arm is both stiff beyond necessity and too compliant.

A non-prehensile way to manipulate the thrown ball is considered in [21]. After the dynamic catch, a balancing
controller is considered to keep the ball on a plate mounted on the robot end-effector.

III. ALGORITHM OVERVIEW

The overview of the proposed method for catching a thrown ball in the 3D space with a monocular visual system is shown in the block diagram of Fig. 1. Once the ball is detected by the visual system, the camera mounted in an eye-in-hand configuration is forced to follow a suitable baseline in the 3D space. A partitioned visual-servoing control is employed to both keep the ball in the field of view of the camera (through the camera orientation) and follow the planned path (camera position). The starting baseline is performed to ensure a well-conditioned estimation problem and collect/process visual information so as to get a first prediction of the ball trajectory with a rough linear estimate. This prediction is employed as a starting point for a more precise nonlinear refinement process of the trajectory, that also considers the case of rolling and bouncing balls. When a new estimate of the robot interception pose is available, the on-line motion planner smoothly switches its target to the new one, always keeping the ball in the camera field of view. Hence, the visual measurements are continuously acquired and processed by the nonlinear optimization algorithm. Finally, when the continuous refinement does no longer improve the prediction of the trajectory significantly, the final catching pose of the robot can be computed. In order to accommodate the ball into the robotic gripper, the robot kinematics is taken into account.

IV. VISUAL BALL DETECTION AND TRACKING

The presence of a ball in the camera’s field of view is evaluated with an algorithm derived from the circular-shape detection proposed in [6], [38]. The main advantages with respect to the classical Hough transform are the absence of decision thresholds or parameters calibration, and the high robustness with respect to changes in lighting conditions.

Let \( I(X_I, Y_I) \) denote the light intensity \((m \times n)\) matrix provided by the camera, where \( X_I \) and \( Y_I \) represent the pixel coordinates of the image sensor. The circular response function \( C_c(X_I, Y_I) \) considered in this paper represents the degree of affinity between the circular region with radius \( r \) of the image at the point \((X_I, Y_I)\) with respect to a radial intensity gradient (i.e. it represents the average fraction of the radial intensity gradient on the circle). In details, this affinity is established with respect to the gradient of the image intensity at the border of the considered circular region with the following expression

\[
C_c(X_I, Y_I) = \frac{1}{2\pi} \int_{0}^{2\pi} \left( \frac{c_\alpha}{s_\alpha} \right) C \left( \frac{X_I + rc_\alpha}{Y_I + rs_\alpha} \right)^2 d\alpha,
\]

where \( c_\alpha = \cos(\alpha) \) and \( s_\alpha = \sin(\alpha) \). The matrix \( C \) represents the contrast-normalized Sobel filter, which is defined as follows [6]

\[
C = \frac{\sqrt{2} \left[ \begin{array}{ccc} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{array} \right] * \left[ \begin{array}{ccc} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{array} \right] ^T}{\sqrt{16 \left[ \begin{array}{ccc} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{array} \right] * I^2 - \left[ \begin{array}{ccc} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{array} \right] ^2 + \epsilon^2}}.
\]

(2)

where \( \epsilon = 1 \) is the discretization unit of pixel intensity preventing singularities in constant image areas, and the operator \( * \) represents the linear convolution. Equation (1) is thus the integral of the squared scalar product of the contrast-normalized Sobel filter with the radial direction. For more details see [6], [38]. Moreover, notice how the matrix \( C \) has two different channels, namely \( C = \left[ \begin{array}{cc} C_X & C_Y \end{array} \right] \), with the same size of \( I \). Figure 2 shows the contrast-normalized Sobel-filter matrix of a ball in an image. Notice the radial distribution of \( C \) close to the border of the ball.

Let be \( A_r \), the affinity matrix for a given radius \( r \), which is defined as follows

\[
A_r = \begin{cases} C_c(X_I, Y_I) & (X_I, Y_I) \in \Xi_r \\ 0 & \text{otherwise} \end{cases},
\]

(3)

where \( \Xi_r = [r, \ldots, m-r-1] \times [n, \ldots, n-r-1] \). The pixels of the image that are possible candidates as the center of a circle are evaluated by applying a threshold \( \alpha_{\text{min}} \) to the set of affinity matrices in the range of radius of interest \( r^- \leq r \leq r^+ \), with \( 0 < r^- < r^+ < (\min(m,n) - 3)/2 \). In Fig. 3 the flowchart

1Differently from the classical Sobel operator, the vector length in the contrast-normalized Sobel filter indicates the gradient purity rather than the gradient intensity.
Fig. 4. Representation of the affinity matrix of an image corresponding to the maximum circular correspondence.

of the ball detection process is represented, while a graphical representation of the affinity matrix of an image corresponding to the maximum circular correspondence is shown in Fig. 4.

A. Affinity matrix evaluation

The evaluation of the affinity matrix \( A_r \) strongly affects the overall performance of the ball detection algorithm. Hence, more details on the adopted approach are provided in this subsection.

Notice that discretizing \( C_r(X_I, Y_I) \), with \( (X_I, Y_I) \in \Xi_r \), is equivalent to the arithmetic mean of the values of \( C \) in correspondence of the points of any circular region \( \Psi_r(X_I, Y_I) \) of radius \( r \) and centered in \( (X_I, Y_I) \). Let \( \tilde{C}_r(X_I, Y_I) \) denote the square region of \( C \) with \( 2r + 1 \) elements for each side and centered at \( (X_I, Y_I) \). Then, the contribution of the element at point \( P \in \Psi_r(X_I, Y_I) \) is \( (\tilde{C}_r(X_I, Y_I) r_P)^2 \), where \( r_P \) is the radial unit vector pointing from \( (X_I, Y_I) \) to \( P \). Notice that the number of unit vectors employed does not depend on the points where \( A_r \) is evaluated. By considering the square dual-channel matrix \( K \) of dimension \( 2r + 1 \) defined as follows

\[
K_r(X_I', Y_I') = \begin{cases} \frac{r(X_I', Y_I')}{r(X_I', Y_I')} & (X_I', Y_I') \in \Psi_r(X_I, Y_I), \\ 0 & \text{otherwise} \end{cases},
\]

the affinity matrix \( A_r \) can be evaluated with a linear convolution of the matrix \( C \) with \( K_r \). Figure 5 shows the evaluation of \( A_3 \) by using the kernel \( K_3 \). On the other hand, the evaluation of the points on a circumference of radius \( r \) is required to evaluate the kernel \( K_r \). To this purpose, the algorithm proposed in [39] is employed with the advantage of considering only integer values and reflection properties.

B. Parallelization

The number of operations required for the evaluation of the affinity matrices rapidly grows with the dimension of the image (e.g., the evaluation of the affinity matrix for an image of \((512 \times 368)\) pixels requires about \(10^9\) operations). In order to reduce the computational time, several arrangements should be introduced. The kernels required for the evaluation of the affinity matrices are pre-computed during the initialization phase of the algorithm. Moreover, the sparse nature of these matrices requires a specific sparse matrix representation.

The evaluation of \( C \) is performed by parallelizing the computation of its two channels. Moreover, each channel can be processed by parallelizing the horizontal and the vertical components of the gradient of \( I \) via the Sobel operator. Finally, also the denominator of (2) is evaluated in parallel with respect to the previous contributes, where the non-constant terms are evaluated by a linear convolution.

By employing specific SIMD (Single Instruction Multiple Data) operations, the evaluation of the affinity matrix is parallelized by rows. Thanks to the symmetry of the employed kernels it is possible to assume, without loss of generality, that the non-zero elements are \( 4p \), with \( p \) any integer number. Hence, for \( r = r^-, \ldots, r^+ \), and \( (X_I, Y_I) \in \Xi_r \), it is possible to obtain

\[
A_r(X_I, Y_I) = \frac{1}{4p} \sum_{k=0}^{4p-1} (u_{k_x} w_{k_x} + u_{k_y} w_{k_y})^2, \tag{5}
\]
where \( u_k = [u_kx, u_ky]^T \) is an element of \( C \), and \( w_k = [w_kx, w_ky]^T \) is an element of \( K_r \), respectively. By denoting with \( q_k = u_k^T w_k \), with \( k = 0, \ldots, 4p - 1 \), Equation (5) can be rewritten as follows

\[
A_r(X_1, Y_1) = \frac{1}{4p} \sum_{i=0}^{p-1} z_i, \quad (6)
\]

where \( z_i = q_{4i}^2 + q_{4i+1}^2 + q_{4i+2}^2 + q_{4i+3}^2 \). This latter term can be evaluated as a scalar product that can be computed by using an intrinsic function available on modern processors.

C. Pyramidal approach and windowing

The localization of a shape into an image with algorithms of template-matching requires a high number of calculations because, in general, it is necessary to explore the entire image and repeat the search several times for different size of the shape. Moreover, with the extension of the shape, the number of calculations rapidly increases. In order to reduce the processing time, a pyramidal approach can be employed. The technique consists in generating a copy in half resolution of the image by selecting the rows and columns of odd place. Iterating this process creates a number of images reproducing the same scene but with a lower level of detail. The loss of resolution can be recovered by using the results achieved on a low-resolution image to recognize the region of interest where perform the search within the original image.

Once that a thrown ball has been detected for the first time, a windowing technique is employed. In details, a suitable region of interest for the searching algorithm can be set up on the base of the position, radius, and velocity of the ball measured in the previous sequence of images. With this approach, a square searching region is positioned on the current image on the basis of the previous ball position and velocity, while its size depends on the previous radius and a suitable safety factor. Also the radius range of search is windowed starting from the previous estimate and ball direction.

D. Sub-pixel refinement

The results achieved with the previous ball detection algorithm can be improved thanks to the use of a post-elaboration process resulting in a sub-pixel accuracy. For each image, it is possible to observe that the intensity of the pixels, lying on an outgoing radial direction from the estimated ball center (see previous subsections), presents a distribution of values with an inflection point (Fig. 6). A linear searching algorithm can be employed to find the exact position \( p_0 = [x_0, y_0]^T \), \( i = 0, \ldots, n_d \), of the ball contour along a finite number \( n_d \) of radial directions spanning the whole circumference.

Once that a number of ball contour measurements are available, a new circle with center \((x_c, y_c)\) and radius \( r_c \), all with sub-pixel accuracies, is evaluated. In detail, the equation of a circle in a plane is represented by \( x^2 + y^2 + \xi_1(x) + \xi_2(y) + \xi_3(x, y, r) = 0 \). Stacking all the data, with \( i = 0, \ldots, n_d \), a least squares approach can be then employed to estimate \( \xi_1 \), \( \xi_2 \) and \( \xi_3 \) from which in turn the unknowns can be estimated as \( x_c = -0.5\xi_1 \), \( y_c = -0.5\xi_2 \), \( r_c = \sqrt{0.25 * (\xi_1^2 + \xi_2^2)} - \xi_3 \). Measurements that are statistically incoherent are discarded, and the estimation process is repeated until all measurements are reliable. In particular, the distance \( d_{\theta_i} \) from each point \( p_{\theta_i} \) and the above estimated circle with center \((x_c, y_c)\) and radius \( r_c \) is evaluated and saved in an array. The mean and standard deviation of such an array are then calculated. The points whose distance \( d_{\theta_i} \) outperforms the standard deviation by a certain factor are discarded and the above described least squares approach is performed again with the surviving points until no elements are rejected.

Several examples of the achieved results are shown in Fig. 7. Moreover, it has been verified during the experimental tests that the proposed algorithm reduces the effect of the blur around the contour of the ball, caused by the high-velocity motion of the ball in the image, and improves the accuracy of the estimation process (see Sections VII and VIII-B). Notice that other methods might be employed for sub-pixel accuracy for moving cameras [7].
V. BALL TRAJECTORY ESTIMATION

Let \( \Sigma_b, \Sigma_c \) and \( \Sigma_c = O_c-x_c,y_c,z_c \) be the robot base frame, fixed with respect to the ground, the end-effector frame and the camera frame, respectively (see Fig. 3). Since \( \Sigma_c \) and \( \Sigma_c \) are fixed with respect to each other, only \( \Sigma_c \) is considered into the remainder of the paper. The camera optical axis is then aligned with the approaching axis of \( \Sigma_c \).

Let \( s = [X \ Y \ T]^T \) be the normalized image coordinates vector of the centroid of the ball, i.e., the center of the circle detected in Section IV. By denoting with \( \tilde{s} = [s^T \ 1]^T \), the position of the ball center with respect to \( \Sigma_c \) is

\[
p_o = [x^e \ y^e \ z^e]^T = z^e [X \ Y \ 1]^T = z^e \tilde{s}.
\]

A. Initial baseline and linear initialization

A classic static triangulation method cannot be adopted in the proposed scenario because of the single employed camera. Hence, the proposed process estimates the 3D trajectory of the ball interpolating 2D visual measurements along the time.

As soon as the ball is detected for the first time, the camera is commanded to move along a straight line, i.e., the initial baseline, in the 3D Cartesian space with high velocity. To gain better results, such initial movement should be orthogonal to the direction of the throw. The orientation of the camera is controlled to keep the ball in its field of view (see Section VII). The visual data collection along this path leads to a well-conditioned problem. Once this first data-set has been acquired, it is possible to start the procedure for the linear initialization. This last is explained in [2], hence only a brief description is here addressed.

Let \( t_k \) be the \( k \)-th visual sample time, \( \tilde{s}_k \) the corresponding measured image feature vector, \( p = [x \ y \ z]^T \) the points belonging to the camera optical ray and passing through the current origin of the camera \( c_k = [c_{x,k} \ c_{y,k} \ c_{z,k}]^T \). The feature vector \( r_k = [r_{x,k} \ r_{y,k} \ r_{z,k}]^T = c_k + R_{c,k} \tilde{s} \) can be defined by the following equations representing a straight line in the 3D space (see Fig. 3)

\[
\begin{bmatrix}
(r_{y,k} - c_{y,k})x + (c_{x,k} - r_{x,k})y + r_{x,k}c_{y,k} - r_{y,k}c_{x,k} - c_{z,k} = 0 \\
(r_{x,k} - c_{x,k})x + (c_{z,k} - r_{x,k})z + r_{x,k}c_{z,k} - r_{z,k}c_{x,k} = 0
\end{bmatrix}
\]

where \( R_{c,k} \) is the rotation matrix of \( \Sigma_c \) with respect to \( \Sigma_b \) at time \( t_k \). Both \( c_k \) and \( R_{c,k} \) are provided by the robot direct kinematics.

A simplified model of the ball trajectory is considered during this initialization phase, i.e., the effect of the air drag to the motion of the ball is neglected:

\[
p_o(t) = p_0 + v_0 t + 0.5 g t^2,
\]

with \( p_o \) the \((3 \times 1)\) position vector of the ball with respect to \( \Sigma_b \), \( g \) the gravity acceleration, \( p_0 \) and \( v_0 \) the \((3 \times 1)\) vectors of the initial position and velocity of the ball \((t = 0)\), respectively, and corresponding to the time of the first ball detection. Notice that, without loss of generality, the gravity acceleration is aligned to the axis \( y \) of the chosen \( \Sigma_b \), i.e.,

\[
g = [0 \ g \ 0]^T \text{ with } g = 9.81 \text{ m/s}^2.
\]

At each \( t_k \), the optical ray intersects the ball trajectory. Folding (9) into (8) yields a system of 2 equations into 6 unknowns, \( p_0 \) and \( v_0 \), that fully describes the trajectory of the ball. Stacking the \( n_i \) measurements into rows yields a system of \( n_i \) equations into 6 unknowns that can be solved through a least squares solution. Additional considerations about this stage are given in [1], [2].

The first interception point candidate is then computed. This allows the robot to reach the predicted interception position, whose computational details are provided in Section VII at the estimated catching time. The robot path is replanned as described in Section VII. The rotational part of the camera, instead, is kept free to track the ball in order to acquire more visual measurements during the movement. The estimate provided by this linear algorithm is employed as a starting point for the next stage.

B. Nonlinear estimation and continuous refinement

Meanwhile the previous linear estimation process gives the result, new visual measurements are collected. Afterwards, both these new visual measurements and the old ones are employed in a nonlinear estimation process that starts initially from the result obtained by the linear method.

In details, let \( s_k \) be the centroid of the ball acquired at a time \( t_k \), the cost function to minimize is

\[
\min_{p_o, v_o} \sum_{k=1}^{n_i} \left\| \frac{1}{z_k} \left[ \tilde{x}_k^e \tilde{y}_k^e \tilde{z}_k^e \right] - s_k \right\|,
\]

with \( n_i \) the current number of available visual measurements at the current estimation time \( t_i \), and \( \tilde{p}_k \) the \((3 \times 1)\) estimated position vector of the ball with respect to \( \Sigma_c \), which is defined as follows

\[
\tilde{p}_k = [\tilde{x}_k^e \tilde{y}_k^e \tilde{z}_k^e]^T = R_{c,k} (\tilde{p}_k - c_k).
\]

The estimated position of the ball \( \tilde{p}_k(p_{0,i-1}, v_{0,i-1}, t_k) \) is numerically obtained by integrating the following ballistic model [17]

\[
\dot{p}_o(t) = g - \frac{c_w \pi d_b^2 \rho_o}{2 m_b} \| \dot{p}_o(t) \| \dot{p}_o(t),
\]

with \( c_w \) a coefficient depending on the thrown object, \( d_b \) the diameter of the ball, \( \rho_o \) the density of the air and \( m_b \) the mass.
of the ball. Hence, the model in (12) includes the air drag, and its numeric integration is sequentially performed in the time intervals \([t_{k-1}, t_k]\), where \(k = 1, \ldots, n_t, t_0 = 0\), with initial conditions \(p_{0,i-1}\) and \(v_{0,i-1}\).

By knowing the altitude of the floor with respect to \(\Sigma_b\), it is possible to detect whether the ball hits the ground during the above mentioned numerical integration. Without loss of generality, let \(p_f = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T\) be a point of the floor surface, which is horizontally placed with respect to \(\Sigma_b\), where \(\gamma\) represents the height of the ground. The adopted bouncing model is the following \([24]\)

\[
\mathbf{v}_{\text{out}} = F_{b} \mathbf{v}_{\text{in}} + \mathbf{b}_k,
\]

in which \(F_{b}\) is a \((3 \times 3)\) diagonal matrix of coefficients representing the loss of energy with respect to the \((3 \times 1)\) vector of the ball velocity \(\mathbf{v}_{\text{in}}\), before the rebound, \(\mathbf{v}_{\text{out}}\) is a \((3 \times 1)\) vector of the ball velocity after the rebound, and \(\mathbf{b}_k\) is a \((3 \times 1)\) vector of coefficients added to refine the model taking into account, for instance, the effects of the air resistance, vision measure error, and the spin caused by friction on the ground. Hence, by defining \(e_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T\), at each integration time the condition \(e_2^T p_k > e_2^T p_f\) is verified during the forward integration of the model \([12]\). When a rebound is detected, the current velocity is changed accordingly to (13).

The purpose of minimizing \((10)\) means that the initial conditions of the ballistic model are tuned to generate an estimated trajectory of the ball that minimizes the distance between the predicted projection of the ball onto the image plane and the corresponding measured observations of the ball along the time.

In practice, the minimization of the cost function \((10)\) is performed using the Levenberg-Marquardt algorithm. The result at the estimation time \(t_i\) is the updated values of \(p_{0,i}\) and \(\mathbf{v}_{0,i}\). As long as the current estimate of the catching time is sufficiently far from the actual time instant, if new measurements have been acquired then the estimation process restarts using the current estimation as starting solution, otherwise \(p_{0,i}\) and \(\mathbf{v}_{0,i}\) become the final estimated results \(p_i\) and \(\mathbf{v}_i\). Hence, such refinement process stops when the current estimate catching time is approaching with respect to the grasping time required by the available gripper. As soon as the new interception point is available, the robot path is replanned as described in Section \([7]\).

C. Statistical reliability test

Differently from \([11]\), a statistical procedure to deal with the presence of image noise is also proposed. During its motion, the ball can be subject to different illumination/shadow conditions that could generate different levels of noise in the measurements dataset. In details, once the minimization process ends at \(t_i\), the mean error, the standard deviation and the contribution of each visual measurement \(s_k\) to the error residual are evaluated. The visual measurements that contribute to the error residual outperforming the standard deviation by a certain factor are temporarily excluded for the next estimation process at time \(t_{i+1}\) (update outliers list in Fig.\([9]\)). In particular, new measurements should be available at the time in which such nonlinear estimation process computes the updated values of \(p_{0,i}\) and \(\mathbf{v}_{0,i}\), i.e. the interception time/point. This new data set (without the measurements which have been classified as outliers at the previous estimation time) is employed during the current nonlinear refinement. Again, once the minimization process ends, all the visual measurements (even the outliers) that contribute to the error residual in a way that is not statistically coherent are excluded for the next optimization. Notice that in this way it should be possible to recover also measures that have previously considered as outliers due to a rough initial estimation.

This arrangement is hence able to improve significantly the accuracy of the estimation process when noisy visual measurements are available without losing significant measurements. Some figures regarding such improvement are given in Section \([8]\).

D. Rolling balls

In parallel to the nonlinear estimation refinement process, a condition is continuously tested to detect whether the balling is rolling or not on the ground. A schematic representation is provided in Fig. \([9]\).

As previously introduced, for each acquired visual measurement \(s_k\) it is possible to associate the equations of the related optical ray \((8)\). By knowing the position and orientation of the floor with respect to \(\Sigma_b\), it is possible to compute the \((3 \times 1)\) intersection point \(p_{\text{int},k}\) between the straight line \((8)\) and the
ground. Afterwards, it is possible to fit the available points \( p_{\text{int}, k} \) with \( k = 1, \ldots, n_i \), through a polynomial curve on the ground plane. Notice that this operation is performed only if each point \( p_{\text{int}, k} \) belongs to the portion of the ground upon which the ball can feasibly roll. In fact, if the ball is flying, the observation angle of the optical ray will produce interception points with the floor that are not located in such portion of the ground.

Without loss of generality, in order to take into account both the ball spin and the effects of the friction on the floor, a parabolic path has been chosen as fitting curve. Hence, the three coefficients of the parabola that best fits all the ground points can be retrieved through a least squares solution. Once the parabola has been defined, the points \( p_{\text{cur}, k} \) closest to the curve at point \( p_{\text{int}, k} \) are computed. Notice that in case of perfect fitting \( p_{\text{cur}, k} = p_{\text{int}, k} \), each point \( p_{\text{cur}, k} \) is then back projected onto the image plane obtaining the feature vector \( \tilde{s}_{\text{cur}, k} \). The norm of the residual error between the visual measurements and these ones derived by the estimated model is employed as a quality measure. If the residual error achieved with this last method is less than the residual of (10), the ball is considered as rolling on the floor.

In case of rolling, the catching candidate point \( p_{\text{c}} \) is computed as the intersection between the ground parabolic path and a predetermined goal line in the robot workspace. The camera’s orientation is kept parallel to the ground, the gripper path and a predetermined goal line in the robot workspace. The computed as the intersection between the ground parabolic path has been chosen as fitting curve. Hence, the ground.

Afterwards, it is possible to fit the available points \( p_{\text{int}, k} \) with \( k = 1, \ldots, n_i \), through a polynomial curve on the ground plane. Notice that this operation is performed only if each point \( p_{\text{int}, k} \) belongs to the portion of the ground upon which the ball can feasibly roll. In fact, if the ball is flying, the observation angle of the optical ray will produce interception points with the floor that are not located in such portion of the ground.

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In case of rolling, the catching candidate point \( p_{\text{c}} \) is computed as the intersection between the ground parabolic path and a predetermined goal line in the robot workspace. The camera’s orientation is kept parallel to the ground, the gripper is positioned at a safety distance from the floor and its fingers camera’s orientation is kept parallel to the ground, the gripper path and a predetermined goal line in the robot workspace. The computed as the intersection between the ground parabolic path has been chosen as fitting curve. Hence, the

\[ \text{VI. ON-LINE PATH REPLANNING} \]

When a new \( h \)-th estimation is available at time \( t_h \), the current robot path has to be modified in a smooth way to reach the new estimated catching point. This means that such new path must guarantee the continuity with the current motion state (position, velocity and acceleration). A fifth-order polynomial has been employed in [1], [2]. In order to lower the polynomial order, reducing as much as possible the oscillations of the planned path, a classic cubic spline [32] employing only third-order polynomials is instead used in this paper.

Hence, the following definition can be given for the desired trajectory of the robot

\[ p_{c,d}(t) = \Pi_h(t) = \begin{cases} \Pi_{h_1}(t) & t_h \leq t \leq t_v1 \\ \Pi_{h_2}(t) & t_v1 \leq t \leq t_v2 \\ \Pi_{h_3}(t) & t_v2 \leq t \leq t_f \end{cases} \] (14)

where \( p_{c,d}(t) \) is a \( (3 \times 1) \) vector denoting the desired trajectory for \( \Sigma_c \) with respect to \( \Sigma_b \), \( t_f = t_x \) (or \( t_f = t_x \), in case of time scaling), \( t_v1 \) and \( t_v2 \) are two time instants referred to two generic virtual points in \( \Pi_h(t) \).

The following 12 equations in 12 unknowns—the 4 coefficients for each of the 3 cubic polynomials in (14)—can be
imposed
\[ \Pi h_1(t_h) = \Pi h_1(t_h), \quad \Pi h_1(t_h) = \Pi h_1(t_h), \quad (15a) \]
\[ \Pi h_1(t_f) = p_c, \quad \Pi h_3(t_f) = v_f, \quad (15c) \]
\[ \Pi h_1(t_f) = 0, \quad (15d) \]
\[ \Pi h_1(t_f) = \Pi h_2(t_v_1), \quad \Pi h_3(t_v_1) = \Pi h_2(t_v_1), \quad (15e) \]
\[ \Pi h_1(t_v_2) = \Pi h_3(t_v_2), \quad \Pi h_3(t_v_2) = \Pi h_3(t_v_2), \quad (15f) \]
where 0 is the (3 x 1) null vector, while \( v_f = \nu_x \) (or \( \nu_x \) if \( \nu_x > v_{max} \)). Notice that the effective location of the virtual points is irrelevant, since their position constraints are exploited for continuity only, and depends on the conditions of \( t_v_1 \) and \( t_v_2 \) (a possible solution is \( t_v_1 = (t_f - t_h)/3 \) and \( t_v_2 = 2(t_f - t_h)/3 \)). Finally, notice that (15a)–(15d) are the equations relative to the initial conditions, (15e)–(15g) are the equations relative to the final conditions, and (15c)–(15d) are the equations relative to the two virtual points. Moreover, notice that for \( h = 1 \) the continuity is intended to be with respect to the planned baseline (see Section V-A).

VII. PARTITIONED VISUAL SERVOING CONTROL LAW

The partitioned visual servoing control law described in this paper belongs to the category named Resolved-Velocity Image-Based Visual Servoing [32], for which it is assumed that the manipulator dynamics is taken into account directly by the low-level robot controller. Such a control law has been introduced in [33–35] and already employed in [1], [3]. Only the key features will be thus here reported: the reader may refer to the original papers for more details.

The (6 x 1) absolute velocity vector of the camera \( \nu_c^e = [p_c^T \omega_c^T] \), the (6 x 1) absolute velocity vector of the thrown ball \( \nu_b^e = [p_0^T \omega_0^T]^T \), both expressed with respect to \( \Sigma_c \), and the (2 x 1) velocity vector of the image feature \( \dot{s} \) in the image plane, are related by the following expression [32]

\[ \dot{s} = L_s \nu_c^e + L_s \Gamma(-\nu_0^e)\nu_0^e, \quad (16) \]

in which \( L_s = [L_{sp}(s, p_0^e) \quad L_{so}(s)] \) is the (2 x 6) interaction matrix of a point image feature defined as follows

\[ L_s = \begin{bmatrix} -\frac{1}{z^2} & 0 & X & Y & -1 & X^2 & Y \\ \frac{1}{z^2} & 0 & 1 & -X & Y & -X \end{bmatrix}, \quad (17) \]

where \( L_{sp} \) and \( L_{so} \) are the (2 x 3) sub-matrices corresponding to the first and last three columns of [17], respectively, and \( \Gamma(\cdot) \) is the (6 x 6) matrix

\[ \Gamma(\cdot) = \begin{bmatrix} -I_3 & S(\cdot) \\ 0 & -I_3 \end{bmatrix}, \quad (18) \]

where \( I_n \) denotes the (n x n) identity matrix and \( S(\cdot) \) the skew-symmetric matrix. Equation (16) can be rewritten as

\[ \dot{s} = L_{sp}(\dot{p}_c^e - \dot{p}_0^e + S(-\dot{p}_0^e)\omega_0^e) + L_{so}(\omega_c^e - \omega_0^e). \quad (19) \]

By exploiting the employed partitioned visual servoing controller, the translational components \( \dot{p}_c^e \) of the robot motion are devoted to move the robot to intercept the path of the ball with the gripper mounted on the robot end-effector [33].

Given the desired trajectory \( \dot{p}_{c,d} \), \( \dot{p}_{c,d} \) and \( \dot{p}_{c,d} \) for the camera frame, on the one hand, the translational components of the velocity input for the camera frame \( \Sigma_c \) can be generated as follows

\[ \dot{p}_c^e = R_c^T (\dot{p}_{c,d} + K_p e_p), \quad (20) \]

with \( R_c \) the rotational matrix of the camera frame \( \Sigma_c \) with respect to the base frame \( \Sigma_b \), \( K_p > 0 \) a diagonal constant (3 x 3) gain matrix, and \( e_p \) the (3 x 1) error vector between the desired trajectory \( \dot{p}_{c,d} \) and the one provided by the robot direct kinematics at time \( t \). On the other hand, the rotational components of the velocity input for the camera frame \( \Sigma_c \) can be generated in the image space as follows:

\[ \omega_c^e = L_{sp}^{-1} [K_{so,eq2}(e_s)\tau_{eq1}(e_s) - L_{sp}(\dot{p}_c^e - \dot{p}_0^e + S(-\dot{p}_0^e)\omega_0^e)] + \omega_0^e, \quad (21) \]

with \( \dot{\tau} \) denoting the pseudo-inverse of a matrix, \( \dot{p}_c^e \) the estimate of the unknown position of the ball in \( \Sigma_c \), and \( \dot{p}_c^e \) evaluated in (20). Notice that the matrix \( L_{sp}L_{sp}^T \) is never singular since its determinant is equal to \( 1/(X^2 + Y^2 + 1)^3 \). The terms \( \dot{p}_0^e \) and \( \omega_0^e \) are the estimates of the unknown absolute translational and angular velocities of the ball with respect to \( \Sigma_c \). Notice that \( L_{sp} \) in (21) is an estimated term since it depends on \( \dot{p}_c^e \). Moreover, the error term \( e_s = -s \) is the image error vector becoming null when the camera is pointed towards the centroid of the ball, while the term \( \tau_{eq1}(e_s) \) is a threshold function defined as [32]

\[ \tau_{eq1}(e_s) = \begin{cases} 0 & \text{if } \|e_s\| \leq e_{b1} \\ (1 - \frac{e_{b1}}{e_s}) e_s & \text{if } \|e_s\| > e_{b1}, \end{cases} \quad (22) \]

with \( K_{so,eq2}(e_s) \) a (2 x 2) gain matrix defined as

\[ K_{so,eq2}(e_s) = \begin{bmatrix} k_oI_2 & \beta_o \left( \frac{\|e_s\|}{e_{b2}} - 1 \right) \frac{I_2}{e_{b2}} \end{bmatrix} \quad \text{if } \|e_s\| > e_{b2}, \quad (23) \]

where \( k_o > 0 \) is a gain factor, \( e_{b2} > e_{b1} > 0 \) are proper thresholds, and \( \beta_o > 0 \) is a restraint factor tuning the increasing rate of \( K_{so} \). The details about how to estimate \( \dot{p}_0^e \) (and then also \( L_{sp} \), \( \dot{p}_0^e \) and \( \omega_0^e \) are given in the next subsection.

Finally, the input to the robot controller, i.e., the joint velocity vector, can be computed as [32]

\[ \dot{q} = J^T(q)T_c \nu_c^e + N_j K_c \dot{q}_r, \quad (24) \]

with \( q \) the vector of joint positions, \( J(q) \) the Jacobian matrix of the robot that is pseudo-inverted as denoted in [32], \( T_c \) the (6 x 6) matrix relating \( \nu_c^e \) to the velocity of the robot end-effector with respect to the base frame, \( N_j \) the projector matrix into the null space of the robot Jacobian, \( K_c \), a diagonal positive definite gain matrix, and \( \dot{q}_r \), a set of joint velocities employed in a possible redundancy management to optimize
some other sub-tasks [40], e.g. joint limits and singularities avoidance, increasing the manipulability.

A. Estimation of the ball-camera relative motion

The necessary quantities to be estimated in (21) are the linear position \( \hat{p}_c \), linear velocity \( \dot{\hat{p}}_c \), and angular velocity \( \omega_c \) of the center of the ball with respect to \( \Sigma_c \). Starting from the current estimate of the position \( p_0 \) and velocity \( v_0 \) of the ball, the ballistic model (12) is numerically integrated in the time interval \([0, t_f]\). In this way, \( \hat{p}_c \) can be obtained at a certain time \( t \). With the same numerical integration, it is also possible to obtain the estimate of the linear velocity \( \dot{\hat{p}}_c \). It is worth noting that the first estimate of \( p_0 \) and \( v_0 \) is obtained after \( n_I \) measurements (see Section VII-A). Before \( n_I \) measurements are collected, in order to compute the above mentioned quantities, an initial value of \( p_0 \) and \( v_0 \) should be anyhow provided. Hence, a statistical calibration has been preliminary realized to retrieve a rough initial estimation of \( p_0 \) and \( v_0 \).

Finally, the angular velocity can be retrieved as

\[
\omega_c = (\hat{p}_c \times \dot{\hat{p}}_c)\|\dot{\hat{p}}_c\|^{-2} = S(\dot{\hat{p}}_c)\|\dot{\hat{p}}_c\|^{-2}.
\]

(25)

B. Stability proofs

Before starting with the proofs, it is necessary to give some bounds about the quantities given in Section VII-A. In particular, taking into account what written in Section VII-A and (12), the following expressions hold

\[
\|\dot{\hat{p}}_c\| \leq B_1 < \infty,
\]

(26)

\[
\|\ddot{\hat{p}}_c\| \leq B_2 < \infty,
\]

(27)

where \( B_1 \) and \( B_2 \) are two positive limited bounds since \( \dot{p}_c \) and \( \ddot{p}_c \) are numerically integrated as described in Section VII-A and this means that they are a finite sum of finite elements. Moreover, they can also be saturated by programming code. For the angular part, taking into account (25) and the property that the Euclidean norm of a skew-symmetric matrix of a vector is equal to the norm of the vector itself, the following bound can be considered

\[
\|\omega_c\| \leq B_3 < \infty,
\]

(28)

with \( B_3 = B_1/B_2 \) a positive limited bound since \( B_1 \) and \( B_2 \) are positive and limited. Finally, the following physical bounds can be considered

\[
\|p_c\| \leq B_4 < \infty
\]

(29a)

\[
\|\dot{p}_c\| \leq B_5 < \infty
\]

(29b)

\[
\|\omega_c\| \leq B_6 < \infty
\]

(29c)

For what concerns the stability proofs of the system, the following theorems hold.

Theorem 1. Provided that \( K_p \) is a positive definite matrix, the control law (20) ensures an asymptotic convergence to zero of the position error \( e_p \).

Proof. The time derivative of the position error can be computed as

\[
\dot{e}_p = \frac{d}{dt}(p_{c,d} - p_c(t)) = \dot{p}_{c,d} - \dot{p}_c, \tag{30}
\]

where \( p_c(t) \) is the \((3 \times 1)\) vector denoting the current position of the camera with respect to \( \Sigma_b \) at time \( t \), computed by solving the direct kinematics of the robot, while \( \dot{p}_c \) is the related translational velocity of the camera with respect to \( \Sigma_b \). Pre-multiplying by \( R_c \), both sides of (20) and folding the result into the previous equation yields

\[
e_p + K_pe_p = 0. \tag{31}\]

Since \( K_p \) is a positive definite matrix, usually a diagonal matrix, the previous system is asymptotically stable and the error \( e_p \) tends to zero along the trajectory with a convergence rate depending on the eigenvalues of \( K_p \).

\[\square\]

Theorem 2. The system (19), equivalent to (10), is asymptotically stable under the control laws (20)–(21), in the presence of a perfect estimate of the unknown terms. Otherwise, only stability can be ensured, where the bounds can be determined by tuning the gain \( k_o \).

Proof. The following analysis is performed by using the direct Lyapunov theorem. Consider the following candidate Lyapunov function \( V(e_s) = e_s^T K_s e_s \), where \( K_s \) is a \((2 \times 2)\) positive definite diagonal matrix. By noticing that \( e_s = -\dot{\hat{\omega}}_o \), computing the time derivative of \( V(e_s) \) and taking into account (19), (20) and (21) yield

\[
\dot{V} = -\alpha_1 - \alpha_2 - \alpha_3, \tag{32}\]

with

\[
\alpha_1 = e_s^T K_s \left(L_{sp} - \dot{L}_{sp}\right) \dot{\hat{\omega}}_o
\]

(32a)

\[
\alpha_2 = e_s^T K_s \left(\sum_{i=1}^{n} e_i \right) \tau_{eb_i}^T e_s
\]

(32b)

\[
\alpha_3 = e_s^T K_s \left(L_i \Gamma(-p_c^o) - \dot{\hat{\omega}}_o^o - \dot{\hat{\omega}}_o^o\right).
\]

(32c)

If each term in (32) is strictly positive, then \( \dot{V} < 0 \). However, no term in (32) is a quadratic form, hence only qualitative considerations can be achieved.

If \( \dot{L}_{sp} = L_{sp} \), the term \( \alpha_1 \) in (32a) vanishes, but there is no guarantee that such condition can happen. Nevertheless, the \( \alpha_1 \) term is bounded since the condition for updating \( \dot{L}_{sp} \) through the estimate of \( \dot{\hat{p}}_c \) seems to be the optimal one during the experiments [41].

By considering (22)–(23), the term \( \alpha_2 \) in (32b) can be bounded as follows

\[
0 \leq \alpha_2 \leq e_s^T K_s \left(\frac{\beta_{o}}{e_{b2}} \left(\frac{\|e_s\|}{e_{b2}} - 1 \right) \right) I_2. \tag{33}\]

By choosing \( K_s = k_o e_s^T \left(\frac{\beta_{o}}{e_{b2}} \left(\frac{\|e_s\|}{e_{b2}} - 1 \right) \right) I_2 \), the last term in (33) is positive definite. Hence, \( \alpha_2 \) is always positive and limited.

The \( \alpha_3 \) term in (32c) vanishes in case of perfect estimate. Otherwise, nothing can be said about the sign of \( \alpha_3 \). Denoting with \( \lambda \) the maximum eigenvalue of \( K_s \) and with \( \sigma_M(\cdot) \) the maximum singular value of a matrix, recalling that \( L_s = [L_{sp}, L_{so}], L_o = [L_{sp}, L_{so}], v_o^o = [\dot{\hat{p}}_c, \dot{\hat{\omega}}_o], v_o^o = [\dot{\hat{\omega}}_o, \dot{\hat{\omega}}_o] \) and taking into account (29), the following
expression for $\alpha_3$ holds $\alpha_3 = \alpha_{3,1} + \ldots + \alpha_{3,6}$, where each term can be bounded as follows:

$$\alpha_{3,1} = e_s^T K_s L_{sp} \hat{p}_{o} \leq \lambda \sigma_M (L_{sp}) B_5 \| e_s \|$$

$$\alpha_{3,2} = e_s^T K_s L_{sp} S(\omega_0^e) \hat{p}_{o} \leq \lambda \sigma_M (L_{sp}) B_6 \| e_s \|$$

$$\alpha_{3,3} = -e_s^T K_s L_{sp} \hat{p}_{o} \leq \lambda \sigma_M (L_{sp}) B_1 \| e_s \|$$

$$\alpha_{3,4} = e_s^T K_s L_{sp} S(-\hat{p}_{o}) \omega_0^e \leq \lambda \sigma_M (L_{sp}) B_2 B_6 \| e_s \|$$

$$\alpha_{3,5} = -e_s^T K_s \sigma (L_{so}) \omega_0^e \leq \lambda \sigma_M (L_{so}) B_3 \| e_s \|$$

$$\alpha_{3,6} = e_s^T K_s \omega_0^e \leq \lambda \sigma_M (L_{so}) B_6 \| e_s \|. \quad (34f)$$

Hence, suppose $\alpha_1$ is positive definite, taking into account (33), the particular choice for $K_s$ and the expressions in (34), the following bounds for $\dot{V}$ hold

$$\dot{V} \leq -e_s^T K_s e_s + \lambda B \| e_s \| \leq -\lambda (\| e_s \| - B) \| e_s \|, \quad (35)$$

where $B = \sigma_M (L_{sp}) (B_5 + B_6 B_4) + \sigma_M (L_{so}) (B_3 + B_6) + \sigma_M (L_{sp}) (B_1 + B_2 B_6)$. Hence, from (35), it is possible to conclude that when $\| e_s \| > B/\lambda$ then $\dot{V} < 0$ and then the system is asymptotically stable. When $\| e_s \| \leq B/\lambda$ nothing can be said about the sign. Hence, the term $B/\lambda$ is the bound for the error $e_s$. To reduce the bound, $\lambda \to \infty$, the gain $k_o$ should be increased as much as possible with respect to the employed controller sample time.

In conclusion, in case of a perfect estimate, the terms $\alpha_1$ and $\alpha_3$ vanish while $\alpha_2$ is positive and limited. Then, the chosen control laws lead to an asymptotically stable system. In case of an imperfect compensation, instead, the error in the image plane is anyway bounded.

For a ball catching task, this stability condition can be considered sufficient, because the visual control goal is mainly in keeping the ball in the field of view of the camera.

VIII. EXPERIMENTS

The employed experimental set-up is depicted in Fig. 10. An USB iDS UEYE UI-1220SE-C camera is mounted in eye-in-hand configuration directly in the center of the base of the gripper. This last is made up of two brushed DC motors, with a metal gearbox and an integrated quadrature encoder. Through a rack-and-pinion mechanism the motion of these motors allows the closure of the gripper fingers. The gripper is in turn mounted on a COMAU Smart-Six robot manipulator standing on a sliding track. The COMAU C4G control unit is in charge of the compensation of the robot dynamics.

An external PC with UBUNTU OS and a patched RTAI real-time kernel generates the references for the robot each 2ms. A second PC with Windows OS is in charge of the visual elaboration process and communicates with the first one through an UDP protocol over a Gigabit dedicated LAN. A high-priority multi-thread programming has been employed to improve the stability of the elaboration time and synchronize the visual measurements with the robot motion.

In order to provide a ground truth for the proposed estimation algorithm, an OptiTrack motion-capture system composed of ten S250e cameras has been employed to track the ball during its motion.

Fig. 10. Architecture of the ball catching system.

A. Technical details

The portion of the camera image employed for the ball detection is limited to $(368 \times 512)$ pixels. On the basis of the available ball, the range of radius of interest for the first detection is $[10, 14]$ pixels, which corresponds to a distance of the ball from the camera in the range of $[5, 6]$ m. For other balls, these parameters have to be retuned. The RoI window is dynamically placed and sized with respect to the current radius of the detected ball. At the first detection, its dimension is set to $(38 \times 38)$ pixels. With this configuration parameter the acquisition frame rate is speeded up to 140 fps.

The available ball has a diameter of 8.5 cm and a weight of about 32 g. Six soft reflecting markers are attached to the ball in order to make the OptiTrack system able to observe it. These markers do not affect the visual detection of the ball in the scene since they do not alter the ball shape. The coefficients of the air drag factor have been tuned to $c_w = 0.45$ and $\rho_s = 1.293$ kg/m$^3$.

On the basis of the available robot, the gain matrix in (20) has been set by experimental tuning to $K_p = 500 I_3$, while the gains in (21) have been tuned to $k_o = 200$, $e_{b1} = 10$ and $e_{b2} = 100$.

By using the OptiTrack system, it has been possible to measure the rebound of the chosen ball on the ground. Hence, a calibration procedure has been adopted to tune the parameters $K_b$ and $b_b$ in (13). In details, the motion-capture system has provided the 3D position of the six markers at a frequency of 250 Hz. By knowing both the geometrical features of the ball and the position of these markers it has been possible to reconstruct the real ballistic trajectory of the ball. A set of 20 trajectories containing rebounded points have been acquired. The ball velocities before and after the rebound have been computed with a filtered derivative of such trajectories. Thereafter, the coefficients $K_b$ and $b_b$ have been retrieved through a least squares solution. Namely, the obtained values are $K_b = \text{diag} [0.2, -0.785, 0.55]$, while $b_b$ has been approximated to a null vector since its values are very small with respect to $K_b$ for the current set-up.

The intrinsic redundancy of the chosen robotic platform has been exploited in (24) to avoid joint limits, kinematic
singularities and to reduce the movements of the sliding track since its dynamics is considerably slower than those of the other joints.

On the basis of the available set-up, the initial baseline has a planned length of 50 cm that should be performed by the camera in 500 ms. The first estimate of the trajectory starts when about \( n_1 = 45 \) samples have been collected, i.e., after about 320 ms. Therefore, typically, the first catching trajectory starts before the end of the baseline path.

Latency periods and delays between the robot control PC, the C4G control unit and the visual elaboration PC have been estimated so as to synchronize at the best the direct kinematic measurements of the robot with the visual data.

**B. Ball detection and tracking results**

In order to evaluate the performance of the proposed sub-pixel refinement algorithm (see Section IV-D), a (fixed) ball with a diameter of 9 cm has been observed at 15 different distances from the camera, equally distributed by a step of 30 cm. Since the camera calibration parameters are known, from the observed radius of the ball in the image it is possible to estimate the distance of the ball from the camera. The error between the real distance and the estimated one, normalized with respect to the ball radius, is depicted in Fig. 11. The errors computed with the measurements taken through the sub-pixel refinement are represented in green, while the not refined measurements are represented in red. In all cases, the refined measure outperform significantly the integer estimations of the ball position and radius.

With respect to a generic throw, the elaboration time of the proposed detection and tracking process with respect to the distance of the ball from the camera (hence, read the picture from right to left) in green the whole elaboration time, in red the time required for the sub-pixel refinement process. The dashed line indicates the time limit corresponding to the camera rate of 140fps.

Fig. 12. Elaboration time of the ball detection and tracking process with respect to the ball distance from the camera (hence, read the picture from right to left). In green the whole elaboration time, in red the time required for the sub-pixel refinement process. The dashed line indicates the time limit corresponding to the camera rate of 140fps.

Fig. 11. Ball distance estimation error normalized to the radius in green (red) with (without) the sub-pixel refinement.

C. Ball Catching Results

Several experiments have been performed with a number of pitchers and varying light conditions. The percentage of caught ball evaluated over a set of 300 shots (100 shots for each of the three trials: rolling, bouncing, flying ball) is about 95%. Namely, the caught balls in the rolling case have been 98, as well as in the flying ball case, while 89 balls have been grabbed in the bouncing case. Hence, the main problems originate from the employed rebound model and the related parameters. Concerning the statistical reliability test, the mean percentage of discarded outliers in each throw is about 11.5%, while the accuracy of the final catching point, on the basis of the OptiTrack measurements, has been improved about 5%. This corresponds to an improvement of the interception point of about 2–3 cm in the available set-up.

An example of a possible ball trajectory for a given throw with the overlay of the motion of the robot is depicted in Fig. 13. Moreover, a multimedia attachment (available also online) shows the performance of the proposed algorithm.

The OptiTrack system has been employed to give a ground truth about the estimate of the ballistic trajectory. Some examples are shown in Fig. 14: in the case of a bouncing ball (Fig. 14(a)), the case of a normal throw in Fig. 14(b), while Fig. 14(c) presents the case of a rolling ball. In all these pictures, the green tube is the space occupied by the ball during its motion towards the robot as measured by the OptiTrack system. The blue line is the final estimated trajectory of the centroid of the ball. It is possible to observe that the blue line is always inside the green tube: this gives a geometric quality measure about the performance of the proposed estimation process. The red line shows the path followed by the camera/gripper mounted on the robot. It is then possible to recognize both the initial baseline and the catching path in which the gripper follows the predicted ball

http://www.youtube.com/watch?v=gT8Zn6L5PEk
trajectory during the catch. In Fig. 14(c) the blue circle denotes the space occupied by the opened gripper that is placed at a safety distance from the ground.

Further to the geometrical path, another quality index to measure the performance of the estimation process is a comparison along the time between the ground truth and the final predicted trajectory. With reference to the throw depicted in Fig. 14(a), the time histories of both the ground truth (i.e., the space traced by the ball) and the predicted trajectories are shown in Fig. 15. Again, the time histories of the predicted trajectory fit inside the ground truth.

The predicted interception points $p_x$ for the throw of Fig. 14(a) at each estimation time $t_i$, projected in both the $(x - y)$ and $(z - y)$ planes of $\Sigma_b$, are represented with a cross point in Fig. 16. The color bar identifies the ordered sequence of such predicted interception points with reference to the number of employed measures, while the biggest brown cross represents the final position $p_x$ in which the estimate has been considered as stable. The dashed lines represent the planned path for the hand (see Section VII), starting from the current motion state and leading to the current estimated interception position, while the continuous line is the real path followed by the gripper, which starts with the baseline (green piece of the path). It is worth noticing that the first estimated point, the green one, is given by the linear estimation process. The big orange circle is the full representation of the ball in the final position measured by the OptiTrack system and projected in the above mentioned planes of $\Sigma_b$. The big blue circle in the background is instead the space occupied by the gripper with all the fingers open.

**IX. Conclusion**

A new technique to catch a thrown ball with a robotic system endowed of only a single camera mounted in eye-in-hand configuration has been provided. Namely, the proposed novelties are: a sub-pixel refinement for the circle detector visual algorithm; a statistical reliability test to reduce image noise and possible outliers; deal with flying, bouncing and rolling balls in the same framework without changing neither the estimator nor the control law. The effectiveness of the proposed approach has been demonstrated in both theory and experimental results on a common industrial robotic set-up. An experimental comparison of the achieved results with the measurements given by an OptiTrack motion-capture system has been provided.
Fig. 15. Time histories of the ground truth (in green) and the final predicted trajectory (in blue) of the throw depicted in Figure 14(a).

Fig. 16. Sequence of the interception points (cross points) projected onto the \((x - y)\)-plane and \((z - y)\)-plane. The dashed lines represent the planned path, starting from the current hand position (circle points) and ending at the current estimated interception points. The continuous lines represent the real path followed by the gripper, starting with the initial baseline (green) and leading to the final catching point (biggest cross). The big orange circle represents the final position of the ball measured by the OptiTrack system. The big blue circle in the background is the estimated final position of the gripper catching base. The color bar on the right identifies the refinements of the interception points, while the related labels represent the number of visual measurements employed by the estimation process of each refinement.

REFERENCES


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