FREQUENCY OF SEEING FUNCTIONS FOR INTENSITY DISCRIMINATION AT VARIOUS LEVELS OF ADAPTING INTENSITY*

BY CONRAD G. MUELLER

(*From the Department of Psychology, Columbia University, New York*)

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In 1941 Hecht, Shlaer, and Pirenne (7) reported energy measurements of a visual stimulus sufficient to provide a specified threshold effect in the dark-adapted eye of a human observer. The threshold effect was defined in terms of the percentage of positive responses occurring in a situation where two responses were possible, one being indicative of "seeing," the other, of "not seeing." Energy measurements at the position of the cornea, coupled with corrections for energy loss between the cornea and the point of absorption of the energy, led the authors to the conclusion that the amount of absorbed energy necessary for detection by the human observer 60 per cent of the time was of the order of 8 to 14 quanta. A second part of the paper demonstrated that the relationship between percentage of positive responses and the logarithm of stimulus intensity could be represented by a cumulative Poisson curve whose slope had a value compatible with an interpretation that 6 to 10 quanta were required to provide a threshold effect. The two lines of evidence, direct and statistical, point to the importance of quantal considerations in a theoretical treatment of the absolute threshold.

Since the publication of the Hecht, Shlaer, and Pirenne paper, many frequency of seeing functions have been determined under a variety of experimental conditions, and many visual phenomena have been treated in quantum terms. For example, Bouman and van der Velden (3) present an account of visual acuity and intensity-time functions in terms of quantum concepts; Baumgardt (2) employs similar principles in his discussion of area-intensity and intensity-time relations; and de Vries (5), Rose (12), and Hendley (8) consider the differential threshold within a framework of quantum theory.

The literature on vision contains many data on the relation of the just detectable difference in intensity, ΔI, to the intensity level, I, but no studies show

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the relation of the slope of the psychophysical or "frequency of seeing" function to the intensity of the adapting stimulus over a large range of the latter. It is likely that data on frequency of seeing functions for the case of stimulus differences will be important in testing any extensions of quantum formulations to intensity discrimination, and the present experiment was designed to obtain these data. In an intensity discrimination situation in which the eye is adapted to one intensity and the threshold for an added intensity of brief duration is determined, five frequency of seeing curves were obtained for foveal observation at each of nine values of adapting intensity covering a range of 5.9 log units. Each frequency of seeing curve shows the percentage of times an increment in intensity was seen as a function of the size of the increment.

Apparatus and Procedure

A diagram of the apparatus used is shown in Fig. 1. The apparatus is similar, in its essential components, to the design used by Baker (1) in studying the course of light adaptation. Two 150 watt bulbs (A and A'), each adjacent to a metal stop containing a small opening, serve as the light sources for the adapting (I) and testing (ΔI) stimuli, respectively. The lights are operated at 115 volts D.C. Two acromatic lenses (C and C'), one in each beam, with focal point at the metal stop, give parallel light through the fixed filters at D and D'. Lenses at E and E' bring both beams to a focus at G where a half-silvered mirror transmits part of the adapting beam and reflects part of the testing beam. From G both beams follow the same path through lenses H and I which focus the light at the eye. A metal stop (J) limits the size of the field of the testing (ΔI) stimulus in such a way that it appears as a circular area in the center of the visual field and subtends a visual angle of 40 minutes. Another stop (K) limits the size of the adapting stimulus to provide a circular area, 12° in visual angle, concentric with the testing field. A stop at the eyepiece limits the size of the pupil to 2 mm. The eyepiece extends into the completely darkened cubicle which houses the subject.

The stimuli were viewed monocularly; H used the left eye; R, the right eye. A headrest was used to reduce gross movements of the eye with respect to the eyepiece. The subjects were instructed to fixate the center of the adapting stimulus field so that the image of the test stimulus fell in the fovea.

Two additional components in the path of the testing stimulus are (1) the variable filter (F) which allows for the continuous adjustment of this intensity and (2) the shutter (L) which consists of a disc mounted on the shaft of a constant speed motor, the disc containing an open sector which yields an exposure of 0.02 second. A mechanical stop, controlled by a relay, provides single, selected revolutions of the disc, and an associated cam controls the current flow through the motor so that the current is cut off when the disc is at rest.

Any one experimental session involved testing at two levels of adapting intensity. Before each experimental session the subject was first dark-adapted for 10 minutes, and then adapted for 5 minutes to the lower adapting intensity to be used in that session. Before the testing session a few preliminary observations established the appropriate range of test intensities; exact values were selected in steps of 0.1 log unit to cover the range from an intensity that was rarely seen to one that was almost always seen. This procedure usually involved four intensities and never less than three. The test intensities were then randomly presented at a rate of one presentation every 10 seconds until ten observations had been made at each intensity. After a rest period of 3 to 5 minutes in the dark, the subject was adapted for 5 minutes to the next adapting intensity, and a similar procedure was followed in securing ten observations at each ΔI value. The experimental session was completed by repeating this routine in the reverse order; *i.e.*, testing first at the higher and then at the lower adapting intensity level. Thus, on the average, each experimental session yielded 160 observations (a few more or less when three or five ΔI values were used), made up of eighty readings at each of two adapting intensities; the eighty readings in turn represented the sum of twenty readings at each of four ΔI values.

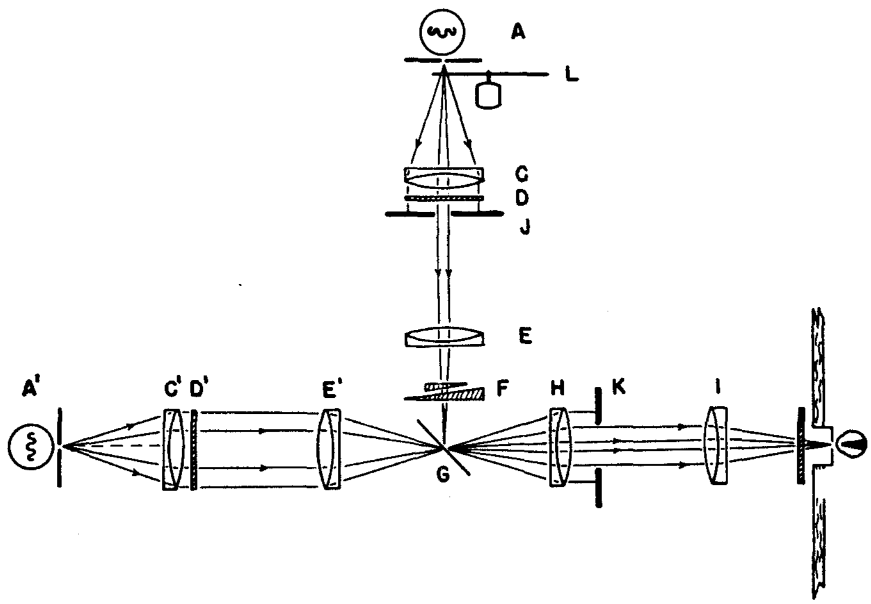

FIG. 1. Diagram of the apparatus as described in the text.

A total of twenty-three experimental sessions on two subjects provided the data to be reported. Nine levels of adapting intensity, ranging from -1.45 to 4.45 log photons, were used.[1] At each of these intensities five sets of data were obtained. A set of

[1] The intensities of the adapting and testing stimuli were calibrated by using a binocular matching technique. Each stimulus beam was stopped down to a diameter of 40 minutes of visual angle and was calibrated against a standard source of the same area and known brightness. Both the left and the right eye systems had exit pupils of equal size (2 mm.). The calibration yielded a measurement in millilamberts. This number was transformed into photons by the relation, photons $= 5D^2/2$ mL., where, in the present experiment, D is the diameter of the artificial pupil, 2 mm. A photon is a unit of retinal illumination and equals 1 candle per square meter passing through a pupil area of 1 square mm.

TABLE I

The percentage of positive responses in twenty stimulus presentations is shown for subects H and R for each of nine adapting intensities and for each magnitude of test stimulus, ΔI.

Log ΔI *photons*	Log-adapting intensity (photons)														
	−1.45					−0.45					0.55				
	Subject H			Subject R		Subject H			Subject R		Subject H			Subject R	
−0.48				5											
−0.38	10	15		25	20	15									
−0.28	50	45	5	45	40	35	20	5	20						
−0.18	75	65	50	80	65	80	35	35	45	20					
−0.08		80	80	90	90	100	80	70	70	40	10	10	20	15	15
0.02		95	85				90	95		60	50	45	15	35	20
0.12										85	90	70	35	55	70
0.22												95	80	85	95
0.32															

	1.55					1.95				
	Subject H			Subject R		Subject H			Subject R	
0.32				5						
0.42				30	20					
0.52	0	5	5	35	40					
0.62	20	55	40	70	95					
0.72	80	100	85	95					0	15
0.82	100					0	10	10	40	30
0.92						10	55	45	85	65
1.02						80	90	100	95	100
1.12						100				

	2.55				
	Subject H			Subject R	
1.22					
1.32	5			20	15
1.42	40	5		35	70
1.52	85	45	25	85	90
1.62		75	60		
1.72		100	95		

	3.25				
	Subject H			Subject R	
1.75	10	10			
1.85	35	15	0		
1.95	80	75	65		
2.05	100	95	95	15	
2.15				60	30
2.25				85	50
2.35				100	75
2.45					100

TABLE I—*Concluded*

Log ΔI	Log-adapting intensity (photons)														
	3.95					Subject H			Subject R		Subject H			Subject R	
	Subject H			Subject R											
photons															
2.55	10	0		5											
2.65	45	25	10	20											
2.75	85	75	60	45	15										
2.85		95	95	75	60										
2.95				95	85										
	4.45														
	Subject H			Subject R											
3.05															
3.15	15	10	10												
3.25	30	65	30	0	5										
3.35	80	95	70	15	30										
3.45	100		100	75	35										
3.55				85	95										

data consists of twenty observations at each of the three, four, or five values of the testing stimulus. Each of these sets of data yielded a psychophysical curve.

RESULTS

Table I summarizes the data. This table shows the percentage of times out of twenty presentations that each stimulus difference was detected. The percentage is given for each of two subjects and each of nine intensities. Three sets of data were obtained at each intensity for subject H; two sets of data at each intensity are available for subject R. Fig. 2 presents the data in the form of plots of the percentage of positive responses against the logarithm of ΔI. Each curve represents the data of one subject at one adapting intensity for one experimental session. There are forty-five such curves: twenty-seven for subject H, eighteen for subject R. Each point in each curve represents the percentage of positive responses in a series of twenty stimulus presentations. The smooth curves are empirical and are drawn to give an indication of slope magnitudes and the changes in slope that occur from session to session and from one adapting intensity to another. The exact form of curve to be used in describing the psychophysical functions is a theoretical matter and will be discussed later.

The data may be summarized in a number of ways. Fig. 3 shows the data in the form of a graph of log $\Delta I/I$ as a function of log I. The threshold ΔI values

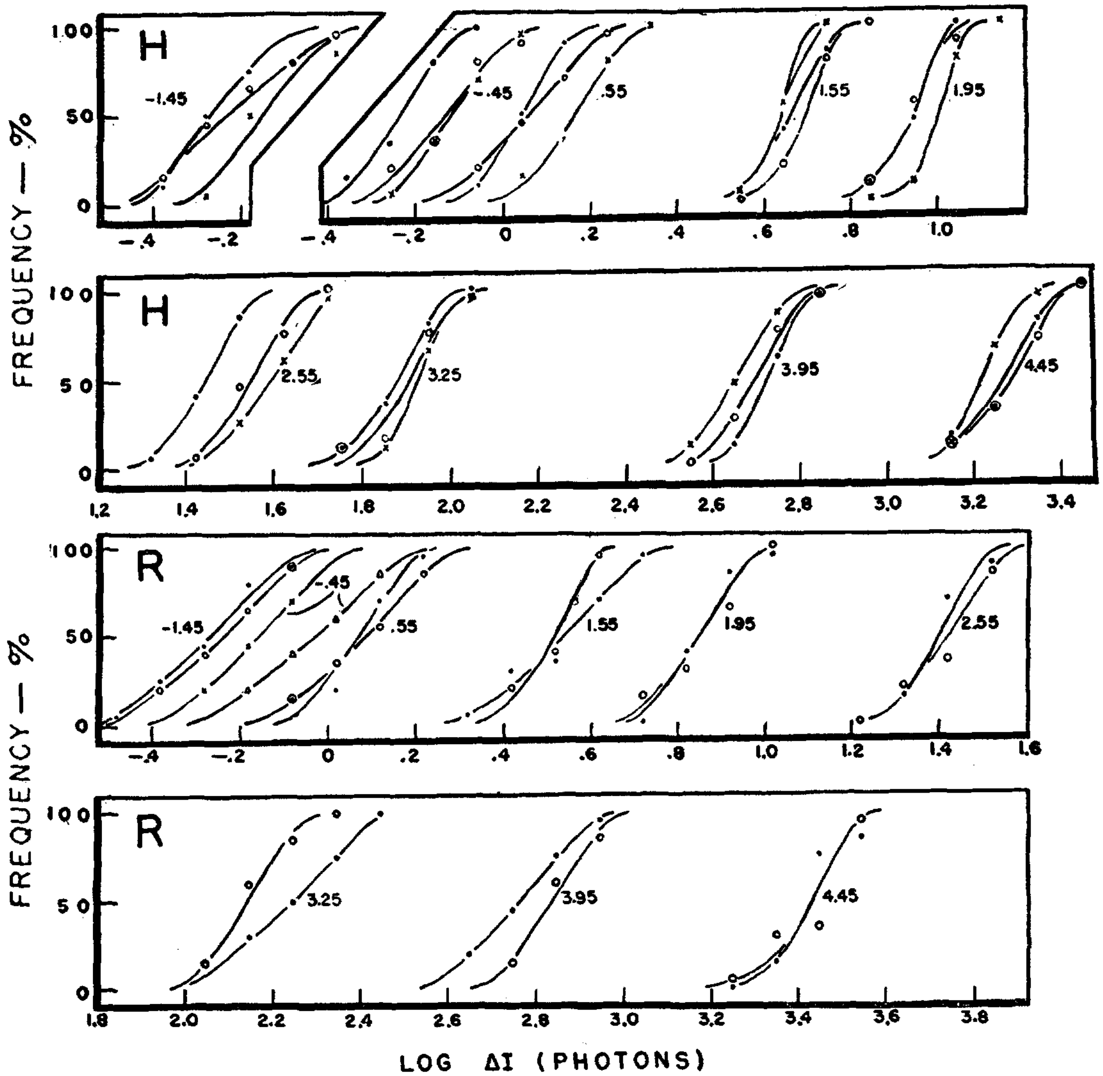

FIG. 2. Percentage of positive responses as a function of the magnitude of the increment in intensity for two subjects and nine adapting intensities. Plot of the data of Table I.

were taken as the values of ΔI at the 60 per cent point of each psychophysical curve. The average of five values from two subjects is shown. Approximately 400 observations enter into each point in Fig. 3, and the great number of observations undoubtedly enhances the regularity of the function. The solid

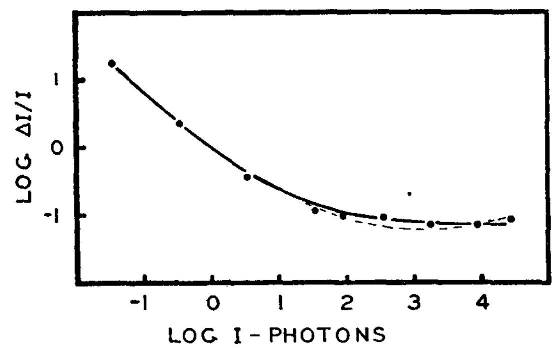

FIG. 3. Log $\Delta I/I$ as a function of log I where the threshold ΔI value for each frequency of seeing curve is taken as the 60 per cent point. Averaged data of two subjects.

line through the data is based on Hecht's hypothesis (6) as it applies to foveal stimulation and is a plot of the equation

$$\Delta I/I = c[1 + 1/(KI)^{\frac{1}{2}}]^2$$

where c and K are constants. The broken line represents Crozier's account (4, 10) and is based on the equation

$$\frac{1}{\Delta I} = \frac{1}{\Delta I_0} - \int_{-\infty}^{\log I} k e^{-[(\log I)^2/2\sigma^2]} \, d \log I$$

where k is a constant, σ^2 the variance of the assumed normal distribution, and ΔI_0, an estimated minimum ΔI. Both curves seen to describe the data in a satisfying manner.

In order to measure the changes in the slope of the psychophysical curves for various adapting intensities, the percentage values in Fig. 2 were plotted on a normal probability grid (log ΔI on the abscissa axis) and straight lines were fitted by eye.[2] A summary of the results is presented in Fig. 4. This

[2] Such a procedure involves the assumption that the data may be fitted by a cumulated normal curve. Some workers, *e.g.*, Hendley (8), Lamar, Hecht, Hendley, and Shlaer (9) have employed the cumulated Poisson curve to fit frequency of seeing functions in the case of intensity discrimination. Mueller (11) has questioned the applicability of the cumulated Poisson curve as a test of any explicit, extant formulation of discrimination using quantum concepts. The present analysis, however, does not depend on the solution to the problem of theoretical fits to psychophysical func-

figure shows the reciprocal of the median slope of the line relating percentage of positive responses and log ΔI as a function of the adapting intensity. The slope values were computed over the interval from 40 to 60 per cent on the probability plot. The data show a slight increase in the slope of the psychophysical curve with increasing intensity of the adapting stimulus. For both subjects there is no overlap in the slopes obtained at the two lowest intensities with the slopes obtained at any of the six highest intensities. When each point is compared with every other point each of the points at the two lowest adapting intensities turns out to be significantly different[3] from each of the points at the four highest values of adapting intensity ($p < .05$). There may be some tendency for the slope to decrease (and the reciprocal of slope to increase) at

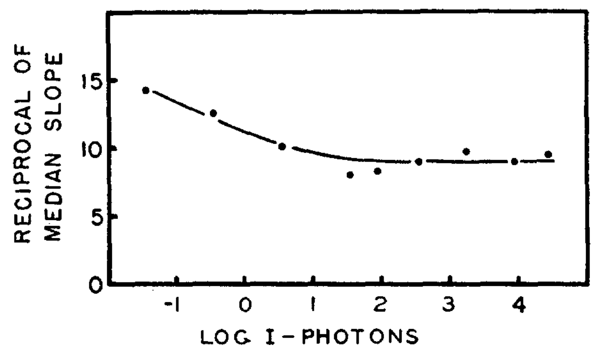

FIG. 4. Median reciprocal of the slope of the frequency of seeing curves as a function of the adapting intensity.

the high intensities, but this trend is not statistically significant in the present data. All the changes are small in magnitude.

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tions, but rests on the fact that there is no evidence in the present data that the plot on probability coordinates is not linear. Since the use of probability coordinates here is a descriptive device for determining the slope, the assumption involved is merely that the numerical value of the slope obtained would apply to either the cumulated Poisson or normal curve within the variability of the data. The answer to the question of the appropriateness of the Poisson and normal curves is not likely to come from curve-fitting procedures if the magnitudes of slopes shown in Fig. 2 are representative of the curves with which we are concerned. The asymmetry of cumulated Poisson functions having the slopes shown in Fig. 2 could hardly be demonstrated experimentally. If differences between Poisson and normal curves are to be tested, other consequences than the shape of the psychophysical function under some specified condition will have to be involved.

[3] The significance test used was one proposed by Festinger (*Psychometrika*, 1946, **11,** 97); it involves no assumptions about the distribution of numbers from which the samples are drawn.

DISCUSSION

The concept of quantum determinants of visual functions has become important in many theoretical considerations of vision. The work of Hecht, Shlaer, and Pirenne (7) emphasizes the usefulness of the concept for the case of absolute threshold. Attempts to extend the Hecht, Shlaer, and Pirrenne account to explain the effects of certain parameters on the absolute threshold have also met with some success (2, 3, 12). The extension of the concept to the case of intensity discrimination, however, probably requires a fuller discussion than it has received heretofore.

Quantum variability is present in every visual experiment. Since this is so we are provided with a "baseline" upon which any and all visual mechanisms must operate and we must be interested in the extent to which quantum variability expresses itself in visual functions. A first question that may be raised is the following: May the quantum concept alone be used to "predict" or "explain" the data of vision? If the answer is *no*, we may then ask whether quantum variability may be shown to be an important determinant of visual function when it acts in conjunction with some photochemical, neural, or other mechanism. Answers to questions of this sort must come from tests of the agreement between data and appropriate theories that involve quantum concepts.

The results of the present experiment allow tests of several quantum formulations of intensity discrimination. We may note at the outset that no existing theory based on quantum variability alone handles the data of Fig. 3 or the data of the many studies antedating the present one and providing similar empirical functions. Simple quantum formulations of the type presented by de Vries (5) and Rose (12) may be eliminated on the basis of Fig. 3 without reference to the frequency of seeing curves from which the $\Delta I/I$ function was obtained. The accounts of de Vries and Rose both require a linear relationship between $\log \Delta I/I$ and $\log I$, a relationship that is not observed over any large range of adapting intensities.

The treatment of intensity discrimination by Hendley (8) and Lamar, Hecht, Hendley, and Shlaer (9) suggests that the slope of a per cent seeing *vs.* $\log \Delta I$ plot is independent of the level of adaptation. Fig. 4 presents detailed data relating to this suggestion. Fig. 4 shows that the slope of the frequency of seeing curve is approximately constant for large values of adapting intensity but decreases at low intensities.

The full implications for quantum formulations of the approximate constancy of slope of frequency of seeing functions have not been fully discussed by Hendley or by Lamar, Hecht, Hendley, and Shlaer. The procedure of fitting a cumulated Poisson curve to frequency of seeing data for intensity discrimination and assigning significance to the parameter of the fitted curve assumes that the variable represented on the abscissa axis (ΔI for Hendley,

$\Delta I/I$ for Lamar, Hecht, Hendley, and Shlaer) is a random variable following the Poisson law. The assumption that $I_2 - I_1$ is such a variable is open to question.

Let us recall that the paradigm for most intensity discrimination experiments has three components. (1) The eye is first adapted to an intensity I_1. (2) During the test or ΔI interval the intensity is increased to I_2. (3) After the test interval the intensity resumes the value I_1. If we assume that light quanta are randomly distributed in time, it follows that the number of quanta in intervals of fixed length will vary according to the Poisson distribution. Therefore, the number of quanta, n_2, absorbed in the test interval varies according to the Poisson law with a mean and a variance proportional to I_2 and the duration of the interval. The number of quanta, n_1, absorbed from I_1 during any fixed period before or after the test interval is similarly distributed with a mean and a variance proportional to I_1 and the duration of the period. If the number of quanta from I_2 and I_1 are Poisson variables the question of whether $n_2 - n_1$ ($\propto \Delta I$) is a Poisson variable is immediately answered in the negative; the distribution of differences arising from two Poisson variables is not a Poisson distribution. We know that the variance of the distribution of differences between two uncorrelated variables equals the sum of the variances of the separate distributions. For the present case this means that, while the mean difference between n_2 and n_1 is the difference between the means, *i.e.* $\bar{n}_2 - \bar{n}_1$ [$\propto (I_2 - I_1)$], the variance of the differences is equal to the sum of the variances; *i.e.*, $\bar{n}_2 + \bar{n}_1$ [$\propto (I_2 + I_1)$]. Since the mean and the variance of any Poisson distribution are equal, $\bar{n}_2 - \bar{n}_1$ is a Poisson variable only if one of two assumptions is made. First, we may assume that I_1 is zero, in which case the mean and variance are equal. This, of course, is the case of the absolute threshold and is not an appropriate assumption for intensity discrimination. Secondly, we may assume that I_1 is not a variable but a constant and that the only variable is that part of I_2 that is not I_1; *i.e.*, ΔI. In this case the mean difference is $\bar{n}_2 - \bar{n}_1$ ($\propto \Delta I$), and since the variance of a variable plus a constant is the variance of the variable we have the variance also equal to $\bar{n}_2 - \bar{n}_1$. The second assumption says, in effect, that ΔI is quantized while I_1 is not. The implication of the preceding paragraph is that a demonstration that the slope of the frequency of seeing curve is approximately constant over a large range of values of adapting intensity does not mean that a constant difference in the number of quanta absorbed is required for a discrimination. On the contrary, the opposite conclusion seems warranted.[4]

[4] One might argue that, to the extent that we observe constancy of slope, we are dealing with some constant number of "events," *i.e.* that the increment in intensity initiates a certain number of events and that a critical number of such events are required for discrimination. Under these circumstances the Poisson curves are empirical and serve as a descriptive device in terms of which the maximum slope of the psy-

An explicit account of intensity discrimination presented by Mueller (11) employs the concept of two variable stimuli and the assumption that a critical difference in number of quanta absorbed from the two stimuli must exist for discrimination. One test of the theoretical account involves a plot of ΔI against the term $g(I_1 + I_2)^{\frac{1}{2}}$, where g is the normal variate equivalent of the percentage of times the difference in intensity is detected, and I_1 and I_2 are the two intensities. If we begin by assuming no adaptation mechanism, photochemical or otherwise, the prediction of equation (12) (in reference 11) is that plots of ΔI against $g(I_1 + I_2)^{\frac{1}{2}}$ yield straight lines whose slope and intercept constants are equal for all levels of adapting intensity. Analysis of the present data fails to confirm the prediction; both "constants" vary with adapting intensity.

If we assume that some adaptation mechanism operates in intensity discrimination, different predictions are made. For example, the action of a photochemical system of the type proposed by Hecht (6) would mean that, for different adapting intensities, differences would exist in a proportionality term, h, relating number of quanta absorbed and the stimulus intensity. We then expect the slope and intercept constants of the line relating ΔI and $g(I_1 + I_2)^{\frac{1}{2}}$ to vary with the level of adapting intensity. The present data confirm this expectation.

Changes in the proportionality between number of quanta absorbed and stimulus intensity imply changes in the absorption characteristics of the eye. The question arises whether the changes in the intercept and slope constants relating ΔI and $g(I_1 + I_2)^{\frac{1}{2}}$ are consistent with any rational expectation of changes in concentration of an absorbing medium with adapting intensity. To answer this question some specific assumption must be made as to the way in which concentration and intensity are related. As a first approximation in the present instance Hecht's formulation (6) was used and the solution for concentration as a function of intensity at the stationary state was used in computing the expected changes in the constant, h. When this solution is applied to the changes in the intercept constant the agreement between data and theory is good; for the case of the slope constant the agreement is less satisfactory. The adequacy of the theory using a combination of quantum variability and changes in the absorption constant and additional data relating to this combination theory will be treated in more detail in a subsequent paper.

In conclusion it is felt that the present experiment provides basic data on

chophysical function may be specified. If a formulation based on events is to prove fruitful as a theory it must permit a specification in terms independent of the observations that are to be explained. The problem remains of providing a rational basis for the expectation that a Poisson distribution will describe the frequency of seeing curves. Events may be distributed in an infinity of ways depending on the properties assigned to the events.

frequency of seeing in intensity discrimination. A review of existing theories shows that, to the present, no quantum formulation has given a completely satisfying account of intensity discrimination over large ranges of relevant variables. The changes in ΔI and in the slopes of the frequency of seeing curves with adapting intensity do not follow the predictions of a general form of the quantum argument (11) unless allowance is made for changes in the absorption constant with adapting intensity. The extent to which such an allowance will yield agreement between theory and data remains for future analysis.

SUMMARY

1. The percentage of times a human subject detects an increment (ΔI) in intensity was determined as a function of the magnitude of the increment and the magnitude of the stimulus (I) to which the increment is added.

2. Foveal stimulation was used, and five frequency of seeing curves were obtained at each of nine values of adapting intensity covering the range from -1.45 to 4.45 log photons. Each frequency of seeing curve shows the percentage of times an increment in intensity is detected as a function of the logarithm of the increment.

3. The slope of the frequency of seeing curve increases slightly with an increase in I and finally becomes independent of I at medium to high intensities.

4. The implications of the results for quantum theories of visual excitation are considered.

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