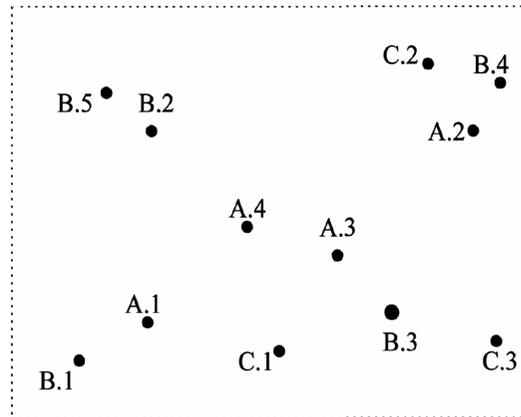


# A Joinless Approach for Mining Spatial Colocation Patterns

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Spring 2007

## Spatial Dataset



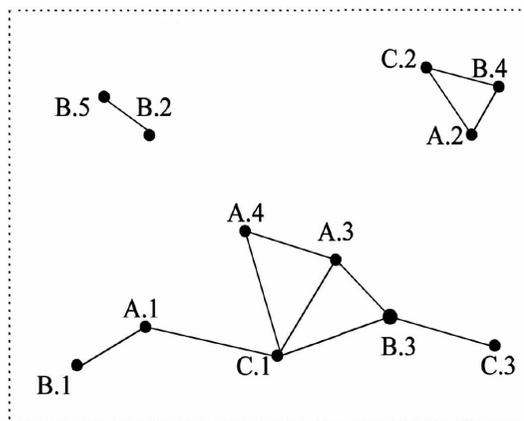
### Spatial Features:

- A
- B
- C

### Feature Instances:

- A.1, A.2, A.3, A.4,
- B.1, B.2, B.3, B.4, B.5,
- C.1, C.2, C.3

## Graph Representation



Given neighborhood distance  $d$ , draw an edge between two feature instances if their distance is  $\leq d$

### Neighbors:

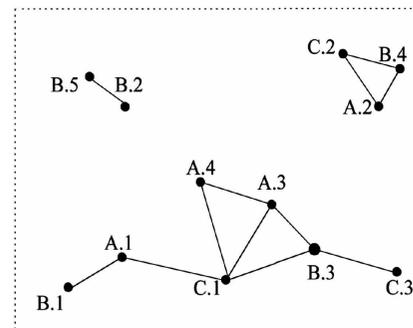
- B.2 - B.5
- B.1 - A.1
- A.1 - C.1
- C.1 - A.4
- ...

### Not Neighbors:

- A.1 - A.4
- A.4 - B.3
- ...

## Graph Representation

**Clique** in an undirected graph  $G$  is a set of vertices  $V$  such that for every two vertices in  $V$ , there exists an edge connecting the two.



### Cliques:

- A.2 - B.4 - C.2
- A.2 - B.4
- B.2 - B.5
- A.3 - A.4 - C.1
- ...

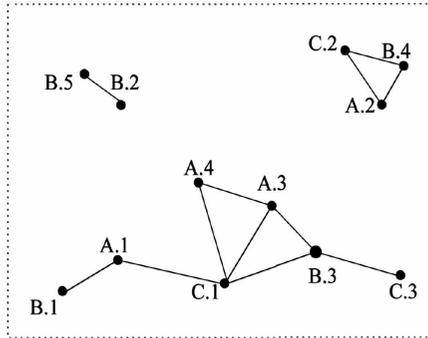
### Not Cliques:

- B.1 - C.1
- A.3 - A.4 - B.3 - C.1
- ...

**Note:** All spatial feature instances in a clique are neighbors ( $\leq d$  distance)

## Colocation Patterns

Colocation is a subset of spatial features, e.g. {A}, {B}, {C}, {A, B}, {A, C}, {B, C}, {A, B, C}



**Colocation {A, B} Instances / Cliques:**

A.1, B.1  
A.3, B.3  
A.2, B.4

**Colocation {B, C} Instances / Cliques:**

B.3, C.1  
B.3, C.3  
B.4, C.2

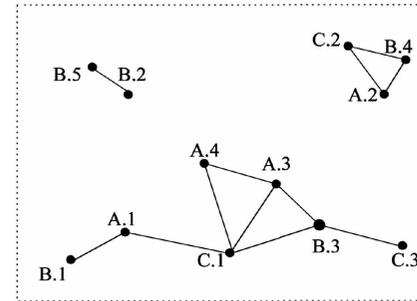
**Colocation {A, B, C} Instances / Cliques:**

A.3, B.3, C.1  
A.2, B.4, C.2

**Note:** Many colocation patterns are possible, we need a way to measure how interesting a colocation pattern is.

## Colocation Interestingness – Participation Ratio

$$\Pr(f_i, C) = \frac{\text{\# of distinct instances of feature } f_i \text{ in instances of colocation } C}{\text{\# of feature instances of } f_i}$$



**Example:**  $\Pr(B, \{B, C\})$

Colocation {B, C} Instances:

B.3, C.1  
B.3, C.3  
B.4, C.2

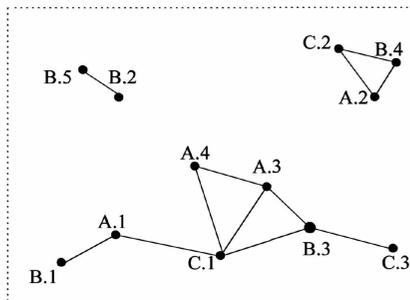
Feature B Instances:  
{B.1, B.2, B.3, B.4, B.5}

Hence,  $\Pr(B, \{B, C\}) = 2/5$

**Note:**  $\Pr(A, \{A\}) = \Pr(B, \{B\}) = \Pr(C, \{C\}) = 1$

## Colocation Interestingness – Participation Index

$$Pi(C) = \text{prevalence}(C) = \min_{f_i} \{\Pr(f_i, C)\}$$



**Example:**  $Pi(\{B, C\})$

$\Pr(B, \{B, C\}) = 2/5$

$\Pr(C, \{B, C\}) = 3/3 = 1$

Hence,  $Pi(\{B, C\}) = 2/5$

**Example:**  $Pi(\{A, B\})$

$= \min \{\Pr(A, \{A, B\}), \Pr(B, \{A, B\})\}$

$= \min \{3/4, 3/5\}$

$= 3/5$

## Colocation Mining Algorithm

**Input:** F = set of spatial features, e.g. {A, B, C}

FI = set of spatial feature instances with coordinates, e.g. {A1, A2, A3, A4, ...}

r = maximum neighbor distance

minPrev = minimum prevalent threshold

**Output:** PC = set of prevalent colocation patterns

**Mine** (F, FI, r, minPrev)

for k = 2 to |F|

$C_k$  = find all candidate colocation patterns of size k

for each candidate colocation pattern P in  $C_k$  ← these sets can be huge!

CI = find all colocation instances of P

prev(P) =  $\min \{\text{pr}(f_1, P), \text{pr}(f_2, P), \dots, \text{pr}(f_k, P)\}$ , where  $f_i$  = a feature type in P

if prev(P)  $\geq$  minPrev

PC = PC  $\cup$  {P}

## Colocation Mining Algorithm Optimization

**Problem 1:** Find all candidate colocation patterns of size  $k$

**Solution:** Use the anti-monotone property of the prevalence measure:

$$\text{prev}(C_k) \leq \text{prev}(C_{k-1}), \text{ w.r.t. subset operator}$$

In other words, if colocation  $\{A, B, C\}$  is prevalent, then colocation  $\{A, B\}$ ,  $\{A, C\}$  and  $\{B, C\}$  which are subsets of  $\{A, B, C\}$  are also prevalent.

Hence, we can use prevalent colocations of size  $k-1$  to construct **candidate colocations** of size  $k$ .

**Note:** We still have to check whether the candidates are really prevalent i.e. they meet the minimum prevalent threshold.

## Colocation Mining Algorithm Optimization

**Problem 2:** Find all colocation instances (cliques) of candidate colocation  $P$

More precisely, how to find cliques efficiently from the spatial data?

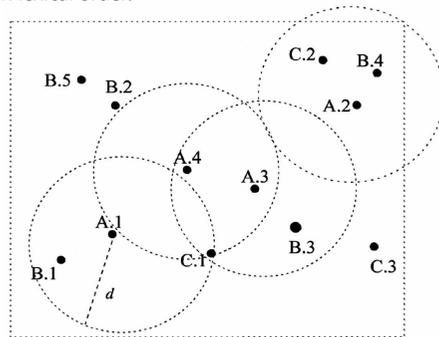
**Solution:** Use some kind of model representation to capture the neighbor relationship of the spatial data.

One possible choice is the **star neighborhood partition model**.

## Star Neighborhood Partitioning

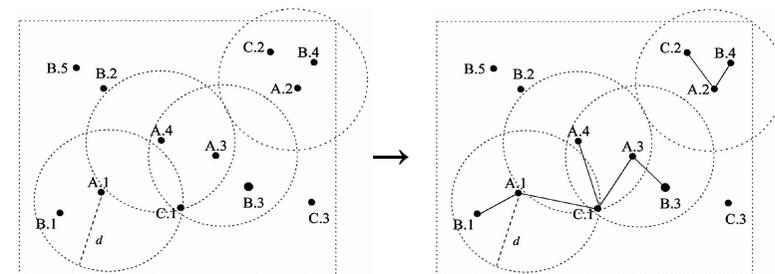
- **Star neighborhood** of a feature instance is:
  - a set consisting of the instance itself plus any other feature instances within the predefined neighbor distance.
  - the feature type of the neighbor instances must be greater than the feature type of the center instance in lexical order.

Star neighborhood area of A.1, A.2, A.3 and A.4 (dashed circles) ▶



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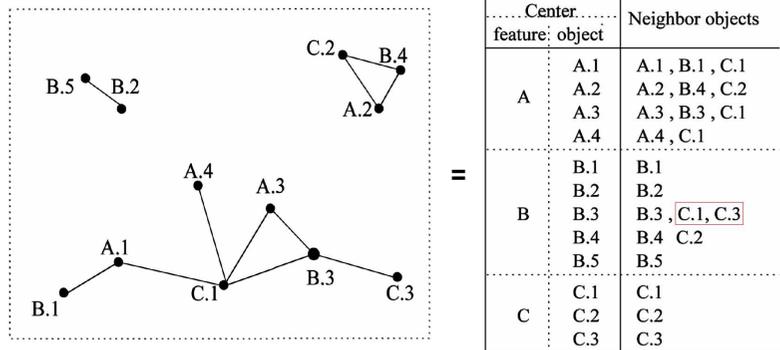


Star neighborhood area of A.1, A.2, A.3 and A.4 (dashed circles)

Star neighbors of A.1, A.2, A.3 and A.4 (edges)

## Star Neighborhood Partitioning

Applying star neighborhood partitioning to our example...



## Star Neighborhood – Advantage #1

- Candidate colocation instances (a.k.a. **star instances**) can be produced quickly.

Center feature	Neighbor objects			
A	A.1, B.1, C.1 A.2, B.4, C.2 A.3, B.3, C.1 A.4, C.1	} Candidate instances for {A}, {A, B}, {A, C}, {A, B, C}		
B	B.1 B.2 B.3, C.1, C.3 B.4, C.2 B.5		} Candidate instances for {B}, {B, C}	
C	C.1 C.2 C.3			} Candidate instances for {C}

## Star Neighborhood – Advantage #2

- Candidate colocation can be coarsely filtered from the participation index of the star instances.

### Example

Given candidate colocation {A, B, C}

A	B	C	
A.1, B.1, C.1	} star instances		
A.2, B.4, C.2			
A.3, B.3, C.1			
3/4	3/5	2/3	◀ Participation Ratio Estimates

If the estimated p.i. is less than minimum prevalent threshold, then discard the candidate.

- In general,  $Pi(\text{star instances of C}) \geq Pi(C)$

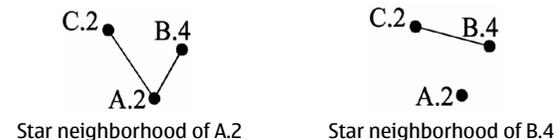
## Star Neighborhood – Advantage #3

- Cliqueness of a star instance can be checked from the star neighborhoods.

### Example

Given star instance {A.2, B.4, C.2}

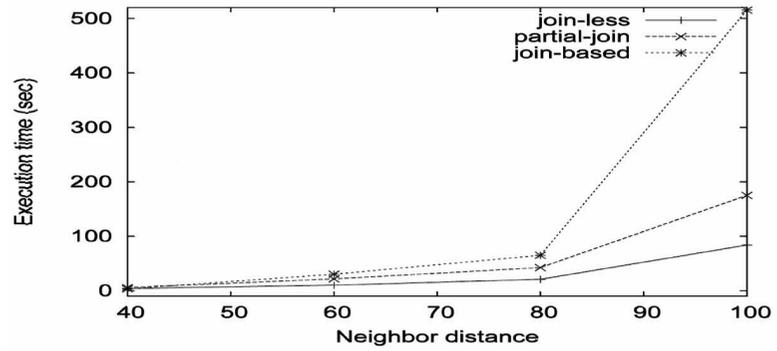
We know A.2's neighbors are B.4 and C.2. If {B.4, C.2} is also a star instance, then {A.2, B.4, C.2} is a clique.



- In general, star instance  $\{o_1, o_2, \dots, o_k\}$  is a clique if subinstance  $\{o_2, \dots, o_k\}$  is a clique.

## Performance Comparison

- The joinless approach which utilizes star neighborhood partitioning is more scalable.



~ The End ~