Accelerated Iterative Transmission CT Reconstruction Using an Ordered Subsets Convex Algorithm

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Abstract—Iterative maximum likelihood (ML) transmission computed tomography algorithms have distinct advantages over Fourier-based reconstruction, but unfortunately require increased computation time. The convex algorithm [1] is a relatively fast iterative ML algorithm but it is nevertheless too slow for many applications. Therefore, an acceleration of this algorithm by using ordered subsets of projections is proposed [ordered subsets convex algorithm (OSC)]. OSC applies the convex algorithm sequentially to subsets of projections. OSC was compared with the convex algorithm using simulated and physical thorax phantom data. Reconstructions were performed for OSC using eight and 16 subsets (eight and four projections/subset, respectively). Global errors, image noise, contrast recovery, and likelihood increase were calculated. Results show that OSC is faster than the convex algorithm, the amount of acceleration being approximately proportional to the number of subsets in OSC, and it causes only a slight increase of noise and global errors in the reconstructions. Images and image profiles of the reconstructions were in good agreement. In conclusion, OSC and the convex algorithm result in similar image quality but OSC is more than an order of magnitude faster.

Index Terms—Maximum likelihood (ML) reconstruction, ordered subsets, transmission computed tomography (TCT).

I. INTRODUCTION

In spite of their computational expense, statistical reconstruction methods are atractive for transmission transmission computed tomography (TCT) for several reasons, for example because they model the Poisson noise in the transmission data, allow incorporation of a priori knowledge of the object being imaged and handle truncated projection data better than Fourier-based reconstruction methods. Interest in statistical iterative reconstruction methods has recently increased markedly because of the need in nuclear medicine (transmission tomography in nuclear medicine [5]). Although much faster than the ML-EM algorithm in [2], the convex algorithm is still too slow for a part of the applications required in clinical routine. Ordered subsets of projection data [6] have been used with great success to accelerate reconstruction in emission tomography (see [7] for an overview), and have also been used previously to accelerate the ML-EM algorithm for TCT [8]. In this paper, ordered subsets of projections are applied to the convex algorithm in order to further accelerate TCT reconstruction. The resulting algorithm, the ordered subsets convex algorithm (OSC), is compared with the convex algorithm using digital and physical thorax phantom data.

II. METHODS

A. OSC Algorithm

The convex algorithm can easily be modified so that it can use ordered subsets of projection data [6]. In OSC, the convex algorithm [1], [3] is simply applied to each subset of projections and the update of the iterant is then used as the starting image for the processing of the next subset. By definition, an iteration is completed when all subsets have been processed once.

The OSC algorithm updates all attenuation coefficients $\mu_{n+1}^k(k)$ belonging to voxel $k$, iteration $n$, and subset $s + 1$ according to

$$
\mu_{n+1}^k(k) = \mu_n^k(k) + \omega^n \frac{\sum_{i \in S(n)} I_{ik}(\mu_n^k) - Y_i}{\sum_{i \in S(n)} I_{ik}(\mu_n^k)}
$$

(1)

where the expected number of counts $\bar{y}_i(\mu_n^k)$ in detector bin $i$ is given by

$$
\bar{y}_i(\mu_n^k) = d_i e^{-\nu(\mu_n^k)}
$$

and $(l_i, \mu_n^k) = \sum_{j} I_{ij}(\mu_n^k(j))$.

In (1), $Y_i$ represents the measured transmission counts in bin $i$, $d_i$ represents the blank scan counts in bin $i$, $l_i$ is the length of projection line $i$ through voxel $j$, $S(n)$ contains the projections in subset $s$, and $\omega^n$ is a weighting factor to ensure a monotonic likelihood increase. It has been shown that the convex algorithm ensures a monotonic likelihood increase for $\omega^n = 1$. This value was also chosen for OSC, but this may not ensure monotonicity. Therefore, at each iteration it was checked whether the likelihood had increased.

Transmission data were reconstructed with the standard convex algorithm and with the OSC algorithm using eight subsets (“OSC-8”) and 16 subsets (“OSC-16”). The subset order was chosen such that the angular distance between successive subsets was maximized. The subsets were ordered according to [9].

B. Phantom Data

The algorithms were compared by reconstructing simulated projections of a digital thorax phantom [mathematical cardiac torso (MCAT) phantom [10], lateral body width 36 cm, Fig. 1(a)] and measured transmission data of a physical phantom [anthropomorphic
thorax phantom. Transmission data of the MCAT phantom were simulated by rotating the object using bilinear interpolation and calculating the attenuation along rays through the object. The projection data contained 64 views, acquired over 180°. The pixel size was 0.3125 cm² × 0.3125 cm² during simulation. After simulation, the pixels were summed in 0.625 cm² × 0.625 cm² pixels and Poisson noise was generated to acquire 25 projection data sets with different noise realizations. The blank scan contained 200 counts/pixel.

Projections of the anthropomorphic thorax phantom were acquired on a two-headed SPECT system, equipped with two scanning line sources (156 mCi each) for transmission imaging. The number of projections was 64, the scan time was 24 s/view, the pixel size was 0.647 cm² × 0.647 cm² and the acquisition arc was 180°. The scaled blank scan contained an average of 225 counts/pixel.

C. Assessment of Image Quality

The following measures were calculated for the reconstructions of the MCAT phantom in order to investigate the quality of the OSC algorithm.

- Contrast, which is defined by

\[
\text{Contrast} = \frac{|I - b|}{I + b}
\]

where \(I\) is the average activity in region 2 and \(b\) is the average activity in region 1 (Fig. 2).

- The normalized standard deviation (NSD) in region 2 (Fig. 2, \(N = 49\) voxels), which is given by

\[
\text{NSD} = \frac{1}{\langle \hat{\mu} \rangle} \sqrt{\frac{1}{N-1} \sum_{k} (\hat{\mu}(k) - \langle \hat{\mu} \rangle)^2}
\]

where \(\langle \hat{\mu} \rangle\) represents the average voxel value in region 2.

- The mean squared error (MSE), which is defined as the mean of the squared differences between the voxels in the reconstruction and the phantom.

\[
\text{MSE} = \frac{1}{N} \sum_{k} (\hat{\mu}(k) - \hat{\mu})^2
\]

- The likelihood that the image estimate generates the projections.

These measures were calculated for all 25 noisy projection data sets and their means were calculated. For further assessment, images and image profiles are shown for reconstructions of both simulated data and measured data. For visual comparison, the reconstructions were post-filtered with a Gaussian filter (full-width–half-maximum = 1.5 cm). To find out whether OSC introduces bias, images and image profiles of reconstructions of noise-free MCAT data are shown.

III. RESULTS

Fig. 3 shows the average contrast, NSD, MSE, and likelihood in the MCAT reconstructions as a function of iteration number. In the graphs, it was assumed that the acceleration factor of OSC over the convex algorithm is proportional to the number of subsets, meaning that the OSC iteration numbers are multiplied by their number of subsets. It can be seen that contrast and likelihood of the OSC reconstructions are very close to those of the convex reconstructions, but NSD and MSE grow slightly faster in OSC reconstructions than in convex reconstructions. It is also observed that OSC-8 is closer to the convex algorithm than is OSC-16. The comparisons in Fig. 4 are not based on any assumption regarding the acceleration of OSC. The figure shows the average NSD and MSE as a function of contrast in the left lung. These graphs also show that compared to the convex algorithm, the OSC algorithm achieves similar contrast in the left
Fig. 3. (a) Contrast as a function of iteration number. (b) NSD as a function of iteration number. (c) MSE as a function of iteration number. (d) Likelihood as a function of iteration number.

Fig. 4. (a) NSD as a function of contrast in the reconstructions. (b) MSE as a function of contrast in the reconstructions.
Fig. 5. Reconstructed images and horizontal image profiles of the MCAT phantom, shown at similar background noise: (top) 96 iterations convex, (second image) 12 iterations OSC-8, (third image) six iterations OSC-16, and (bottom frame) image profiles.

Fig. 6. Reconstructed images and horizontal image profiles of the MCAT phantom data for noise-free data: (top) 96 iterations convex, (second image) 12 iterations OSC-18, (third image) six iterations OSC-16, and (bottom frame) image profiles.

lung at the cost of a slightly increased NSD and MSE. An interesting observation concerning these graphs is that OSC-8 and OSC-16 perform equally well.

Evaluation of the likelihood function during OSC reconstruction showed that the likelihood increases monotonically as a function of iteration number using weighting factors $\omega^n = 1$. This indicates that the OSC algorithm does not require a time-consuming search for an optimal $\omega^n$ that ensures a likelihood increase. Fig. 5 provides reconstructions and profiles of a noisy MCAT phantom data set. Small differences between the convex and the OSC generated images can be seen, but image quality appears to be equally good. The reconstructions are shown at comparable image noise. Results of noise-free MCAT data are shown in Fig. 6. The profiles of the convex and the OSC generated images are almost indistinguishable,
indicating that OSC and the convex algorithm generate the same bias in the reconstructions. Fig. 7 compares the convex algorithm and OSC for measured data. Again, only very small differences are visible.

IV. CONCLUSION

An acceleration of the convex algorithm that uses OSC is proposed. OSC was compared with the convex algorithm using digital and physical thorax phantom data. Results show that the OSC algorithm is significantly faster than the convex algorithm. The degree of acceleration is approximately proportional to the number of subsets in OSC and is achieved with only a slight increase of noise and global errors in the reconstructions. Reconstructions of noise-free data indicate that OSC and the convex algorithm result in the same bias. The tremendous acceleration achieved by OSC could bring the advantages of iterative ML algorithms for TCT within the possibilities of wide clinical application.

REFERENCES


Fig. 7. Reconstructed images and horizontal image profiles of the anthropomorphic phantom, shown at similar background noise: (top) 96 iterations convex, (second image) 12 iterations OSC-8, (third image) six iterations OSC-16, and (bottom frame) image profiles.