Iterative Decoding and Equalization for 2-D Recording

Naveen Singla
Advisor: Prof. J.A. O'Sullivan
ESSRL

Agenda

- Problem description
- LDPC codes
  - Introduction
  - Ensemble of codes
  - Decoding
  - Encoding
- Solutions and Results
- Future work

Motivation

- Why 2-D?
  - 1-D recording approaching saturation so move to 2-D to increase storage capacity.
  - LDPC and Turbo codes used as ECC codes for recording, work good for long lengths.

    1-D, Sector is 4096 bits~1.3dB from capacity.

    1024x1024=1048576~0.13dB from capacity.

Problem Description

- Read-back on recording channels suffers from ISI
- We consider 2-D recording media, 2-D ISI
- Use equalization techniques with LDPC code as ECC
- Why LDPC?
  - Random connection between coded bits
  - ISI is only local

LDPC Codes

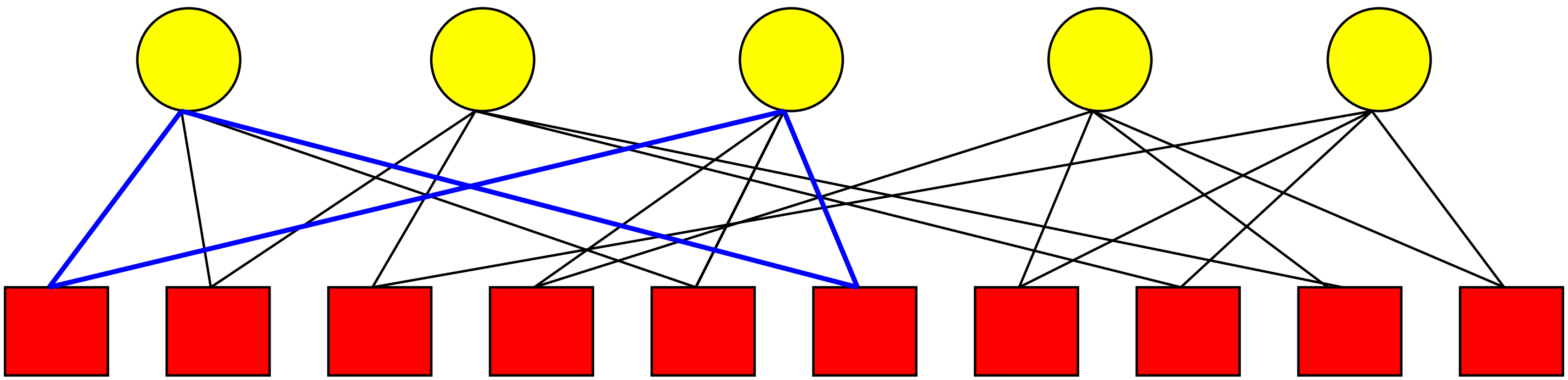
- Linear block codes
- Completely defined by sparse $H_{m \times n}$

  c is codeword iff Hc = 0

  Rate = $1 - \frac{m}{n}$
- Regular and Irregular codes

Bipartite Graph

- Bipartite graph representation



Ensemble of LDPC codes

- Degree Polynomials

$\lambda(x) = \sum_{i>1}^{d_v} \lambda_i x^{i-1}$ λ_i :number of edges emanating from variable nodes of degree i

$\rho(x) = \sum_{i>1}^{d_c} \rho_i x^{i-1}$ ρ_i :number of edges emanating from check nodes of degree i

$C^n(\lambda, \rho)$ Ensemble of length n codes with (λ, ρ)

Decoding

- Maximum A Posteriori decoder

  N(v)=all c s.t. H(c,v)=1 (1's in a column)

  N(c)=all v s.t. H(c,v)=1 (1's in a row)

  r=c+x : received word

  Hr=z=Hx : syndrome

  Calculate $P(x \mid z) = \dfrac{P(x)P(z \mid x)}{P(z)}$

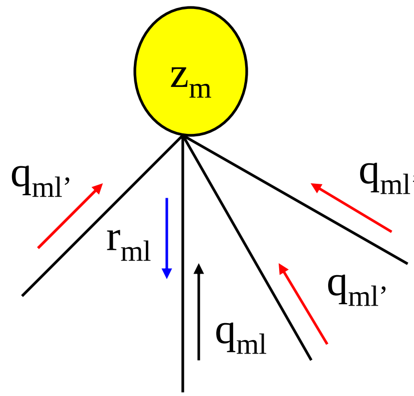
Message Passing

- Priors from channel output
- Check to variable messages

$$r_{ml}^{x} = P(z_m \mid x_l = x)$$

$$= \sum_{x_{l'}: l' \in N(m) \setminus l; x_l = x} P(z_m \mid x_{l'}) P(x_{l'} \mid x_l = x)$$

$$= \sum_{x_{l'}: l' \in N(m) \setminus l} P(z_m \mid x_{l'}; x_l = 0) \prod_{l' \in N(m) \setminus l} q_{ml'}^{x_{l'}}$$

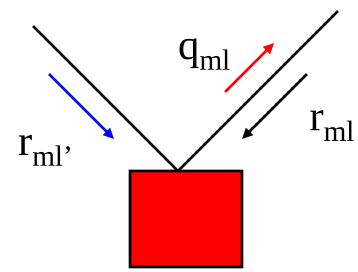


Message Passing contd…

- Variable to check messages

$$q_{ml}^{x} = P(x_l = x \mid z_{m'}) = \frac{P(x_l = x)P(z_{m'} \mid x_l = x)}{P(z_{m'})} \qquad m' \in N(l) \setminus m$$

$$q_{ml}^{x} = \alpha_{ml} p_l^0 \prod_{m'} r_{m'l}$$



- Posterior probabilities

$$q_l^{x} = \alpha_l p_l^0 \prod_{m \in N(l)} r_{ml}$$

- Complexity proportional to n

Encoding

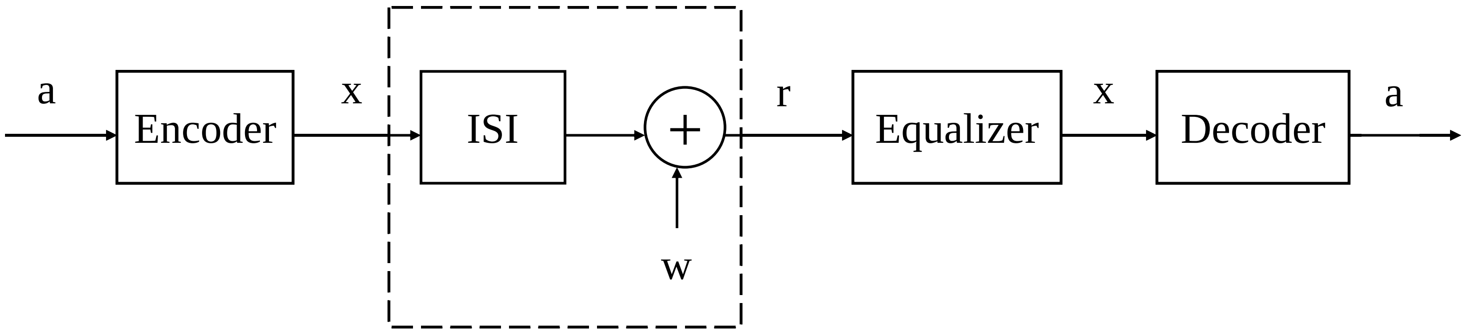
- Encoding is $O(n^2)$ – drawback
- Richardson and Urbanke, efficient encoder O(n) for large enough block lengths
- Kavcic, LDPC coset codes

$$Hc = z;\ z_i \in \{0,1\}$$

Cycles

- Decoding algorithm converges to exact APP if done on a tree
- Our graph has cycles
  - Remove short cycles
- Richardson and Urbanke, performance concentrates around cycle free case with high probability as n gets large

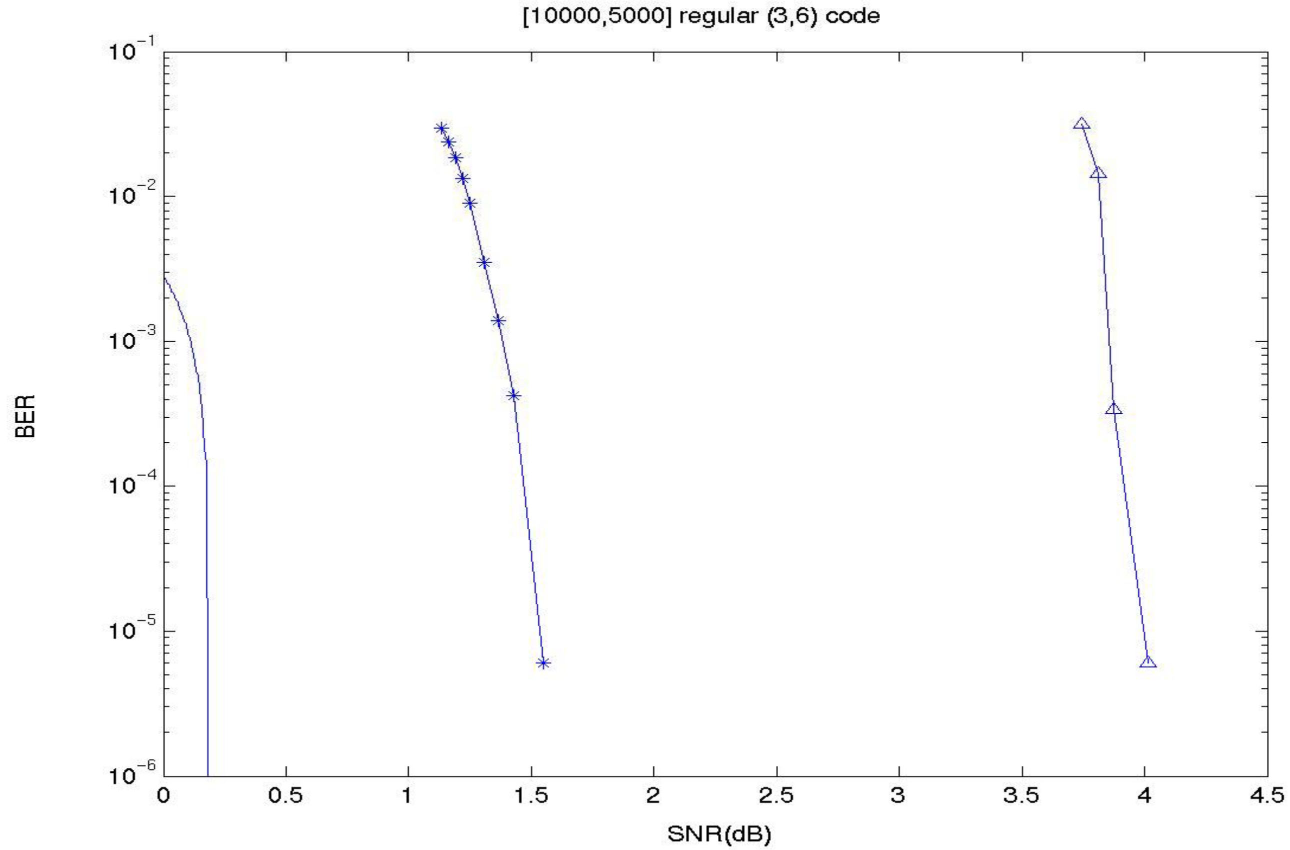
Approach I



- r = h**x + w

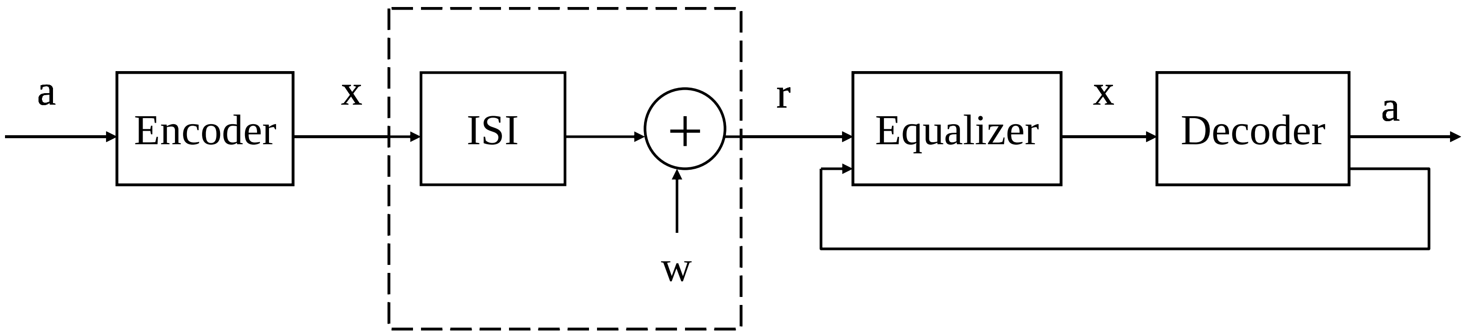
  $h=\begin{bmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{bmatrix}$, w AWGN
- Assume x is Gaussian and apply Wiener filter

Results I



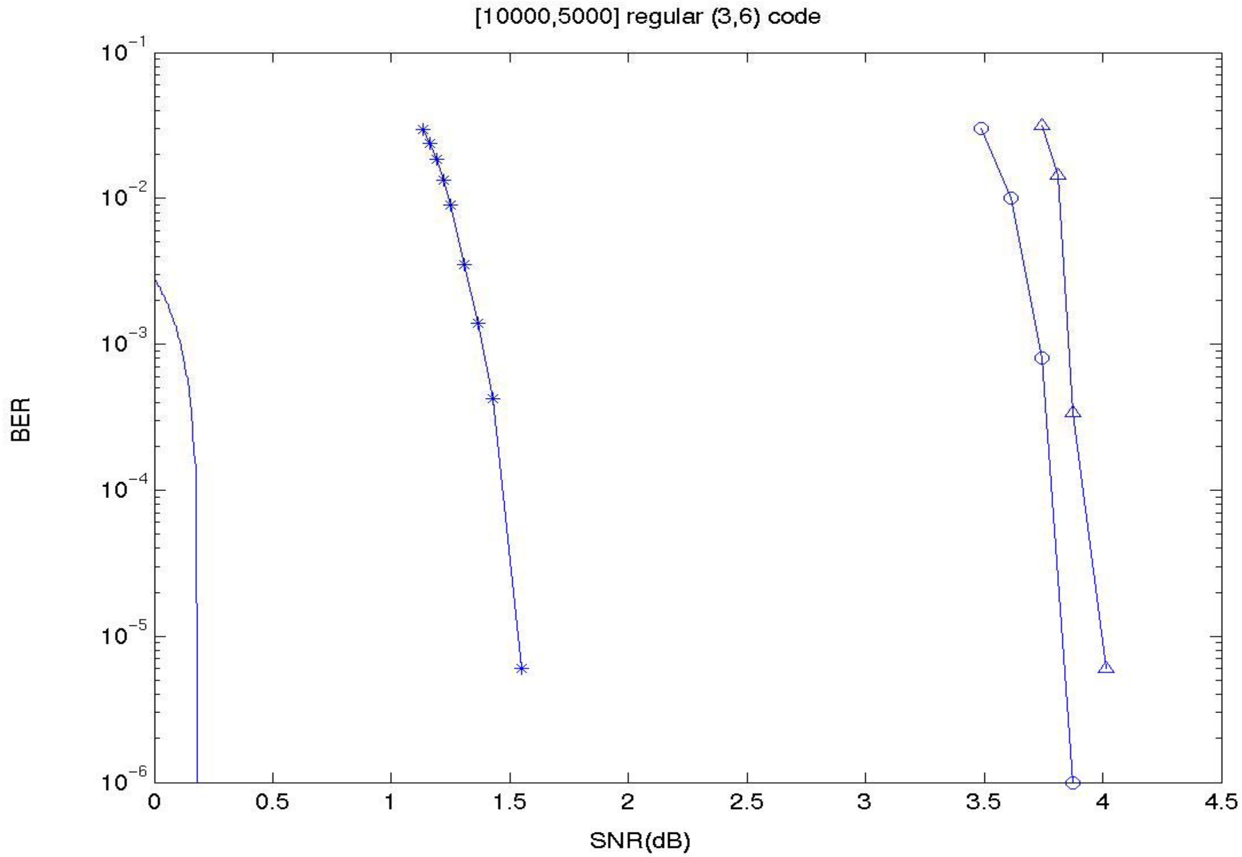
[10000,5000] regular (3,6) code
BER
SNR(dB)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5

Approach II



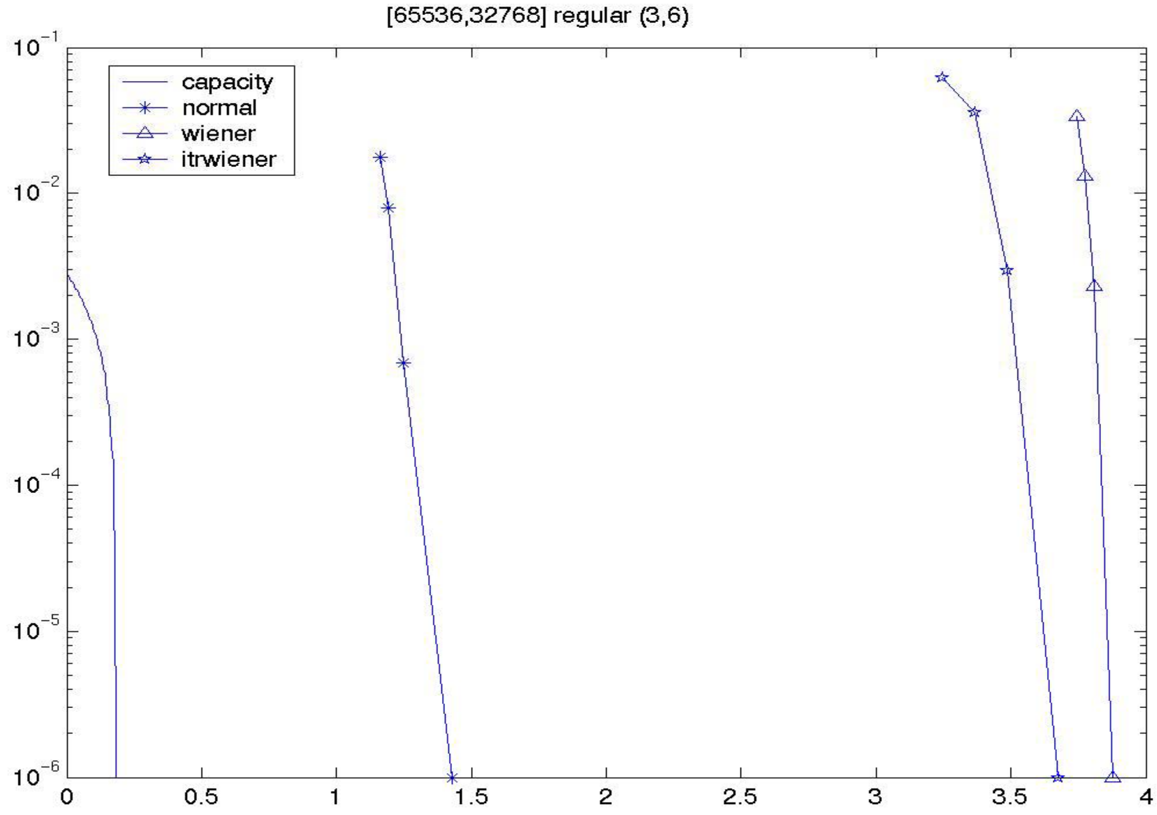
- Iterate between Wiener filter and LDPC if single application fails

Results II

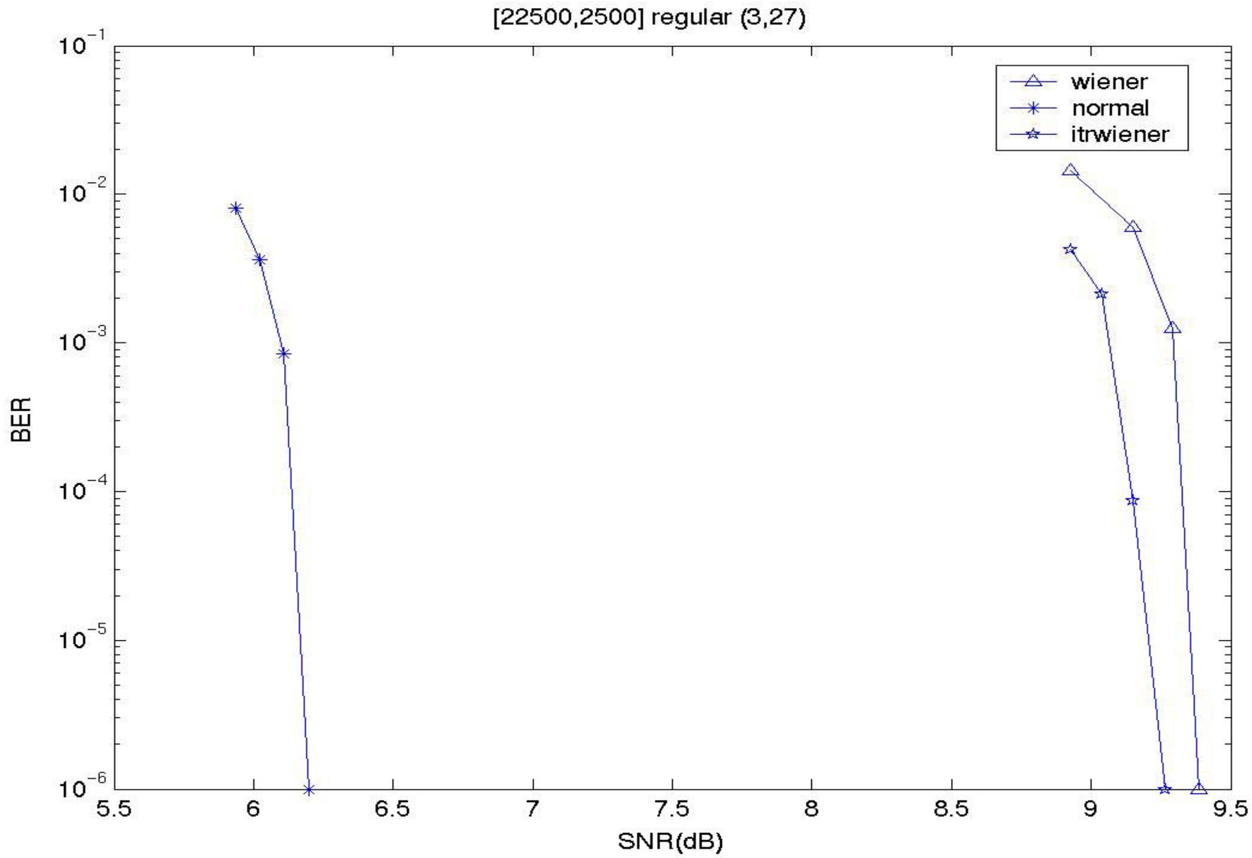


[10000,5000] regular (3,6) code
BER
SNR(dB)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5

More Results

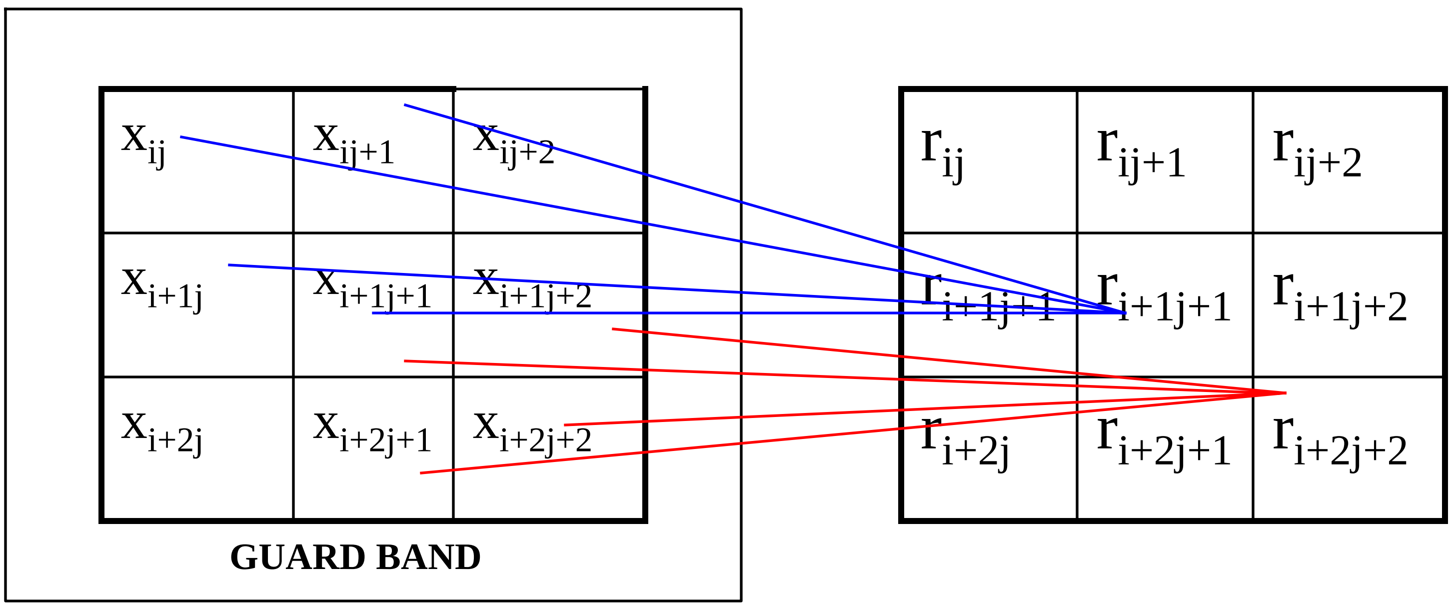


More Results contd…



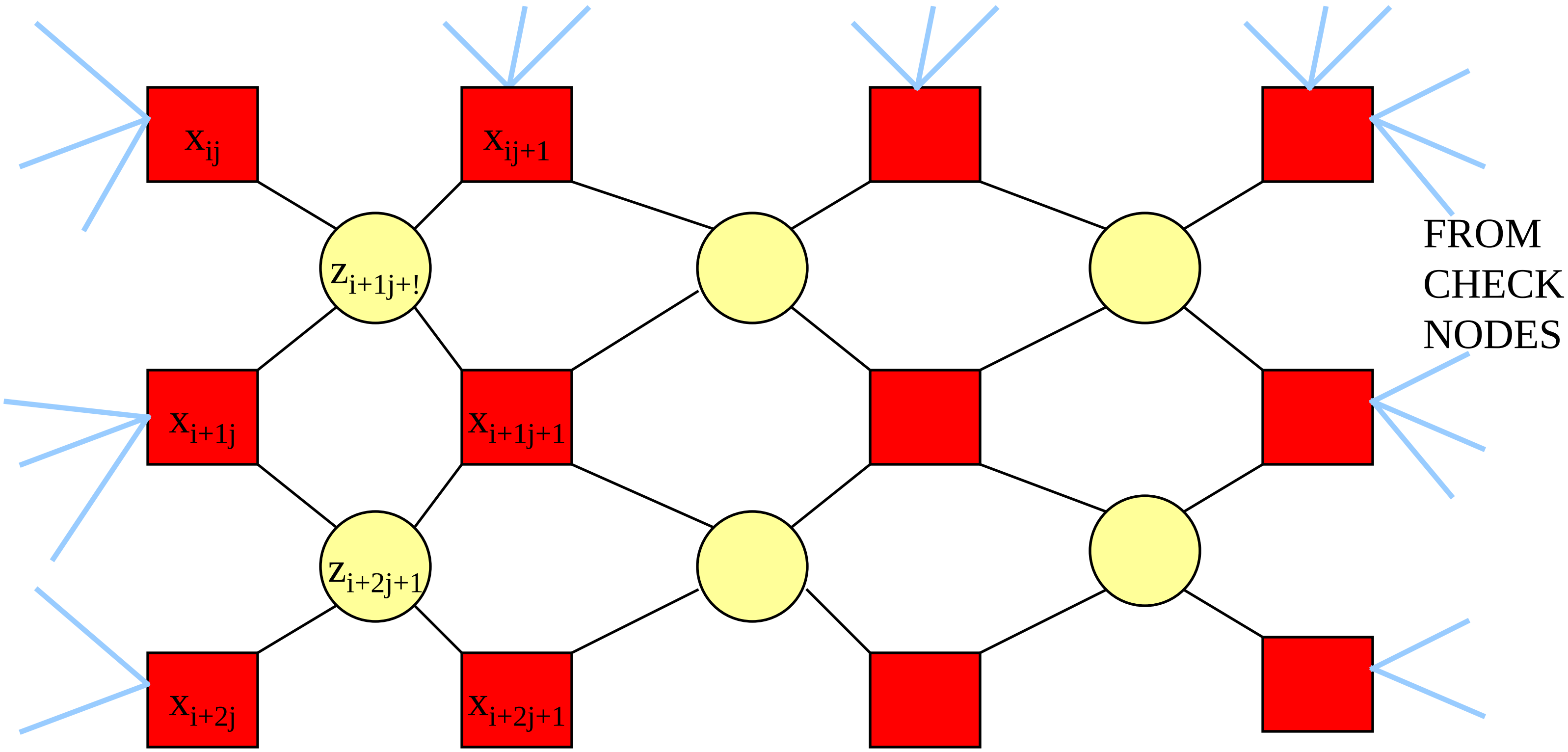
Approach III

- Full graph decoder

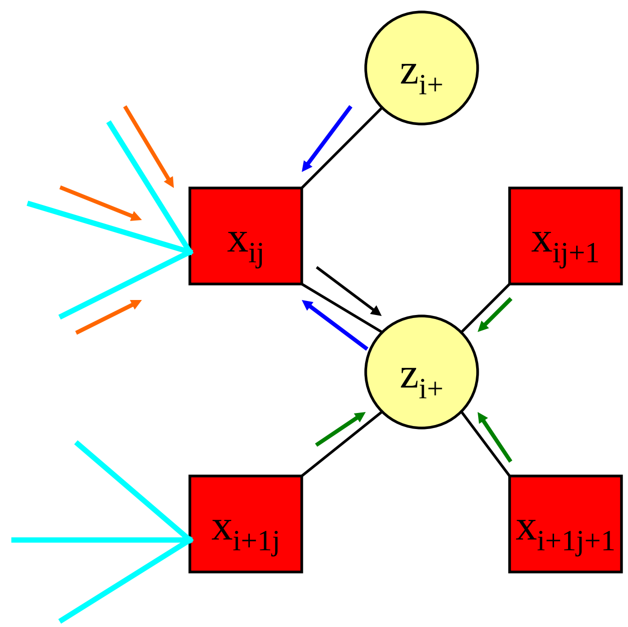


Full Graph

- $r_{ij} = h_{22}x_{i-1j-1} + h_{21}x_{i-1j} + h_{12}x_{ij-1} + h_{11}x_{ij} + w_{ij}$



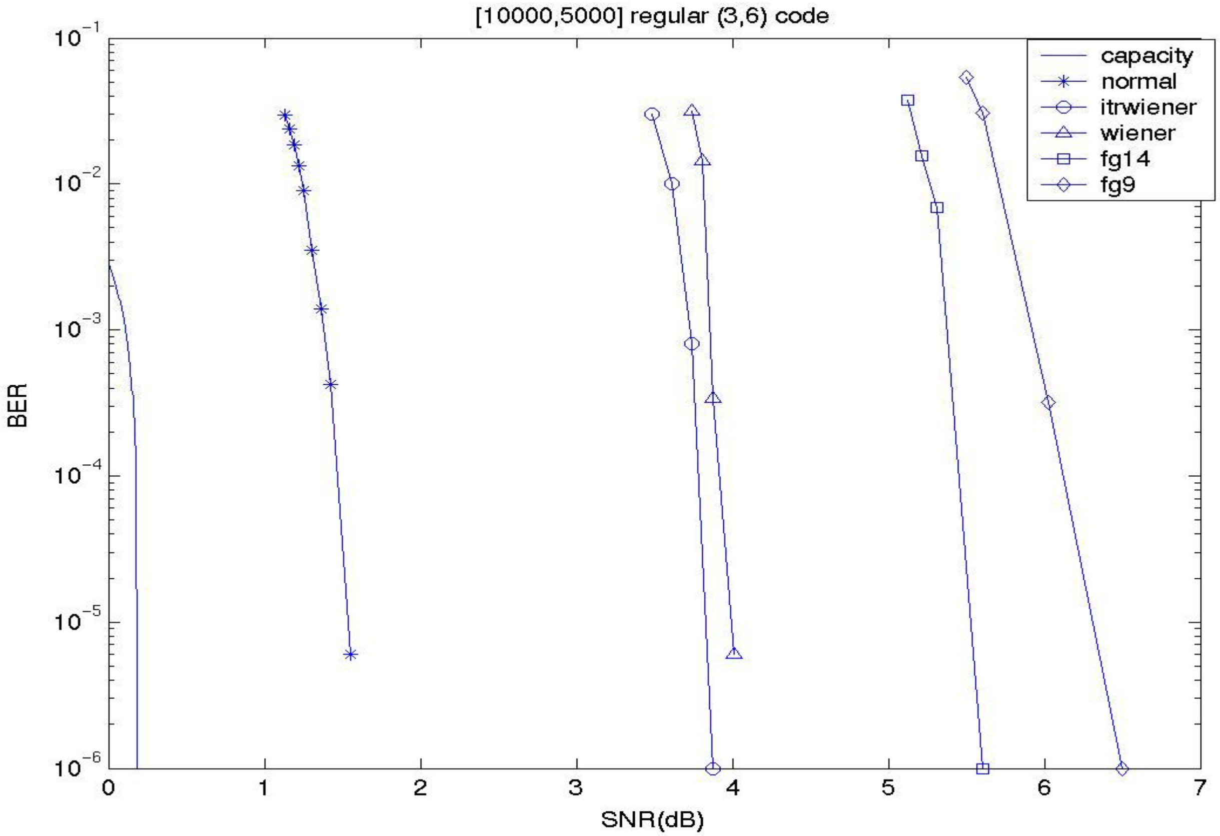
Full Message Passing



Full Graph contd…

- If LDPC fails then do full graph for fixed number of iterations.
- Scheduling : $v \rightarrow c \rightarrow v \rightarrow r \rightarrow v$.

Results III

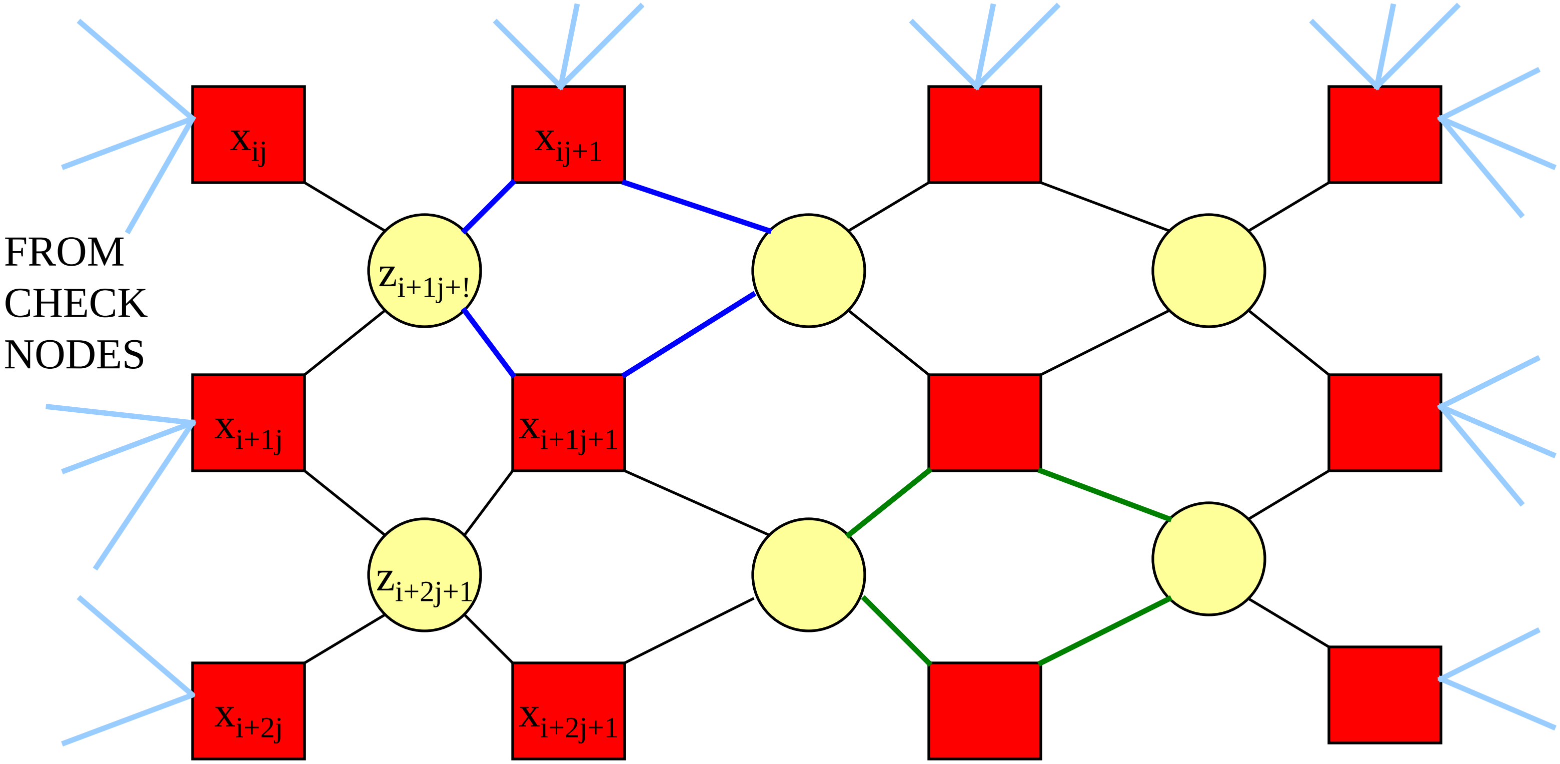

[10000,5000] regular (3,6) code
BER
SNR(dB)
capacity
normal
itrwiener
wiener
fg14
fg9
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
1
2
3
4
5
6
7

Comments

- Length 4 cycles present, degrade the performance.
- Use modification to ‘forget’ loops.

Full Graph

- $r_{ij} = h_{22}x_{i-1j-1} + h_{21}x_{i-1j} + h_{12}x_{ij-1} + h_{11}x_{ij} + w_{ij}$



Comments

- Length 4 cycles present, degrade the performance.
- Use modification to ‘forget’ loops.

Future Work

- Irregular LDPC codes
- Capacity for binary input ISI channel
- Construct capacity approaching sequences