

Effective depth of regular wave on submerged submarine

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This paper evaluates the effective depth of waves on the submarine at the depth of water. Ocean surface wave has huge energy that causes dynamic variations on the floating and submerged bodies. Water wave is an orbital wave in which particles move in orbital path. These waves transmit energy along interface between two fluids of different densities. Circular orbital motion dies out quickly below the surface. At some depth below the surface, the circular orbits become so small that the movement is negligible. This depth is called the Wave Base. The decrease of orbital motion with depth has many practical applications. For instance, submarines can avoid large ocean waves simply by submerging below the wave base. This paper is aimed to recommend a safe depth for calm and stable movement of a submarine. This safe depth is not equal to wave base necessarily. For this study, a torpedo-shaped submersible is analyzed in several depths accompanied by regular surface wave. By increasing the depth, the reduction of submarine movements is evaluated. The results of this research can be used for AUVs, research submersibles and submarines. This analysis is performed by CFD tools of Flow-3D (V.10) software based on solving the RANS equations and VOF method.

[Keywords: CFD; Regular wave; submarine; Dynamics; Flow3D; VOF.]

Introduction

Water wave is an orbital wave in which particles move in orbital path. These waves transmit energy along interface between two fluids of different densities. Circular orbital motion dies out quickly below the surface. At some depth below the surface, the circular orbits become so small that the movement is negligible. This depth is called the "wave base". Wave base can be regarded equal to one-half the wavelength ($\lambda/2$) measured from still water level (Fig.1). Only wave length controls the depth of the wave base, so the longer the wave, the deeper the wave base. The decrease of orbital motion with depth has many practical applications. For instance, submarines can avoid large ocean waves simply by submerging below the wave base. Even the largest storm waves will go unnoticed if a submarine submerges to only 150 m¹. Floating bridges and floating oil rigs are constructed so that most of their mass is below wave base, so they will be unaffected by wave motion. In fact, offshore floating airport runways have been designed using similar principles.

Additionally, seasick scuba divers find relief when they submerge into the calm, motionless water below wave base¹. Therefore, deep water defines as depth more than $\lambda/2$. The hydrodynamic forces of ocean surface wave on the submerged bodies are studied in several different fields of engineering:

Offshore engineering

Wave effects on the vertical and horizontal fixed cylinders such as the structural members of platform leg. Several extended studies have been performed to

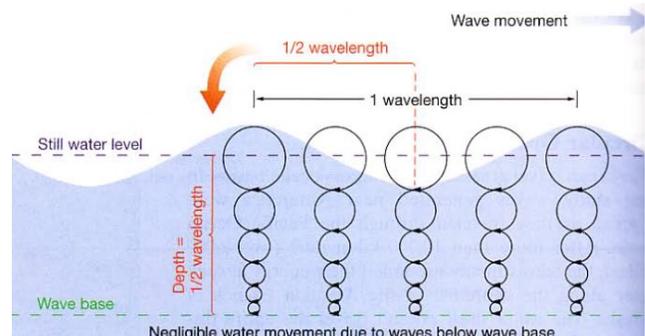


Fig. 1— Orbital motion in waves¹

analyze diffraction around a submerged fixed cylinder. Thus Dean (1948)² used a linearized potential theory, for showing the reflection effects. Ursell (1949)³ and later Ogilvie (1963)⁴ presented the formulation of wave steepness up to the second order. Chaplin (1984)⁵ measured the nonlinear force on a fixed horizontal cylinder beneath waves by experimental method. He analyzed the influence of the Keulegan-Carpenter number on the harmonics of the applied force.

Wave energy converter (WEC)

Wave effects on the moored or prescribed motions of cylinders of energy converter just near the surface. It is interesting in offshore engineering for moored semi-submersibles^{6,10}. Wu (1993) presented a formulation for calculating the forces exerted on a submerged cylinder undergoing large-amplitude motions. The free surface condition is linearized and the body surface condition is satisfied on its instantaneous position. The solution for the potential is stated as multi-pole expansion. Wu obtained results for a circular cylinder in purely vertical motion and clock-wise circular motion in a wave field (Wu, 1993).

Submarine and submersible design

Wave effects on the non-moored free submerged body near the free surface and at the snorkel depth.

The aim of this paper is the third category. This paper is aimed to recommend a safe depth for calm and stable movement of a submarine. This safe depth is not equal to wave base necessarily. For this study, a torpedo-shaped submersible is analyzed in several depths accompanying by regular surface wave. By increasing the depth, the reduction of submarine movements is evaluated. The results of this research can be used for AUVs, research submersibles and submarines. General discussion and specifications about submersible and submarine hydrodynamics are presented^{11,14}. In the field of submarine hydrodynamic near the free surface effect or in snorkel depth (or periscope depth), three general categories could be considered:

Resistance

By focus on the wave making resistance of a submarine traveling below the free surface in still water (without ocean wave)^{15,22}.

Dynamic in still water

By focus on the submarine dynamic equations and coefficients affected by free surface of water. General

dynamic equations of marine vehicles and submarines are presented^{23,24} as the most famous and comprehensive references in these fields. Revised standard submarine equations of motion were also presented^{14,25,27}. An interesting common study about submarine control is designing a control system for a submarine running near the free surface or snorkel depth. Refs. The controller design and maneuvering in still water is presented in several studies^{28,32}.

Dynamic under surface waves (seakeeping)

By focus on the submarine dynamic equations under ocean wave exciting is assessed^{33,41}. A collective experimental helpful result for wave forces on submerged bodies are presented⁴² for several different wave conditions.

Finally after literature survey, it can be stated that approximately all references are based on potential flow for inviscid fluid. For modeling the 3D object and calculating their hydrodynamic coefficient, some methods such as strip theory and conformal mapping should be used which are basically incompatible to submerged body (without water plane area). Other activities for adjusting these potential flow solutions to submerged bodies can be useful only in the early stages of design³³. In the early stage of design, some estimated and approximated values are sufficient. For the next stages and earning better careful results and exactly modeling the 3D form of submarine, numerical prediction of CFD method can be a good selection. Special explanation of numerical methods for modeling the submarine near the free surface is presented³². These methods are more time consuming than analytical methods but have better results. Accordingly our study and manner of this paper is focused on the CFD method. There are several CFD softwares which can model the ocean waves (regular and irregular waves), namely, Flow-3D, IOWA and OpenFOAM. This study uses Flow-3D software.

Materials and Methods

CFD Method of Study

In this study, the dynamic pressure fluctuation has been evaluated by a commercially available CFD solver, Flow-3D, developed by Flow Science Inc⁴³.

Governing equations

To solve the governing equations of fluid flow, Flow-3D uses a modification of the commonly used Reynolds-average Navier-Stokes (RANS) equations. The modifications include algorithms to track the free surface. The modified RANS equations are shown as:

$$\text{Continuity: } \frac{\partial}{\partial x}(uA_x) + \frac{\partial}{\partial y}(vA_y) + \frac{\partial}{\partial z}(wA_z) = 0 \quad \dots(1)$$

Momentum:

$$\frac{\partial U_i}{\partial t} + \frac{1}{V_F} \left(U_j A_j \frac{\partial U_i}{\partial x_j} \right) = \frac{1}{\rho} \frac{\partial P'}{\partial x_i} + g_i + f_i \quad \dots(2)$$

Turbulences model

More recent turbulence models are based on Renormalization-Group (RNG) methods. The empirically predicted coefficients of k- ϵ model are explicitly derived in RNG model.

Numerical methodology

The commercially available CFD package Flow-3D uses the finite-volume method to solve the RANS equations⁴³. The computational domain is subdivided using Cartesian coordinates into a grid of variable-sized hexahedral cells. The average values for the flow parameters (pressure and velocity) for each cell are computed at discrete times by staggered grid technique (Versteeg and Malalasekera 1995). The free surface is computed using a modified volume-of-fluid (VOF) method⁴³.

Obstacle generation

The FAVOR (Fractional area/volume obstacle representation) method, outlined by Hirt and Sicilian (1985) and Hirt (1992), is a porosity technique used to define obstacles.

Free surface modeling

One of the most accurate methods to track the free surface of water is the VOF method. The VOF method is evolved from the marker-and-cell method (Harlow and Welsh 1965), but is more computationally efficient. The VOF method is described in Nichols and Hirt (1975), Nichols et al. (1980), and Hirt and Nichols (1981).

Specifications of Model

In this study, a torpedo-shaped submersible (Persia110) is considered. The general form and dimensions of this model is shown in Figure 2. This model has 1 DOF, free to pitch. The model has a volume of 8.38 l, total area of 0.36 m² and weight of 8.38 kg and transverse moment of inertia (I_{yy}) of 1.3 kg m². This model is the same in several depths in CFD method and is the same for validation experiment in towing tank marine laboratory.

Validation and Verification

For validating the results of Flow-3D modeling, an experimental test was performed on the model Persia-110 in the towing tank of Admiral Makarov University, which has 33 m length, 2.5 m width and 1.3 m draft (Fig. 3). The towing tank is equipped with a trolley which operates in 0.05-6 m/s speed with

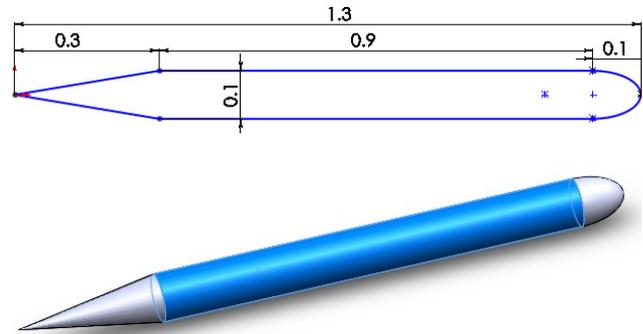
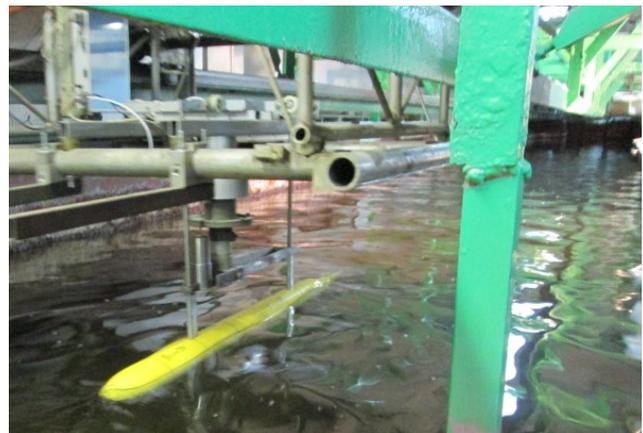


Fig. 2 — General configuration of the model Persia-110



a) Marine laboratory of Admiral Makarov University



(b) Model Persia-110 according to specifications of Fig. 2

Fig. 3 — Towing tank and model

accuracy of ± 0.02 m/s. A three degree of freedom dynamometer is used for force and moment measurements. The calibration of dynamometer was performed by calibration weights and several case studies. The model is fixed without any DOF. The test is in still water and water inside the tank is fresh water. The experiment was performed in surface condition at the draft of 7 cm and speed of 1 m/s. The CFD modeling was adjusted exactly according to the experimental conditions. Comparison of Figures 4-a and 4-b shows a good agreement between experimental and CFD results. The variations of free surface have a good compatibility. The comparison of the resistance in these two conditions is presented in Table 1.

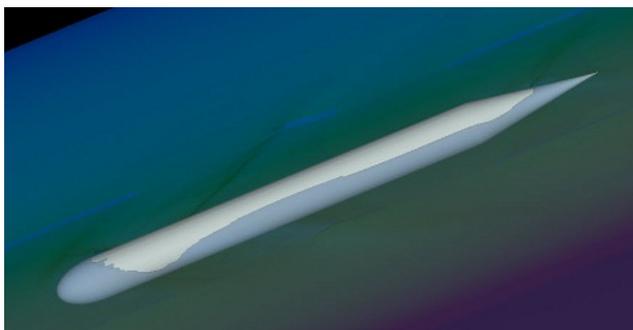
The difference of about 6.6 percentages is reasonable and acceptable. This validation case clearly shows the capability of a CFD tool, Flow-3D to reasonably predict the hydrodynamic problems of incompressible flow.

Table 1 — Comparison of resistance

Resistance in experiment	1.67 (N)
Resistance in CFD	1.79 (N)
Difference	6.6 %



(a) Test in the surface draft of 7 cm and speed of 1 m/s

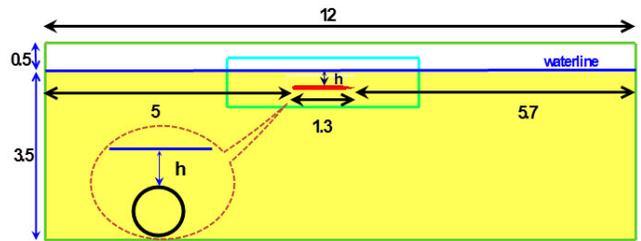


(b) CFD modeling in the same conditions of experiment

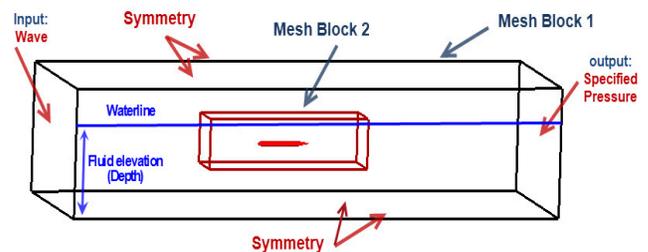
Fig. 4 — Comparison of the results of the experiment and CFD method

Domain and Boundary Conditions

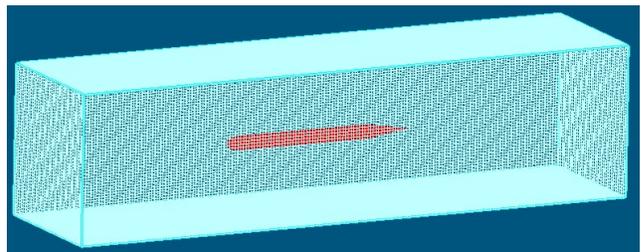
The general configuration and dimensions of domain are shown in Figure 5. The length and width are 12 and 2.6 m. Depth is 4 m (3.5 for draft and 0.5 for freeboard). There boundary conditions are: Input: wave, Output: Specified pressure, and other sides are symmetry. The model is situated in different depths of "h" according to Figure 6a. There are two mesh blocks: One block for the total domain with coarse meshes and the other block for fine meshes around the object body. The accuracy of the shape of body depends on the fine meshes (Figure 5b & c). For producing the wave, the input boundary condition is



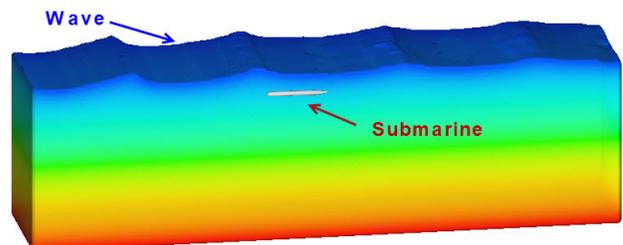
(a) Dimensions of domain (m)



(b) Boundary conditions in domain



(c) Fine meshes in Mesh Block 2



(d) Generated wave and position of submarine

Fig. 5 — Schematic of

"Wave". Flow-3D can produce regular and irregular waves. The produced wave and the situation of object under the waves are shown in Figure 5d. The reason of selecting each parameter is mentioned below.

Meshes

As mentioned above, there are two mesh blocks. The dimensions of mesh block 2 are: 4*1*1 m. By changing the location of the model, the situation of the mesh block 2 was changed. For selecting the proper mesh numbers and mesh independency evaluation, several mesh numbers were analyzed (Fig.6). This diagram shows that for mesh numbers after 500.000 the variation is very little and after 800.000 it remains almost constant. Therefore in all conditions of analyses in this study, the mesh number is considered 800.000 which is 300.000 for mesh block1 and 500.000 for mesh block 2. Therefore mesh block 2 contains fine meshes around the object. Generally, it should be notified that in wave problem, it doesn't need for very fine meshes modeling the boundary layer because the frictional forces are very small compared to the wave pressure forces. All meshes are hexahedral and without skew. Aspect ratio is 1, expansion factor between mesh block 1 and 2 equals to 2 and inside every block it is 1. Mesh planes coincide in the adjacent meshes.

Wave Modeling

The defined input boundary condition is regular wave. General definition of regular wave is represented in Figure 7. Here these parameters are defined in Flow-3D: wave amplitude 0.18 m, wave period 1 s and mean fluid depth (according to the depth of domain) is 3.5 m and current velocity is regarded zero. Based on these definitions, deep water condition is compatible because $d/\lambda > 0.5$. For deep waters, according to the formula of $\lambda = 1.56T^2$ the wave length is 1.56 m. Wave speed according to $C = 1.25\sqrt{\lambda}$ is 1.56 m/s. The orbital radius of wave

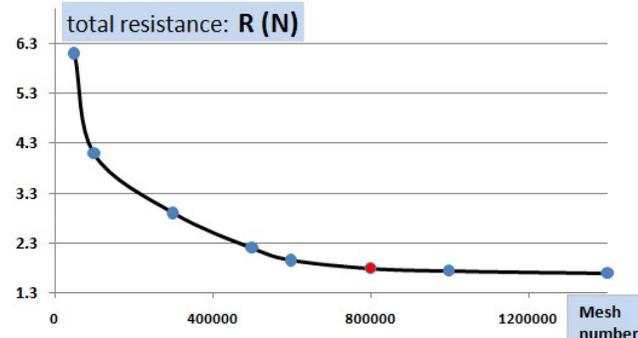


Fig. 6 — Mesh independency evaluations

articles path (R) according to this formula depends on the distance from water surface (h): $R = A \cdot e^{-kh}$ and $k = 2\pi/\lambda$. The variations of article radius versus depth are shown in Figure 1 and here can be stated as: 1) At the water surface, $h=0$ and $R=A$ which means at the surface, the radius of orbital movement of articles equals wave amplitude. 2) At $h=\lambda/2$, $R=0.043A$. 3) At $h=\lambda$, there is $R=0.002A$. It is obvious that at the depth equal to $\lambda/2$, the circle radius is only 4% of the surface value and at the depth equal to λ , it is only 0.2% of the initial value at the surface. Therefore, at the depths more than $\lambda/2$, the wave will be damped out.

Simulation time

For selecting the proper simulation time, the time history of variation of pitch angle was studied in 100 s (Fig. 8). This diagram shows that there are two overshoot points (maximum and minimum) and

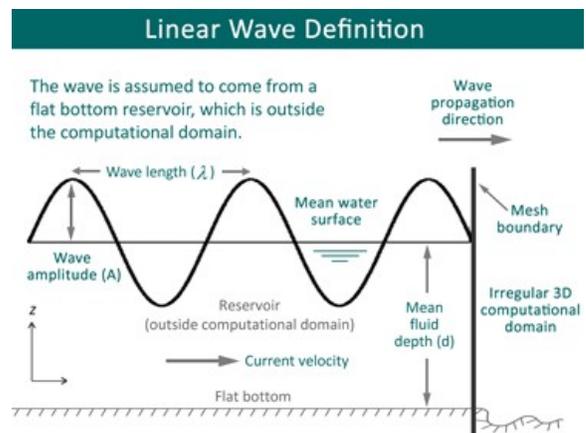


Fig. 7 — Linear wave definition

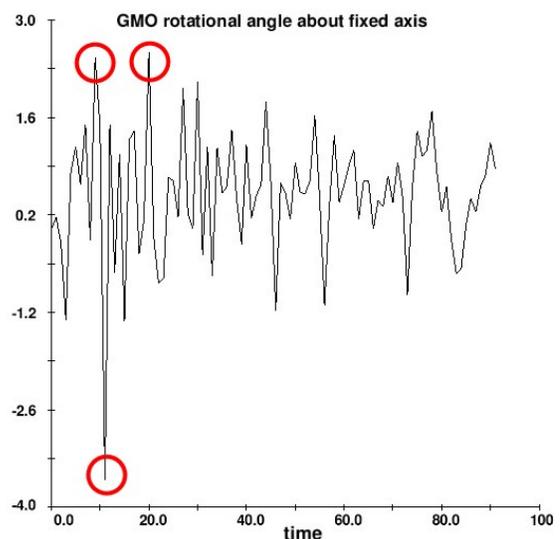


Fig. 8 — Evaluation of time history of pitch angle in 100 seconds for condition $h=D$

except these values, other variations are smooth and inside a certain limit. These overshoots happen because of initial momentum of inertia. For saving the time, these overshoot points were eliminated and simulation time was considered 20 s.

Domain Dimensions

In this problem, the specifications of wave are very important for determination of domain dimension.

Length: The considered wave length is about 1.56 m. For better forming of wave before arriving to the object, more than two complete waves are considered in 5 m. About the same value is considered after the object is equal to 5.7 m. By considering the 1.3 m length of the object, the total length of domain is achieved 12 m.

Breadth: The considered breadth is equal to object length (L=1.3 m) to each side and the total breadth is 2L.

Breadth: As mentioned above, the wave base is approximately equal to $\lambda/2$. This study aims to evaluate the wave effects on the submarine at the depth of 2λ . For avoiding the bottom effects, the draft of domain is considered 3.5 m. The wave amplitude (A) is 0.18 m, thus the freeboard above water level is considered 0.5 m. Therefore the depth of domain is considered 4 m.

Settings of simulation are abstracted in Table 3.

Considered Conditions for Analyses

For studying the wave effects on the submarine, several depths for submarine situation (h) are considered according to Figure 5a and Table 2.

Results and Discussion

Method of extracting the results

According to Figure 8, there is disordered and irregular variation of pith angle versus time. Usually in

sea keeping studies, Root Mean Square (RMS) analysis is used. Therefore, here the RMS value of pitch angle is calculated in every depth, after eliminating the overshoot points. RMS is calculated as Eqn. 9:

$$RMS = \sqrt{\frac{\theta_1^2 + \theta_2^2 + \dots + \theta_n^2}{n}} \quad \dots(3)$$

Results

The time history of pith angle in 12 conditions is analyzed. Figure 9 shows two samples of time history for h=0.1 and 0.35 m. Table 3 represents the results for each depth. The percentage of decrease in last column is based on comparison to h=0, therefore average= $((h_0-h_i)/h_0*100)$. It should be notified that the static pitch angle of this submarine is 0.34 degree.

Discussion and analysis

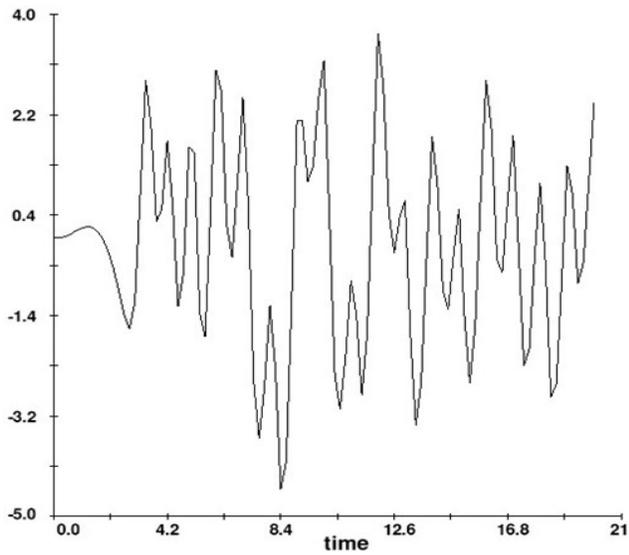
It is obvious that by increasing the depth, the wave effect decreases and pitch angle approaches static trim

Table 2 — Considered conditions for analyses

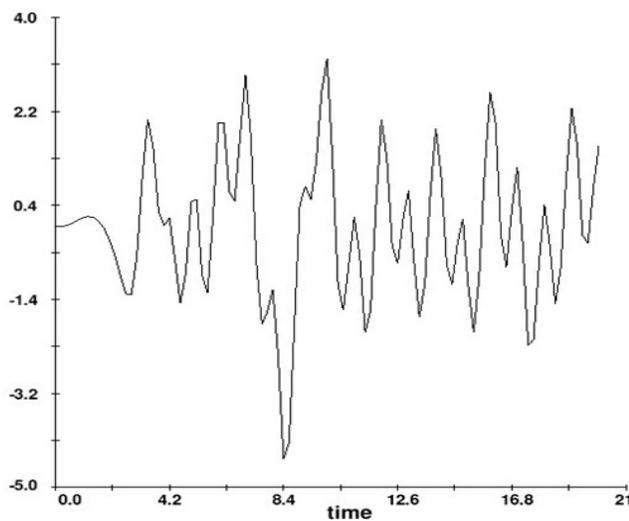
Submarine depth (m)	Description (equivalent to)
1	0 Body tangent to free surface
2	0.05 R_s (or) 0.03λ
3	0.1 D_s (or) 0.06λ
4	0.15 $1.5D_s$ (or) 0.09λ
5	0.25 $2.5D_s$ (or) 0.16λ
6	0.35 $3.5D_s$ (or) 0.22λ
7	0.55 $5.5D_s$ (or) 0.35λ
8	0.75 $7.5D_s$ (or) 0.48λ
9	0.95 $9.5D_s$ (or) 0.61λ
10	1.6 $\cong \lambda$
11	2.4 $\cong 1.5\lambda$
12	3 $\cong 3\lambda$

Table 3 — Settings of simulation

Element	Boundary conditions	Element conditions	Boundary conditions	Descriptions
Domain	Cubic	dimensions	grid	with free surface and linear wave - domain with inlet, outlet and symmetry - without heat transfer- without current velocity $L*B*D=12*2.6*4$ m- draft 3.5 m structured grid- hexahedral cells-without skew- two mesh block- more fine meshes in mesh block 2 around the object- Mesh number of 800.000, aspect ratio 1, expansion factor between blocks 2 and inside blocks 1.
Fluid	-	settings		Simulation time: 20 sec- Time step=0.005-0.013 sec Incompressible fluid (fresh water)- tempreture:20 deg- $\rho=999.841$ kg/m ³ - turbulent modeling: Standard k- ϵ
Object	GMO			Submarine, length:1.3m, Diameter:0.1, 1DOF free to pitch angle
Input	Inlet			Linear wave, wave amplitude 0.18 m, wave period 1 sec, mean fluid depth 3.5 m
Output	Outlet			Specified pressure (Specified fluid level: 3.5 m)
Symmetry	Symmetry			In 4 faces
Initial conditions				Fluid level: 3.5 m



(a) $h=0.1$, RMS=1.67 deg



(b) $h=0.35$, RMS=1.22 deg

Fig. 9 — Time history of pith angle

angle. The last column of Table 4 can smoothly describe the percentages of reduction in pitch angle. In depth of 0.03λ , there is 33% reduction and in depth of 0.06λ there is 51% reduction. Intense gradient of pitch angle will be continued until the depth of 0.09λ which experiences 59% reduction. After this depth, there is a gentle variation. Values of RMS at the depths of λ , 1.5λ and 2λ are equal to static trim angle which meant no effect of waves on the submarine. Almost around the depth of $\lambda/2$ the wave effect is negligible. The reason of this phenomena is based on the principle of "wave base" described in Introduction. It means that if a submarine dives to the

Table 4 — RMS values for considered conditions

	depth (m)	depth (λ)	RMS (degree)	Percentage of Decrease (%)
1	0	0	3.43	0
2	0.05	0.03	2.29	33
3	0.1	0.06	1.67	51
4	0.15	0.09	1.42	59
5	0.25	0.16	1.38	60
6	0.35	0.22	1.22	64
7	0.55	0.35	1	71
8	0.75	0.48	0.82	76
9	0.95	0.61	0.44	87
10	1.6	1	0.1	97
11	2.4	1.5	0.03	99
12	3	2	0	100

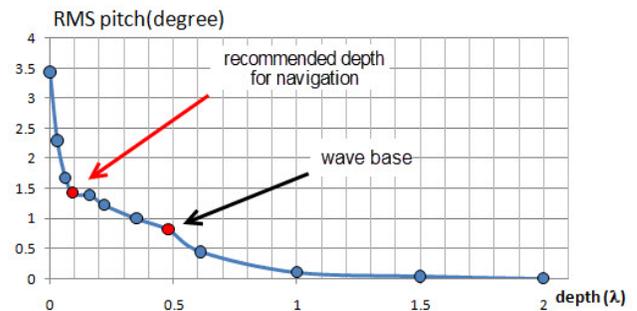


Fig. 10 — Gradient of RMS pitch versus submergence depth of submarine

depth more than $\lambda/2$, it doesn't experience the wave effects. For long swell waves, the value of $\lambda/2$ may be more than the collapse depth of the submarine and be impossible. In this condition, if submarine dives to the depth about 0.1λ , it can avoid 60% of movements and shakes. For instance, in a swell wave (which is very similar to regular waves) with a period of 15 s, the wave length is 351 m. The half wave length is about 175 m which may be dangerous depth for a submarine and it can be catastrophic. Despite, if submarine dives to the depth of 0.1λ equal to about 35 m, can navigate in very calm and more stable conditions.

Conclusion

In conclusion, the results could be abstracted in Figure 10 which fairly shows the gradient of movements versus depth of submergence. Depth of $\lambda/2$ could be considered as the absolutely calm depth but the depth of 0.1λ could be recommended as an operational safe and approximately calm depth for submarines.

Nomenclature

λ	Wave length (m)
θ	Pitch angle (degree)
A	Wave amplitude (m)
AUV	Autonomous Underwater Vehicle
CFD	Computational Fluid Dynamics
d	Depth of water (m)
Ds	Diameter of submarine body
DOF	Degree Of Freedom
GMO	General Moving Object
h	Distance from top of the object (submarine) to the water surface (m)
IHSS	Iranian Hydrodynamic Series of Submarines
L	Length of object (submarine)
R	orbital radius of wave articles path (m)
Rs	Radius of submarine body
RMS	Root Mean Square
VOF	Volume Of Fluid

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