ASYMPTOTIC ANALYSIS AND TOPOLOGICAL DERIVATIVES FOR LINEAR ELASTICITY

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Asymptotic analysis with respect to singular perturbations of geometrical domains is used in shape optimization in order to improve the performance of numerical methods, in particular of the level set method. The first term of asymptotic expansion of a given shape functional with respect to small parameter which measures the size of the perturbation is called topological derivative of the shape functional under study. Such notion is introduced in (2) for elliptic boundary value problems, and it is based on the works of Russian school on the asymptotic analysis of PDE’s in the sense of Il’in as well as of Mazja, Nazarov and Plamenievskii, the list of references can be found e.g., in (4).

We present a general framework for construction of asymptotic expansions of shape functionals in two dimensional case. We restrict ourselves to the case of plane elasticity and singular geometrical perturbations of the domain of integration. The method of analysis of elasticity boundary value problems is based on complex variable approach developed by Muskhelishvili. In this way we are able to construct explicit solutions of elasticity boundary value problems in a ring and perform the asymptotic analysis of the solutions with respect to the radius of the interior hole, when the radius tends to zero.

The asymptotic analysis results in expansions of arbitrary order with respect to the small parameter which measures the size of the small opening inserted into the domain of integration, i.e. of the small hole in an elastic body. The results are given not only in an explicit form, but also in the form useful for numerical methods. The presented formulae can be used in shape and topology optimization in structural mechanics. This means that we are able to derive the form of higher order topological derivatives in linearized elasticity, which seems to be new in the literature on the subject. Roughly speaking, the first order topological derivative indicates the place where a small hole can be injected in order to improve the value of the shape functional, and the second order topological derivative indicates the size of the hole, we refer to the presentation of A. Novotny in our minisymposium for the results in this direction in the case of the Laplacian.

Some numerical results are also included in our presentation.

REFERENCES