An Analytical Model of Spectrum Fragmentation in a Two-Service Elastic Optical Link

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Abstract—Elastic Optical Networks (EONs) enable optical circuits to be assigned distinct numbers of spectrum slices. Individual circuits can then be assigned an optimal number of slices to best match their target transmission rates. A well-known drawback of EONs is spectrum fragmentation and its resulting uneven blocking probability, which circuit requests experience when the available spectrum slices in the fiber are insufficient or not contiguous. Capturing this spectrum fragmentation problem analytically is a challenging problem. Not surprisingly, most of the existing studies at this time mainly use simulation-based techniques to quantify blocking probability in EONs. In this paper, the authors present a Markov Chain (MC) model that attempts to characterize the fragmentation problem in a simplified scenario, i.e., only two types of circuit services are allowed over a single fiber link. Despite its limited scope, this initial analytical effort is able to accurately capture the non-monotonic behavior of the blocking probability in EONs for the first time.

I. INTRODUCTION

Elastic Optical Networks (EONs) enable uneven assignments of the fiber spectrum — logically divided to form multiple consecutive equal slices — to best match traffic bandwidth requirements between network nodes [1]. Consider for simplicity an EON offering only two types of circuit service. In the former service type, low rate circuits are established between node pairs by reserving a single slice of spectrum. In the latter service type, high rate circuits are established between node pairs by reserving \( n > 1 \) consecutive slices. This second type of service is often referred to as superchannel.

By allocating (consecutive) slices adaptively, EONs can match spectrum allocation against requested circuit transmission rates and yield up to 30\% improved spectrum utilization when compared to conventional fixed grid systems [2], [3]. However, the uneven spectrum allocation technique enabled by EONs is likely to cause spectrum fragmentation. The main drawback of spectrum fragmentation is the existence of isolated and unused slices that cannot be assigned to superchannels, which require a predetermined number of consecutive slices. This problem is exacerbated by the spectrum continuity constraint, according to which, a circuit must be assigned the same group of slices in every fiber in the network that belongs to the circuit end-to-end path. Due to spectrum fragmentation, in many practical cases the network utilization remains somewhat limited. More importantly, superchannels are more likely to be blocked due to the lack of consecutive available slices when compared to single-slice circuits.

The fragmentation problem in EON has been extensively studied [1], [4]–[6]. In these studies, a number of Routing and Spectrum Allocation (RSA) algorithms are proposed [1], [4], [5] with the aim of reducing the adverse effects originating from spectrum fragmentation. A typical objective of the RSA algorithm is to compute the end-to-end path and the group of slices that must be assigned to each optical circuit in order to minimize the number of circuit requests that are blocked, i.e., circuits that cannot be provisioned in the EON due to the insufficient availability of slices in the fibers. The study in [4] proposes to combine the \( K \) Shortest Paths (KSP) and First-Fit slice assignment algorithms in order to reduce blocking probability. The study in [6] defines two ways to quantify how serious the spectrum fragmentation problem is, instead of estimating the effect of fragmentation on the network performance. Simulation-based studies have shown that blocking probability in EON can be uneven, i.e., high-rate circuit requests are more likely to be blocked when compared to low-rate requests due to the shortage of contiguous available slices, e.g., [7]. Due to the complexity of the fragmentation phenomenon, a few analytical models have been proposed so far to rigorously quantify this problem [8]. For example, the study in [9] presents a simple but not always accurate analytical model for computing the blocking probability of both single-slice and \( n \)-slice circuits in a single fiber link.

In this paper, the authors present an accurate Markov Chain (MC) model for quantifying blocking probability in a two-service elastic fiber link. Two distinct traffic groups coexist in the link and are referred to as group 1 and group 2, respectively. Low rate circuits (group 1) are established by reserving a single slice of spectrum. High rate circuits (group 2) are established by reserving a superchannel, \( n > 1 \) consecutive slices. The blocking probability is analytically estimated and a number of numerical experiments are conducted to document the model accuracy. The model is able to quantify the blocking probability for each type of service and their differences. In addition, the model is used to detect and quantify the non-monotonic pattern of the blocking probability values for the single-slice circuit requests. Network-wide simulation results show similar trends of the blocking probability, both in terms of unfairness and oscillating behavior.

II. SYSTEM DESCRIPTION AND MODELING ASSUMPTIONS

The proposed model accounts for a single fiber link with an elastic spectrum (simulation results for multiple fibers comprising a network are discussed in Section IV-B). Let the fiber spectrum be divided to form \( N \) slices, which are numbered...
from 0 to \(N - 1\). A request in group 1 requires \(m = 1\) slice and a request in group 2 requires a superchannel which consists of \(n \times m\) contiguous slices, \(n > 1\). The \(N\) slices can be logically grouped to form \(N / m\) superchannels\(^1\) as shown in Fig. 1. This semi-flexible grid technique is effective in reducing both spectrum fragmentation and blocking probability [1].

The following superchannels or group of slices are predefined: \((0, n - 1), (n, 2n - 1), (2n, 3n - 1), \ldots, (N - n, N - 1)\). A group 1 request is randomly assigned one of the empty single slices.\(^2\) A group 2 request is randomly assigned one of the completely empty (unused) superchannels. A request is blocked when it cannot be assigned the required number of (contiguous) slices, \(m = 1\) or \(n\), respectively. Generated requests are handled First Come First Serve (FCFS) and they are discarded if blocked. Requests are generated according to a Poisson arrival process, and their holding time (service time) is modeled as a random variable with an exponential probability distribution. Let the arrival rate of group 1 (2) requests be \(\lambda_1\) (\(\lambda_2\)) and the average service time of group 1 (2) requests be \(1/\mu_1\) (\(1/\mu_2\)), respectively.

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\(1\) \(\frac{N}{m}\) is assumed to be an integer.

\(2\) For tractability of the analytical model, only the random slice assignment algorithm is considered.

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Fig. 1. An example of semi-flexible grid \((m = 1, n = 2, N = 16)\).

III. TWO-SERVICE ANALYTICAL MODEL FOR SEMI-FLEXIBLE GRID

A. An Accurate Markov Chain Model

A continuous-time Markov chain model is defined in this section with the aim of estimating the blocking probabilities that are experienced by group 1 and 2 requests when reserving their slices in a single fiber link. Let \((i, j, e)\) be the state description for the MC, defined as follows: \(0 \leq i \leq N\) counts the number of group 1 requests that are currently reserved one slice in the fiber, \(0 \leq j \leq \frac{N}{m}\) counts the number of available superchannels that are each currently reserved \(n\) contiguous slices in the fiber, and \(0 \leq e \leq \frac{N}{m}\) counts the number of available superchannels in the fiber. A superchannel is available or unused as long as all of its \(n\) slices are available. The MC states are subject to the following constraints: \(i \geq 0, j \geq 0, e \geq 0\) and \(i + j \times n + e \times n \leq N\). Naturally, if \(e = 0\), any group 2 incoming request is blocked. On the contrary, group 1 requests may still be accepted as long as there is one single slice available in the fiber. The MC state transition diagram is shown in Fig. 2. Its transition rates are shown in Fig. 3 and described next.

When a group 2 request is reserved one of the available superchannels, the following state transition takes place: \((i, j, e) \rightarrow (i, j + 1, e - 1), e > 0\). The rate of this transition is \(\lambda_2\). When a group 2 request departs (end of service), the following transition takes place: \((i, j, e) \rightarrow (i, j - 1, e + 1)\). The rate of this transition is \(j/\mu_2\).

The state transitions associated with group 1 requests are not straightforward. According to the random slice assignment algorithm, a group 1 request can be assigned either a single slice of one of the available superchannels or a single slice of one of the partially used superchannels. Two distinct and mutually exclusive transitions are possible:

- Transition \((i, j, e) \rightarrow (i + 1, j, e - 1)\): i.e., the request is assigned one slice of one of the available superchannels.
- Transition \((i, j, e) \rightarrow (i + 1, j, e)\): i.e., the request is assigned one slice of one of the superchannels, which are already partially used by group 1 request(s).

The number of available superchannels is either decremented in the former case or remains unchanged in the latter. For each case, we need to compute the corresponding transition rate as follows. Let \(\lambda_1(i, j, e, e_{\text{next}})\) be the transition rate associated to incoming group 1 requests, where \(i, j\) and \(e\) are as defined for the MC state and \(e_{\text{next}}\) is either \(e\) or \(e - 1\). Let \(j = \frac{N}{m} - j\) be the number of superchannels that are not used by group 2 requests. The number of superchannels either partially or completely used by group 1 requests is \(K = j - e\). The number of unused (available) slices in these \(K\) superchannels is \(s_{\text{unused}} = K \cdot n - \ldots\)
Recalling that the slice assigned to the request is randomly chosen from those available, it is straightforward to obtain the following equations:

$$\lambda_1(i, j, e, e_{\text{next}}) = \begin{cases} \frac{e\cdot n}{s_{\text{unused}}+en} \lambda_1 & (i, j, e) \rightarrow (i + 1, j, e - 1), \\ \frac{e\cdot n}{s_{\text{unused}}+en} \lambda_1 & (i, j, e) \rightarrow (i + 1, j, e). \end{cases}$$

(1)

A simple example is provided next to illustrate how these equations are applied. Assume that $m = 1$, $n = 2$, $N = 8$ and the current state is $1, 0, 3$, i.e., the number of group 1 requests is $1 (0)$ and the number of unused superchannels is $3$. Consequently, the number of superchannels partially used by group 1 requests is $K = \frac{7}{2} - 0 - 3 = 1$, $s_{\text{unused}} = 1 \cdot 2 - 1 = 1$ and the total number of available slices in the spectrum is $s_{\text{unused}} + e \cdot n = 1 + 3 \cdot 2 = 7$. The transition rate from $(1, 0, 3) \rightarrow (2, 0, 3)$ is $\frac{1}{2} \lambda_1$. The transition rate from $(1, 0, 3) \rightarrow (2, 0, 2)$ is $\frac{1}{3} \lambda_1$.

Upon departure of a group 1 request, the following transitions are possible:

- If $i = K$, state $(i, j, e)$ will transit to state $(i - 1, j, e + 1)$, which means a departure will always free one superchannel.
- If $i \geq (K - 1) \cdot n + 2$, state $(i, j, e)$ will transit to state $(i - 1, j, e)$, which means a departure will never free a superchannel.
- If $K < i < (K - 1) \cdot n + 2$, state $(i, j, e)$ may transit to either state $(i - 1, j, e + 1)$ or state $(i - 1, j, e)$ — in other words, a departure may or may not free a superchannel. For this case, it is necessary to evaluate the probability that a superchannel is freed or not by a departure.

In the first case, the $i$ group 1 requests are assigned exactly one slice in $i$ distinct superchannels, i.e., $i = K$. When any one of the $i$ group 1 requests departs, one of the used superchannels becomes available, hence the number of unused superchannels is incremented. The transition rate for this case is $i \mu_1$.

In the second case, each of the $K$ superchannels is used by two or more group 1 requests, i.e., $i \geq (K - 1) \cdot n + 2$. When any one of the $i$ group 1 requests departs, the number of unused superchannels remains unchanged. The transition rate for this case is also $i \mu_1$.

In the third case, two are the possible outcomes — either $e$ is incremented or $e$ remains unchanged. States that belong to this third case are characterized by $K < i < (K - 1) \cdot n + 2$. For this case, it is necessary to compute the probability that upon departure of one of the $i$ group 1 requests, the $e$ value is incremented or left unchanged, respectively. We first need to solve a simpler but related problem, i.e., the computation of how many ways $i$ balls can be placed in $K$ boxes, i.e., function $\text{cnt}(i, K)$, with the additional constraints that each box has $n$ compartments, each compartment can hold up to 1 ball and each box must have at least one ball. Let $K_{\text{min}} = \lceil i/n \rceil$, which represents the minimum number of boxes that must be used to host the $i$ balls — in this case $K - K_{\text{min}}$ boxes are completely empty. We need to demonstrate that

$$\text{cnt}(i, K) = \sum_{w=0}^{K-K_{\text{min}}} (-1)^w \binom{K}{w} \binom{K-w}{i} \cdot n$$

(2)

The first term ($w = 0$) in the summation, $\binom{K}{K_{\text{min}}}$, counts how many ways there are to place $i$ balls in $K \cdot n$ containers. This count includes ball distributions that leave some of the $K$ boxes completely empty, i.e., the number of empty boxes in the $K$ boxes can be $E = 0, 1, 2, \ldots, K - K_{\text{min}}$. We then need to remove from the count the cases in which $E > 0$. This removal step is performed iteratively, starting with case $w = 1$, and continuing till case $w = E = K - K_{\text{min}}$ is accounted for. Consider case $w = E = 1$ first. One of the $K$ boxes must be completely empty (there are $\binom{K}{1}$ ways to choose one of the $K$ boxes to be the empty one), and then we count how many ways there are to place $i$ balls in $K - 1$ boxes, i.e., $\binom{(K-1)}{i}$. The product of these two functions represents the second term $(w = 1)$ of the summation in (2). This term is subtracted from $\binom{K}{i}$ in order to remove from the count all the cases with $E = 1$. However, term $\binom{K}{i}$($\binom{(K-1)}{i}$) also accounts for cases $E = 2, 3, \ldots, K - K_{\text{min}}$. More precisely, these additional cases are counted multiple times, i.e., case $E = 2$ is counted twice, case $E = 3$ is counted three times, etc. To compensate for this the extra counting of these cases in the second term $(w = 1)$ of the summation, additional correcting terms are needed in the summation with alternating signs. Once all the terms in the summation are taken into account, it can be demonstrated that all of the $E = 1, 2, \ldots, K - K_{\text{min}}$ cases are correctly counted and removed from $\text{cnt}(i, K)$. A way to prove this claim is to note that the terms in the summation counting the number of occurrences of case $E = 1, 2, \ldots, K - K_{\text{min}}$ are

$$\sum_{w=0}^{E} (-1)^w \binom{E}{w} = 0, \quad \forall E = 1, 2, \ldots, K - K_{\text{min}}$$

(3)

indicating that these cases are not extra-counted in $\text{cnt}(i, K)$, as intended.

We next compute how many ways there are to distribute $i$ group 1 requests to make use of exactly $K$ superchannels out of $j$ available superchannels. This function — which counts how many cases are represented by state $(i, j, e)$ — is given by

$$\text{num}(i, K, j) = \binom{j}{K} \cdot \text{cnt}(i, K)$$

(4)

where the binomial indicates how many ways one can choose $K$ superchannels from $j$ available superchannels. We then compute how many of the $\text{num}(i, K, j)$ cases have at least one superchannel with exactly one group 1 request in it. If a case has two or more superchannels with exactly one group 1 request each, this case is counted multiple times, one per such request, whose departure would turn its superchannel into an empty one. In other words, the total number of cases and possible group 1 request departures that would empty one superchannel is given by

$$\text{emp}(i, K, j) = \binom{j}{1} \binom{n}{1} \cdot \text{num}(i - 1, K - 1, j - 1)$$

(5)

where the first binomial represents the number of possible ways a superchannel can be chosen out of $j$ options, the second binomial represents the number of possible ways one can place one group 1 request in that superchannel, and $\text{num}(i - 1, K - 1, j - 1)$ represents the number of possible ways one can distribute $i - 1$ requests over exactly $K - 1$ superchannels.
superchannels chosen from a total of \( j \) superchannels (one is already assigned to host one request only). We can finally compute the transition rate from state \((i, j, e)\) to \((i-1, j, e+1)\) and from state \((i, j, e)\) to \((i-1, j, e)\) as follows:

\[
\mu_1(i, j, e, e_{next}) = \begin{cases} 
\frac{\text{emp}(i, K, j)}{\text{num}(i, K, j)} \cdot \mu_1 & e_{next} = e + 1, \\
(i - \frac{\text{emp}(i, K, j)}{\text{num}(i, K, j)}) \cdot \mu_1 & e_{next} = e.
\end{cases}
\]

(6)

The following example illustrates how (2)-(6) are applied. Assume \( m = 1, n = 4, N = 20, \) and MC state \((4, 0, 2)\). Then \( i = 2, j = 5, K_{\text{min}} = 1. \) Note that \( K = 3 < i = 4 < (K-1) \cdot n + 2 = 10. \) Hence, upon departure of a group 1 request, one of these two states will be reached: state \((3, 0, 3)\) or \((3, 0, 2)\). Table I shows how many times specific cases are counted in the summation of (2). The number of possible permutations when assigning the \( i = 4 \) group 1 requests to exactly \( K = 3 \) superchannels is \( \text{num}(i, K, j) = \text{num}(i, K, j) = \frac{5!}{4!} \sum_{w=0}^{w=1} \frac{3!}{3!(w-1)!} = 2,880. \) In this equation, when \( w = 0 \), the number of permutations is \( \frac{3!}{4!} \), which counts how many ways there are to assign \( i = 4 \) requests to \( 3 	imes 4 = 12 \) slices. More precisely, this term will account for the following cases: all 3 superchannels are used \((E = 0)\), only 2 superchannels are used \((E = 1)\), and only 1 superchannel is used \((E = 2)\). All of these cases are counted only once, as indicated in the second column of the table. We then need to remove from the count the cases in which \( E > 0 \). In this example, the removal step starts with \( E = 1 \) and ends with \( E = 2. \) Consider case \( E = 1 \) first. One of the 3 superchannels must be completely empty — there are \( \frac{5!}{4!} \) ways to achieve that. Then the \( i = 4 \) group 1 requests are assigned to the 2 superchannels left — there are \( \frac{2!}{4!} \) ways to do that. Their product \( \frac{5!}{4!} \cdot \frac{2!}{4!} \) represents the second term in (2) \( w = 1 \). This term is subtracted from \( \frac{3!}{4!} \). However, according to (3), case \( E = 1 \) is removed once and case \( E = 2 \) is removed twice (as indicated in the third column of the table). In order to compensate for the extra removal of case \( E = 2 \), the third and final term in the summation \( (w = 2) \) is needed (leftmost column in the table). Continuing now with (4), the number of possible permutations using only \( i - 1 \) group 1 requests and exactly \( K - 1 \) superchannels is \( \text{num}(i-1, K-1, j) = \text{num}(3, 2, 4) = \frac{5!}{4!} \sum_{w=0}^{w=1} \frac{2!}{2!(w-1)!} = 288. \) Then \( \text{emp}(i, K, j) = \frac{3!}{1!} = 6 \cdot 288 = 5,760. \) Finally, the departure rate from state \((4, 0, 2)\) to \((3, 0, 3)\) is \( \frac{\text{emp}(i, K, j)}{\text{num}(i, K, j)} \cdot \mu_1 = 5,760 \cdot 288 = 2,880. \) The departure rate from state \((4, 0, 2)\) to \((3, 0, 2)\) is \( (i - \frac{\text{emp}(i, K, j)}{\text{num}(i, K, j)}) \cdot \mu_1 = (4 - 5,760) \cdot 288 = 2.880. \) Rate from state \((4, 0, 2)\) to \((3, 0, 2)\) is \( (i - \frac{\text{emp}(i, K, j)}{\text{num}(i, K, j)}) \cdot \mu_1 = (4 - 5,760) \cdot 2.880.

### Table 1. Number of Times Each Case is Counted in (2) for \( m = 1, n = 4, N = 20, \) and MC State \((4, 0, 2)\).

<table>
<thead>
<tr>
<th>K</th>
<th>w</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>-2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

B. Steady State Equations and Blocking Probability

Let \( \pi_{i,j,e} \) be the steady state probability of state \((i, j, e)\). These probabilities are derived by solving the following global balance equations:

\[
\begin{align*}
\lambda_1(i-1, j, e)\pi_{i-1, j, e} & + \lambda_1(i, j, e+1)\pi_{i, j, e+1} + \\
\mu_1(i-1, j, e)\pi_{i-1, j, e+1} & + \mu_1(i, j, e)\pi_{i, j, e+1} + \\
\lambda_2\pi_{i+1, j+1, e+1} & + (j + 1)\mu_2\pi_{i, j+1, e+1} = [(i + 1) + j]\mu_2 + \lambda_2]\pi_{i, j, e}.
\end{align*}
\]

Let \( BP_1 \) \((BP_2)\) be the blocking probability experienced by group 1 \((2)\) requests. These probabilities are \( BP_1 = \sum_{(i,j,e) \in N} \pi_{i,j,e} \) and \( BP_2 = \sum_{(i,j,e) \in N} \pi_{i,j,e}. \) The former equation accounts for all possible states that do not have any available slice, hence any request of group 1 is blocked. The latter equation accounts for all possible states that do not have any available superchannel, hence any request of group 2 is blocked. The overall blocking probability is obtained by \( BP_{\text{ave}} = (\lambda_1 \cdot BP_1 + \lambda_2 \cdot BP_2)/(\lambda_1 + \lambda_2). \)

C. Model Complexity

The total number of states in the MC model is one possible way to assess the model complexity. Let \( S(N,n) \) represent the number of states in the proposed MC model, given as \( S(N,n) = \sum_{m=1}^{N} \frac{N}{n}. \)

IV. Numerical Results

This section provides numerical results obtained using both the MC model and a custom simulator. An existing discrete event-driven simulation platform for EONs [7] has been modified to account for semi-flexible spectrum allocation. The EON simulator accounts for both the semi-flexible algorithm in [1] — according to which circuit requests are assigned slices in the spectrum using a first-fit assignment policy — and a random-fit policy, as described earlier in this paper. The simulation platform is used to obtain results for both a single elastic fiber link and an entire EON consisting of a number of nodes and fiber links connected to form an arbitrary mesh. In conducting the simulation experiments, the following assumptions are made. The simulation platform is used to estimate the blocking probability of two types of service (i.e., circuit requests for either single-slice channel or superchannel consisting of \( n > 1 \) slices) in both a single elastic fiber link and an EON. The spectrum of each fiber link is divided to form \( N = 80 \) slices. The circuit (service) requests are generated according to a Poisson arrival process, whose rate is varied in order to yield blocking probability values in the [10^{-7}, 1] range. Offered traffic load is defined as \( \rho = \lambda_1/\mu_1 = \lambda_2/\mu_2. \) The request holding time (service time) is described by a random variable with an exponential distribution.

A. Single Fiber Case

Results generated from the MC model are reported using continuous curves, while simulation generated results are reported using circles. As defined in Section III-B, \( BP_1 \) and

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3For contained computational complexity of the analytical model, \( N = 80 \) is assumed. However, the model also works for \( N > 80. \)
BP2 represent the blocking probability of group 1 and 2 requests, respectively. BP_ave represents the average blocking probability experienced by all requests.

The first group of figures refers to the case of a single fiber link being used to assign spectrum slices to circuit requests. Fig. 4 shows the blocking probability versus traffic load when using \( m = 1 \) and \( n = 2 \), i.e., superchannels consist of two contiguous slices of spectrum. Fig. 5 shows the same chart, this time using \( m = 1 \) and \( n = 4 \). Fig. 6 shows blocking probability versus traffic load when using \( m = 1 \) and \( n = 8 \), with logarithmic scale applied to traffic load. Fig. 7 shows blocking probability versus traffic load when using \( m = 1 \) and \( n = 16 \), with logarithmic scale applied to traffic load. Results from both the MC model and simulation are in good accord in all cases. Even at light load, the curves are well matched as clearly indicated by the two logarithmic scale plots in Fig. 6 and Fig. 7.

Non-monotonic curves are noted for group 1 requests (BP1) when \( n = 8, 16 \). In the \( n = 8 \) case, the BP1 curves show a pronounced oscillation (more than one order of magnitude) when the offered load is in the \([10, 100]\) range. Similarly, in the \( n = 16 \) case, the BP1 curves show an even more pronounced oscillation (more than four orders of magnitude) when the load is in the \([5, 130]\) range. This counter-intuitive result needs to be further clarified. A possible explanation for the existence of a negative slope in these charts is as follows. As \( \rho \) increases, both groups of requests offer higher arrival rates. While trying to secure either a slice or a block of \( n \) slices, group 1 and group 2 requests are racing against each other. However, group 1 requests have the advantage of “stealing” unused superchannels (and their slices) away from group 2 requests by simply reserving one of the \( n \) slices of a superchannel. The stolen superchannel can then be used by only additional incoming group 1 requests as it cannot be assigned to any group 2 request. The higher the number of stolen superchannels, the higher the number of slices that are stolen away from group 2 requests. This effect creates a reinforcing cycle for group 1 requests, which experience a lowering of blocking probability till slices quickly run out as \( \rho > 100 \) for case \( m = 1, n = 8 \) and \( \rho > 130 \) for case \( m = 1, n = 16 \). These oscillations disappear when \( n = 2, 4 \) as the number of “stolen” slices is limited.

Table II reports the accuracy of the MC model when compared to the single fiber simulation results, using four distinct experiments. The relative error function in [10] is applied. The relative error reported in the table is comparable to the confidence interval of the results obtained from simulation.

**Table II. The Analytical Model Accuracy**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>BP1</th>
<th>BP2</th>
<th>BP_ave</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m = 1, n = 2 )</td>
<td>0.0197</td>
<td>0.0112</td>
<td>0.0106</td>
</tr>
<tr>
<td>( m = 1, n = 4 )</td>
<td>0.0509</td>
<td>0.0013</td>
<td>0.0013</td>
</tr>
<tr>
<td>( m = 1, n = 8 )</td>
<td>0.0823</td>
<td>0.0084</td>
<td>0.0085</td>
</tr>
<tr>
<td>( m = 1, n = 16 )</td>
<td>0.2137</td>
<td>0.0054</td>
<td>0.0046</td>
</tr>
</tbody>
</table>

**B. Network Case**

It is interesting to investigate the potential presence of oscillations in the blocking probability curves when considering a network-wide scenario. The NSF topology with 14 nodes and 21 links is used for this investigation. Each link in the topology...
represents two unidirectional fibers (one per direction) and each fiber spectrum is divided into \( N = 80 \) slices. For each node pair in the topology, 5 shortest paths are computed using hop-count as the metric. One of the five paths is chosen for each incoming circuit request — generated by a Poisson arrival process with source and destination pairs randomly and uniformly chosen, and with a circuit holding random time described by an exponential probability distribution — based on two RSA algorithms, i.e., the semi-flexible algorithm (Semiflex) [1] and the \( K \) Shortest Paths with first-fit policy (KSP_FF) algorithm [7]. Spectrum or wavelength continuity is enforced along every circuit path.

![Fig. 8. Network blocking using semiflex \((m = 1, n = 8)\).](image1)

![Fig. 9. Network blocking using KSP_FF \((m = 1, n = 8)\).](image2)

![Fig. 10. Network blocking using Semiflex \((m = 1, n = 16)\).](image3)

![Fig. 11. Network blocking using KSP_FF \((m = 1, n = 16)\).](image4)

Blocking probability curves for the NSF network are reported in Fig. 8 and Fig. 9 for the \( n = 8 \) case, and in Fig. 10 and Fig. 11 for the \( n = 16 \) case. The oscillation depth is confirmed to be proportional to the value chosen for \( n \), i.e., larger values of \( n \) magnify the phenomenon. The two RSA algorithms show similar oscillation patterns. It is interesting to notice that the network blocking probability curves for group 1 requests present multiple oscillations, compared to the single oscillation reported for the single fiber case in Section IV-A. A possible explanation for this difference is as follows. The fiber links in the network are not subject to the same amount of traffic load. Each fiber load is determined by the paths that are computed between node pairs. Some fibers are then more likely to be used than others, their load depending on their location in the network, the topology structure and the traffic distribution offered by the network nodes. Consequently, fiber links tend to reach critical loads (which are responsible for creating negative slopes in the blocking probability curve of group 1 requests) at different values of the overall network offered load. Having the fiber link critical loads occurring at distinct network offered loads has two consequences: multiple cycles of oscillation take place and the depth of these oscillations are decreased in intensity (when compared to the single fiber case).

V. SUMMARY

Elastic Optical Networks (EONs) enable optical circuits to be assigned flexible spectrum resources (slices) and to best account for varying data traffic demands. A drawback of EONs is their resulting spectrum fragmentation, which limits the achievable network utilization and generates uneven blocking probability of circuit requests. A Markov Chain (MC) model is presented in this paper with the aim of gaining further insight into this drawback. The MC model accounts for two types of service, the former offering circuits of \( m = 1 \) slice, the latter offering superchannels consisting of \( n \times m \) slices. The MC model is able to accurately capture the blocking probability of the two service types in a single fiber link and reveals a non-monotonic blocking behavior as a function of traffic load when the superchannel size is \( n = 8 \) and \( n = 16 \). This oscillating behavior of the blocking probability is further observed network-wide through simulation. We expect this initial model to be generalized in future studies to account for additional scenarios of interest.

ACKNOWLEDGMENT

This research was supported in part by NSF Grants CNS-1111329, CNS-1405405 and CNS-1409849.

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