Kinematics and Dynamics of a Hybrid Serial-Parallel Mobile Robot

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Abstract- In this study, kinematics design, dynamics modeling and verification of a compounded serial-parallel wheeled mobile robot is elaborated. The proposed novel kinematic structure is best suited to fulfill stable motion of the robotic system when handling heavy objects by manipulators mounted on mobile platforms. The proposed system is made of a differentially-driven wheeled platform, a planar parallel manipulator, which is called here as Star-Triangle (ST) mechanism, and a serial Puma-type manipulator arm. The suggested structure adopts the advantages of both serial and parallel robots, to move the base point of the serial robot with respect to the mobile platform to fulfill the system stability after grasping heavy objects. In order to investigate the comprehensive kinematics model of the robot, after introducing its novel structure it is divided into three modules, i.e. a mobile platform, a parallel ST mechanism, and a serial robot. Next, a closed-form dynamics model is derived for the whole hybrid system based on a combined Newton-Euler and Lagrange formulation. The proposed method presents the mutual dynamic interaction wrenches between the integrated platform and the serial manipulator which can be exploited for the tip-over stability analysis of the mobile robotic system. Then, to verify the obtained mathematical model, several benchmark actuating inputs are applied to the model and the system responses are analyzed.

Index Terms: Wheeled Mobile Robot- Dynamics modeling-Planar Parallel Manipulator.

I. INTRODUCTION

Mobile robots have an unlimited workspace. This may be the main reason for the vast growth of their applications. Such systems are used in different kinds of fields such as fire fighting, forestry, dismantling bombs, toxic waste cleanup, transportation of nuclear materials, and even in military. Attempts by robotics research community for attaining the dynamical equations of motion for mobile robots have led to successful results. However, most of these results suppose that one or more manipulator arms are attached atop the platform at a base fixed point. For instance, a systematic method for the kinematics and dynamics modeling of a two degree-of-freedom (DOF) Automated Guided Vehicle (AGV) has been presented by Saha and Angeles, [1]. The Lagrangian formulation has been applied to generate the explicit dynamics model of space free-flying robots, [2]. The kinematics of differentially-driven and car-like platforms has been described by Papadopoulos and Poulakakis, [3]. Moosavian and Eslamy have investigated the dynamics as well as Multiple Impedance Control of a differentially-driven multiple-arm robotic system to manipulate a common object, [4]. Forward Recursive Formulation for the dynamics of multibody systems has been employed to obtain the governing equation of the nonholonomic mobile manipulator systems, [5]. The Kane's approach has been reported for the dynamics of cooperating mobile manipulators moving on flat surface while transporting an elastic object, [6].

On the other hand, the dynamical analysis of parallel manipulators is more complicated due to the existence of multiple close-loop chains. Several approaches have been proposed, including Newton-Euler formulation, [7], Lagrangian formulation, [8], and the principle of virtual work, [9].

In this paper, a new hybrid robot is suggested which exploits the advantages of both serial and parallel manipulators mounted on a mobile platform, Fig. 1, where the planar parallel manipulator can move the base point of the serial robot with respect to the mobile platform to fulfill the system stability. The suggested ST mechanism has no singularity in its workspace while precisely executes positioning the base point of the serial robot. The general structure of the robotic system is described, and the kinematics of the system is demonstrated in Section II. Then, in section three a combined Newton-Euler and Lagrange
formalism is developed to derive the closed form dynamics model of the system as well as mutual interaction wrenches. Next, the verification procedure is detailed and finally, several concluding remarks are presented.

II. KINEMATICS OF THE HYBRID SERIAL-PARALLEL WHEELED ROBOTIC SYSTEM

To date, mobile manipulators which consist of one or more manipulator arms mounted on a mobile base have been extensively studied, [4- 7]. Although the workspace of such systems due to the mobility of the base is very large, most of them utilize serial manipulator(s) attached at a fixed point of the base. However, in this research the attachment point of the serial arm to the platform is allowed to move in the base plane. Such a feature provides a higher dexterity for the robot and enhances its postural stability in the emergent and hazardous situations, [11]. A parallel manipulator (parallel platform) has the advantages of higher stiffness, higher payload capacity, and lower inertia to the manipulation problem than a comparable serial manipulator, at the price of a smaller workspace and more complex mechanism. Based on the aforesaid advantages, a novel planar parallel manipulator (as depicted in Fig. 2) is selected as an interface between the serial manipulator and the mobile base which can move the attachment point of serial arm accurately. This parallel manipulator is made from a reference Triangle (T) and a moving Star (S). These two elements are connected through three limbs while each of them consists of three joints, i.e. Prismatic-Revolute-Prismatic (PRP) joints. The serial robot is mounted at the center of star part which is called hereafter point ‘F’. In order to analyze the kinematics of the system, the overall robotic system is divided into three modulus including mobile base, parallel manipulator and serial arm.

A. Differentially-Driven Platform

Motion of a reference point ‘P’ on the mobile platform is used to describe the system translation with respect to an inertial frame of reference, (XYZ), see Fig. 3. The wheeled platform is differentially-driven, and so the two left and right wheels are active and independent, both driving and steering the system, while the other two front and rear wheels are passive. The point ‘G’ denotes the platform’s center of gravity. Herein, the goal is to calculate the velocity of points ‘P’ and ‘G’ in terms of the actuating velocities of wheels, i.e. \( \dot{\theta}_r, \dot{\theta}_l \). To this end, the velocity of point ‘G’, described in platform attached frame, is written in two manners in which once, \( \dot{^i\vec{V}}_G \) is calculated in terms of the velocity of point ‘B’ and it is then computed versus the velocity of point ‘A’. Next, by equating these obtained relations the following results are achieved:

\[
(1) \phi_1 = \frac{r}{b}(\dot{\theta}_r - \dot{\theta}_l)
\]

\[
(2) \dot{^i\vec{V}}_G = \left[ \frac{r}{2}(\dot{\theta}_r + \dot{\theta}_l) 0 0 \right]^T
\]

where \( \phi_1 \) and \( r \) represent the platform angular velocity and the radius of the robot’s wheels respectively.

Notice that the left superscript denotes the frame in which the variable has been described; e.g. \( \dot{^i\vec{V}}_G \) represents absolute linear velocity of point ‘G’ which has been described in platform attached frame. If one wishes to find the description of linear velocity of point ‘G’ in inertial frame then \( \dot{^i\vec{V}}_G \) is pre-multiplied by \( ^I\vec{R}_p \) (the rotation matrix of platform with respect to inertial frame), which gives

\[
\dot{\vec{V}}_G = \dot{^i\vec{V}}_G \left( ^I\vec{R}_p \right) \]

Also, the velocity of point ‘P’ can be written as

\[
\dot{^i\vec{V}}_r = \dot{\vec{V}}_G + \dot{^i\vec{V}}_{G/P} = \frac{r}{2}(\dot{\theta}_r + \dot{\theta}_l)i + (\phi_1 \hat{k} \times \hat{i}) = \frac{r}{2}(\dot{\theta}_r + \dot{\theta}_l)i + \phi_1 \hat{j} \]

In a similar manner, using \( ^I\vec{R}_p \), the velocity of point ‘P’, in matrix form, can be written as

\[
\dot{\vec{V}}_P = \begin{bmatrix}
\frac{r \cos(\phi_1)}{2} + \frac{lr \sin(\phi_1)}{b} \dot{\theta}_r \\
\frac{r \sin(\phi_1)}{2} - \frac{lr \cos(\phi_1)}{b} \dot{\theta}_r \\
r \frac{\dot{\theta}_r}{2} + \frac{lr \dot{\theta}_r}{b} \\
0
\end{bmatrix}
\]

Furthermore, the velocity of point ‘G’ can be shown in terms of velocity components of point ‘P’ as below

\[
\dot{^i\vec{V}}_G = \dot{\vec{V}}_P + \dot{^i\vec{V}}_{G/P} = X_P \hat{i} + Y_P \hat{j} + (\phi_1 \hat{k} \times \hat{i}) \]

or in the matrix from as

\[
\dot{^i\vec{V}}_G = \begin{bmatrix}
X_P + l \phi_1 \sin(\phi_1) \\
Y_P - l \phi_1 \cos(\phi_1) \\
0
\end{bmatrix}
\]
Next, using IRP the velocity of point ‘G’ can be described in the platform attached frame as
\[
\vec{V}_G = R \vec{V}_G = \begin{bmatrix} \dot{X}_G \cos(\phi) + \dot{Y}_G \sin(\phi) \\ -\dot{X}_G \sin(\phi) + \dot{Y}_G \cos(\phi) - l \dot{\phi} \\ 0 \end{bmatrix} \tag{7}
\]
Moreover, as it is assumed the platform does not slip sideways, it can be stated that the velocity of ‘G’ is along the longitudinal axis of the vehicle. This makes that the second entry of \(\vec{V}_G\) equals zero, that is
\[
\dot{X}_G \sin(\phi) - \dot{Y}_G \cos(\phi) + l \dot{\phi} = 0 \tag{8}
\]
The above equation is a non-integrable equation and cannot be formed into an algebraic constraint.

![Fig. 4: The ST mechanism and the associated coordinates.](image)

![Fig. 5: The auxiliary triangle connecting points \(D_1, D_2,\) and \(D_3.\)](image)

### B. Parallel ST Mechanism

In this subsection, the forward differential kinematics of suggested planar parallel manipulator is studied. It should be mentioned that for the first time the Double Triangle (DT) mechanism was suggested by Daniali et al., [12]. The DT mechanism is made of a fixed and moving triangle. Then, ST mechanism as a modified form of DT mechanism was suggested by Hunt et al., [13] and herein for the first time is adopted in the context of mobile manipulators. To ease of discussion and formulation, the whole relations are calculated with respect to the platform attached frame, see Fig. 4. The triangle has three equal sides. Each of which is 1m.

Since, the whole sides of triangle are equal and each of them is 1 (m) long, then the coordinates of points \(D_1, D_2,\) and \(D_3\) can be calculated as
\[
\begin{align*}
D_1 &= \left[ \frac{-\sqrt{3}}{2} q_1 \quad \frac{1}{2} q_1 \quad 0 \right]^T \tag{9a} \\
D_2 &= \left[ \frac{-\sqrt{3}}{2} \quad \frac{1}{2} - q_2 \quad 0 \right]^T \tag{9b} \\
D_3 &= \left[ \frac{-\sqrt{3}}{2} + \frac{\sqrt{3}}{2} q_3 \quad \frac{-1 + \frac{1}{2} q_3}{2} \quad 0 \right]^T \tag{9c}
\end{align*}
\]
Note also that \(\varphi_2\) denotes the rotation angle of ST mechanism with respect to the platform. Hence, if \(q_1=0\) then point \(D_1\) coincide with point ‘\(P\)’, see Fig. 4. Furthermore, in this configuration \(\beta = 180^\circ\) and \(\varphi_2=0\), therefore
\[
\varphi_2 = \beta - 180^\circ \tag{10}
\]
Considering triangle \(D_1D_2D_3\) and Fig. 5, in triangle \(FD_3D_1\) the following equation can be written based on law of sines
\[
\frac{FD_3}{\sin(D_1')} = \frac{D_1D_1'}{\sin(120^\circ)} \tag{11}
\]
In the same manner, one can write
\[
\frac{FD_3}{\sin(D_2')} = \frac{D_2D_2'}{\sin(120^\circ)} \tag{12}
\]
If one equates Eqs. (11) and (12) then it gives
\[
\frac{D_1D_1'}{\sin(D_1')} = \frac{D_2D_2'}{\sin(D_2')} \tag{13}
\]
Besides, in triangle \(D_1D_2D_3\), using law of cosines gives
\[
D_1 = \cos^{-1} \left[ \frac{D_1D_1^2 + D_2D_2^2 - D_3D_3^2}{2D_1D_2D_3} \right] \tag{14a}
\]
The variable ‘\(k\)’ is defined as
\[
K = D_2 + D_1 - 60^\circ \tag{15}
\]
Also, the following relations can easily be obtained
\[
D_1' = D_1 - D_1^* \tag{16}
\]
\[
D_2' + D_2^* = 60^\circ \tag{17}
\]
Using Eqs. (15), (16) and (17) the following is obtained
\[
D_1' = K - D_2^* \tag{18}
\]
Also, exploiting Eqs. (13), (15) along with (17) leads to
\[
D_2^* = \tan^{-1} \left[ \frac{D_3D_3' \cdot \sin(K)}{D_2D_2' + D_1D_1' \cdot \cos(K)} \right] \tag{19}
\]
On the other hand, the angle of \(D_2D_3\) with respect to the positive direction of \(X_p\) can be written as
\[
\angle(D_2D_3, X_p) = \alpha = \tan^{-1} \left( \frac{y_{D_3} - y_{D_2}}{x_{D_3} - x_{D_2}} \right) \tag{20}
\]

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On the basis of Fig. (4), the following relations can be written
\[
\beta_2 = \alpha + D_2^* \quad (21a)
\]
\[
\beta_3 = \beta_2 + 120^\circ \quad (21b)
\]
Based on Fig. (4), and relations (14), the coordinates of point ‘F’ can be written as
\[
\begin{bmatrix}
\frac{-\sqrt{3}}{2} + \frac{\sqrt{3}}{2} q_3 + FD_1 Cos(\beta_3) \\
\frac{1}{2} - \frac{1}{2} q_3 + FD_1 Sin(\beta_3) \\
0
\end{bmatrix}
\quad (22)
\]
If Eqs. (9) to (21) are utilized in a chain form then the coordinate of point ‘F’ relative to ‘P’ is obtained versus the variables \(q_1, q_2\), and \(q_3\) analytically. Other than \(PF\), the other above mentioned variables can also be written in terms of \(q_1, q_2\), and \(q_3\) in a closed-form. In order to complete the forward kinematics of the ST mechanism the linear velocity of point ‘F’ as well as the absolute angular velocity of parallel platform should be obtained. The absolute velocity of point ‘F’ which has been described in the platform frame can be written as
\[
\begin{align*}
\dot{p}_F^P &= \dot{p}_P^P + \dot{p}_{rel}^P + \dot{\phi}_p \times PF \\
&= \begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2 \\
\dot{q}_3
\end{bmatrix}
\end{align*}
\quad (23)
\]
where \(\dot{p}_{rel}^P\) represents the velocity of point ‘F’ with respect to a rotating observer at point ‘P’. Also, \(\dot{\phi}_p\) denotes the absolute angular velocity of platform which is equal to \(\phi_\hat{K}\). Moreover, \(\dot{p}_P^P\) can be substituted from Eq. (4). In order to find \(\dot{p}_{rel}^P\), it is sufficient to simply differentiate the entities of Eq. (22) with respect to time. In this manner, the linear velocity of point ‘F’, i.e. \(\dot{p}_P^F\), is obtained versus five actuating velocities, i.e. \([\theta_1, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3, \theta_2]^T\). Next, the absolute linear velocity of point ‘F’ is projected into the inertial frame as follows
\[
\ddot{q}_1 = R_i \dot{p}_P^F 
\quad (24)
\]
It is emphasized that due to lots of computations needed in the above mentioned procedures, the software Maple 6, is adopted to accomplish the required nested form calculations. Now, the calculation of the angular velocity and acceleration of ST mechanism is detailed. Remembering Eq. (10), \(\varphi_2 = \beta_2 - 180^\circ\), and noting that \(\beta_2 = \beta_2 + 240^\circ\), the following result is obtained
\[
\varphi_2 = \beta_2 + 60^\circ
\quad (25)
\]
where \(\beta_2\) is obtained from Eq. (21a). Hence, it is possible to compute this angle versus the variables \(q_1, q_2\), and \(q_3\) analytically via nested formulas 9-22. Therefore, the angular velocity and acceleration of ST mechanism (and equivalently frame \(x_0 y_0 z_0\)) are obtained as
\[
\begin{align*}
\ddot{\phi}_b &= \begin{bmatrix} 0 & 0 & \phi_1 + \phi_2 \end{bmatrix}^T \\
\ddot{\phi}_b &= \begin{bmatrix} 0 & 0 & \phi_1 + \phi_2 \end{bmatrix}^T
\end{align*}
\quad (26, 27)
\]

C. Serial Manipulator Arm

To analyze the kinematics of the manipulator arm, first several coordinate frames are attached to the various links of the arm, where corresponding DH parameters can be obtained.

Next, via outward kinematics formulations [11], the linear and angular velocity/acceleration of various links can be computed. Therefore, it can be observed that by performing the outward analysis (from star part of ST mechanism to the end-effector of serial robot) the linear/angular velocity and acceleration of all parts of the serial robotic arm are calculated versus actuating velocities and accelerations, i.e.
\[
\begin{align*}
\ddot{\theta}_1 &= \begin{bmatrix} \ddot{\theta}_1, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3, \theta_2 \end{bmatrix}^T \\
\ddot{\theta}_1 &= \begin{bmatrix} \ddot{\theta}_1, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3, \theta_2 \end{bmatrix}^T
\end{align*}
\]

After that, it is possible to compute the inertial forces/torques exerted to the whole bodies of the serial robot in terms of actuating velocities and accelerations as will be discussed in the next section.

III. DYNAMICS OF THE HYBRID SYSTEM

In this section, the dynamics of the system is derived. To this end, the whole system is divided into two main modules including the serial robotic arm and the integrated platform and ST mechanism. Therefore, the mutual interaction wrenches between the serial robot and the rest of the system is obtained explicitly. Moreover, these interaction dynamic wrenches can be employed for tip-over stability analysis of the overall robotic system, [11]. So, the Newton-Euler formulation is adapted for the serial robot module. Note that for this system the generalized coordinates used for the system description are as follows
\[
\ddot{q} = [X_p, Y_p, \varphi_1, q_1, q_2, \varphi_2, \theta_1, \theta_2, \theta_3]^T
\]

A. Serial Manipulator Arm Dynamics

After the kinematical computations performed in the previous section, it is possible to calculate the inertial forces/torques exerted to the whole bodies of the serial robot. Next, by using the obtained inertial forces/torques and inward Newton-Euler’s iterations, [11], the interaction wrenches generated at the whole joints of the serial robot are computed as follows
\[
\ddot{\theta}_1 = \begin{bmatrix} \ddot{\theta}_1^111 + \ddot{\theta}_1^1 \end{bmatrix} + \ddot{\theta}_1 \quad (28a)
\]
\[
\ddot{\theta}_1 = \begin{bmatrix} \ddot{\theta}_1^111 + \ddot{\theta}_1^1 \end{bmatrix} + \ddot{\theta}_1 \quad (28b)
\]
where, \( i \hat{f}_i \) and \( i \ddot{n}_i \) represent the force and torque exerted on the link \( 'i' \) from link \( '(i-1)' \) which have been described in \( i \)-th link attached frame. It is pointed out that the above required calculations are performed via Maple 6. Finally, the required torque for the desired motion of each joint is obtained as
\[
\tau_i = \hat{n}_i \times \ddot{Z}_i
\]  
(28c)

Moreover, the force/torque exerted by the robotic arm to the ST mechanism is obtained as
\[
\bar{F} = - R_1 \hat{f}_1 \\
\bar{N} = - R_1 \hat{n}_1
\]  
(29a)

\( B. \) Integrated Platform and ST Parallel Manipulator Dynamics

To derive the dynamics of this part of the system the Lagrangian approach is used. As the whole integrated subsystem moves parallel to the horizontal plane and during the motion its center of mass height does not change, hence the potential energy taken equal to zero. The platform’s kinetic energy can be written as
\[
T_p = \frac{1}{2} m_p \ddot{V}_p \cdot \ddot{V}_p + \frac{1}{2} J_\phi \dot{\phi}_i^2
\]  
(30)

where \( m_p \) and \( J_\phi \) are the mass and mass moment of inertia of platform about the point ‘G’ respectively. Also, the ST’s kinetic energy can be written as
\[
T_S = \frac{1}{2} m_z (\ddot{V}_z + \dot{\phi}_z)^2
\]  
(31)

where \( m_z \) and \( J_\phi \) are the mass and mass moment of inertia of Star about the point ‘F’ respectively. Consequently, the kinetic energy of the integrated subsystem is
\[
T = T_p + T_S
\]  
(32)

Notice that the generalized coordinates needed to describe the pose of the integrated subsystem is as follows
\[
\hat{q}_i = [X_p, Y_p, \phi_i, \dot{q}_1, \dot{q}_2, \dot{q}_3]^T
\]  
(33)

The Lagrange equations of motion for the integrated parts can be written as
\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{r}_k} \right) - \frac{\partial T}{\partial r_k} = \sum_{i=1}^{m} \lambda_i a_{ik} \quad (K = 1, \ldots, 6)
\]  
(34)

where \( r_k \) is the \( k \)-th generalized coordinate and \( Q_k \) is the corresponding generalized force. Also, \( \lambda_i \)'s are Lagrange multipliers and ‘\( m \)’ is the number of system constraints. Herein, \( m=1 \) due to the existence of nonholonomic constraint. The nonholonomic constraint, i.e. Eq. (8), can be written in the matrix form as
\[
A \ddot{q}_i = 0
\]  
(35)

where
\[
A = \begin{bmatrix}
\sin(\phi_i) & -\cos(\phi_i) & 1 & 0 & 0 & 0
\end{bmatrix}
\]  
(36)

Substitution of (32) into (34) gives
\[
M \ddot{q}_i + C + \lambda A \dot{q}_i = \tau
\]  
(37)

where \( M \) is a \( 6 \times 6 \) mass/inertia matrix, \( C \) is \( 6 \times 1 \) vector of nonlinear velocity dependent terms.

On the other hand, the generalized speeds can be written in terms of the actuating velocities as
\[
\dot{q}_i = S \dot{\nu}
\]  
(38)

where
\[
S = \begin{bmatrix}
\frac{r}{2} \cos(\phi_i) + \frac{l_r}{b} \sin(\phi_i) \\
\frac{r}{2} \cos(\phi_i) - \frac{l_r}{b} \sin(\phi_i) \\
0 \\
\frac{r}{2} \sin(\phi_i) + \frac{l_r}{b} \cos(\phi_i) \\
0 \\
\frac{r}{b}
\end{bmatrix}
\]  
(39)

\[
\dot{\nu} = [\dot{\theta}_1, \dot{\theta}_e, \dot{q}_1, \dot{q}_2, \dot{q}_3]^T
\]  
(40)

Substituting Eq. (38) into (35) leads to the following result
\[
A S = O_{6 \times 5}
\]  
(41)

Now, using the Natural Orthogonal Complement method, [1] and pre-multiplying Eq. (37) by \( S^T \) and using (41) gives
\[
S^T M \ddot{q}_1 + S^T C = S^T \tau
\]  
(42)

In the above equation, the LHS (Left Hand Side) is completely known. Yet, the RHS has not been determined. Moreover, Eq. (42) represents five equations. While the number of generalized variables of the integrated subsystem is \( 6 \). Consequently, the nonholonomic constraint is differentiated with respect to time as
\[
\sin(\phi_1) \ddot{X}_p - \cos(\phi_1) \ddot{Y}_p + \frac{\partial \phi_i}{\partial \phi_1} \ddot{X}_p \cos(\phi_i) + \frac{\partial \phi_2}{\partial \phi_1} \ddot{Y}_p \sin(\phi_i) = 0
\]  
(43)

The RHS of Eq. (42) can be determined by noting the fact that the infinitesimal work of generalized forces should be equal to that of the external forces/torques acted on the integrated subsystem.

Finally, by augmenting the serial robot Eqs. 28, the final equations of the robot is obtained as
\[
\ddot{\mathbf{r}}_i = \ddot{\mathbf{r}}_i(\mathbf{q}, \dot{\mathbf{q}}) + C_{\mathbf{q} \dot{\mathbf{q}}} = \tau_{\mathbf{q} \dot{\mathbf{q}}}
\]  
(44)

where
\[
\tau = [\tau_1, \tau_2, \tau_3, F_1, F_2, F_3, \tau_1, \tau_2, \tau_3]^T
\]  
(45)

IV. VERIFICATION OF THE OBTAINED DYNAMICS MODEL

In order to investigate validation of the obtained model several rational inputs are applied, and the obtained output results are analysed. It will be seen that the obtained outputs are logical and expected and hence the soundness of modeling approach is guaranteed.

A. First Maneuver

In this Manoeuvre, two different values of actuating torques for the platform are considered as cases one and two. The initial values for the robot pose in both cases are depicted in Table I. In addition, the whole initial velocities are set to be zero. Besides, the whole actuating torques for both cases are set to be zero excluding the ones for the wheels of the platform. For the first case \( \tau_e = 2000 (N.m) \) while the
corresponding ones for the second case are \( \tau_i = \tau_r = 4000 \, (N.m) \).

### Tables I: Initial values for the two cases.

<table>
<thead>
<tr>
<th>( X_P )</th>
<th>( Y_P )</th>
<th>( \varphi_1 )</th>
<th>( q_1 )</th>
<th>( q_2 )</th>
<th>( q_3 )</th>
<th>( \theta_1 )</th>
<th>( \theta_2 )</th>
<th>( \theta_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{3}/2 ) (m)</td>
<td>0</td>
<td>0.5 (m)</td>
<td>0.5 (m)</td>
<td>0</td>
<td>90°</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The responses of the system for both cases are shown in Figs. 6 and 7. Since, the actuating torques for both wheels are the same in both the sense and value, it is expected that \( X_P \) is increasing, whereas \( Y_P \) remains constant. Additionally, in the second case, \( XP \) increases more as the actuating torque is greater as well.

![Fig. 6: The comparative diagram of \( X_P \) versus time for two cases.](image1)

![Fig. 7: The trajectory of \( Y_P \) versus time.](image2)

### B. Second Manoeuvre

In this section, the correctness of the obtained dynamics for the ST platform is examined. To this end, again the robotic system is considered with the initial pose mentioned in Table I. Also, in this manoeuvre two similar cases are analysed. In these cases, the whole actuating torques are set to be zero except for those of ST mechanism, i.e. \( F_1, F_2 \) and \( F_3 \). In the first case \( F_1 = F_2 = F_3 = 200 \, (N) \) while in the second case \( F_1 = F_2 = F_3 = 400 \, (N) \). The variations of \( q_1 \) and \( q_2 \) are shown in Fig. 8. As it is seen due to the positive forces all \( q_i \)'s have been increased. Also, since the amount of actuating forces in the second case is greater, hence the corresponding amount of \( q_i \)'s is greater.

![Fig. 8: The response of the ST mechanism in the second manoeuvre.](image3)

### V. CONCLUSIONS

A novel serial-parallel wheeled mobile robot was proposed to fulfill stable motion for handling heavy objects. A planar parallel manipulator was used as an interface between the serial manipulator and the mobile base which can move the attachment point of serial arm with respect to the mobile base. To develop kinematics and dynamics model of the system, first the whole system was divided into three parts and the kinematics of the overall system was analysed in a modular manner. Next, in order to derive the dynamics of the system, the robot was divided into two parts, i.e. the serial arm and the integrated platform together with the ST parallel platform. A combined Newton-Euler and Lagrangian formalism was developed to derive the system dynamics model. Besides, the presented technique provides the mutual interaction wrenches between the serial arm and the integrated platform which can be utilized for postural stability evaluation of the system. Finally, the obtained model was verified, using some benchmark inputs, where the obtained outputs of the system were analysed.

### REFERENCES


