High resolution LFMCW radar system using model-based beat frequency estimation in cable fault localization

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Abstract: A linear frequency modulated continuous wave (LFMCW) radar is introduced to localize the impedance discontinuities on the instrument cable used in nuclear plants. The LFMCW reflectometry uses a phenomenon that electromagnetic pulses are reflected at the impedance discontinuities to localize impedance discontinuity points on the cable. For localizing impedance discontinuities, time delays between the incident signal and the reflected signals from the impedance discontinuities have to be measured. The LFMCW is modeled by time-varying auto-regressive (AR) model. From the coefficients of the AR model, instantaneous frequency is estimated by the Kalman filtering to calculate the time delays. The performance of the proposed method is verified by experiments.

Keywords: LFMCW radar, reflectometry, chirp signal, auto-regressive model, fault locating, instantaneous frequency estimation

Classification: Electronic instrumentation and control

References

1 Introduction

In the diagnosis of a cable system, fault localization is crucial in order to maintain the integrity. Therefore, various cable fault localizing methods have been developed. Most of the fault localization techniques are based on reflectometry, a type of range measurement system. The system transmits the designed electromagnetic wave to the cable and measures the reflected signal from any impedance discontinuities present. Using the time delay between the incident signal and the reflected signal, the system calculates the distances from the measurement point to the impedance discontinuities. Time domain reflectometry (TDR) [1], frequency domain reflectometry (FDR) [2], and joint time frequency domain reflectometry (JTFDR) [3] are the predominant reflectometry techniques for cable fault localization. Compared to TDR and FDR, JTFDR, which uses a Gaussian chirp as the incident signal and time-frequency correlation as the receiver, has the advantages of simultaneously improving the range resolution and the SNR. However, JTFDR is disadvantageous in that it requires a significant amount of computational burden for calculating time-frequency correlation, a difficulty generating a Gaussian chirp signal, which has high frequency and power, and the range resolution degradation due to blind spots. Therefore, we introduce a linear frequency modulated continuous wave (LFMCW) radar system [4] for localizing faults in a cable system.

LFMCW radar is a technique used to obtain range information from a frequency modulated continuous signal. Conventional LFMCW radar systems use a fast Fourier transform (FFT) to estimate the beat frequency, which contains range information [4]. The resolution of FFT is limited by the data length, and the spectral leakage can be misinterpreted as the location of the reflected signal [5]. In a cable fault localizing application, the cable being tested can be a lossy dispersive medium. The propagation characteristics of the cable induce the nonlinearity in the frequency sweep, resulting in beat frequency variation with time [6]. Therefore, we used the time-varying auto-regressive (AR) model [7] and Kalman filtering to estimate the instantaneous beat frequency of the LFMCW radar system and calculated the fault distance using the estimated beat frequency.

2 LFMCW radar system for cable fault localization

The incident signal for the LFMCW radar is the linear chirp signal. The instantaneous frequency (IF) of the incident signal is varied like a sawtooth wave. The linear chirp signal is given by:

\[ s_k = A \cos(0.5\xi k^2 + \omega_0 k + \phi) \]  

(1)
where $k = 1, 2, \ldots, N$, $A$ is the amplitude, $\xi$ is the normalized angular frequency sweep rate, $\omega_0$ is the normalized initial angular frequency and $\phi$ is the angle. The schematic of the LFMCW radar system for cable fault localization is shown in Fig. 1. We used directional couplers to make a homodyne receiver [2]. The incident signal propagated through the instrument cable is reflected from the impedance discontinuities in the cable. The reflected signal is represented as:

$$r_k = \sum_{i=1}^{p} \Upsilon_i A \cos(0.5\xi(k - d_i)^2 + \omega_0(k - d_i) + \phi_i)$$

where $p$ is the number of impedance discontinuities, $\Upsilon_i$ is the parameter related to the attenuation and reflection, $d_i = 2L_i/v_p$ is the time delay between the incident and the $i$th reflected signal, $L_i$ denotes the distance from the measurement point to the $i$th impedance discontinuity point, and $v_p$ is the propagation velocity of the cable.

The reflected signal is mixed with the incident signal. The mixed signal consists of two sinusoids which occupy different frequency bands. One of the sinusoids is the second harmonic of the incident signal, and the other sinusoid has the beat frequency. The second harmonic signal can be removed using lowpass filtering, and the output of the lowpass filter contains the sinusoid which has the beat frequency. The lowpass filter output is given by

$$m_{LF}^{i,k} = M_i \cos(\omega_{i,b}k - \phi_{i,m})$$

where $M_i = \frac{1}{2} \Upsilon A^2$ is the amplitude, $\omega_{i,b} = \xi d_i$ is the normalized angular beat frequency, and $\phi_{i,m}$ is the angle of the lowpass filter output. The beat frequency is also shown in Fig. 1, where the interval B-C denotes the beat frequency related to the range information. The beat frequency depends on the time delay between the incident and reflected signals.

### 3 AR modeling for estimating instantaneous beat frequency

In order to estimate the beat frequency of the LFMCW radar system, we modeled the lowpass filter output as a time-varying AR model and used Kalman filtering to estimate the coefficients of the AR model. The time-varying AR model for the lowpass filter output is given by:

$$m_{LF}^{i,k} = - \sum_{i=1}^{q} a_{k,i} m_{LF}^{i,k-i} + v_k$$

where $a_{k,i}$ is the time-varying AR coefficient at time $k$, $q$ denotes the model order, and $v_k$ is the observation error. The time-varying AR model can be represented as the measurement equation

$$y_k = H_k x_k + v_k$$

where $H_k = [-m_{LF}^{k-1}, \ldots, -m_{LF}^{k-q}]$ and $x_k = [a_{k,1}, \ldots, a_{k,q}]^T$.

In our model, the AR coefficients are set as state variables. The transition between the states is modeled as a random walk model, since there is no a
priori information for the state transition. The random walk model for the states is given as:

$$x_{k+1} = x_k + w_k$$  \hspace{1cm} (5)

where $w_k$ is the zero-mean white Gaussian noise with known covariance $Q$. In order to estimate the time-varying AR coefficients, we used Kalman filter, which is an unbiased minimum variance estimator. The Kalman filtering equations are as follows:

$$P_{k|k-1} = FP_{k-1|k-1}F^T + Q,$$

$$K_k = P_{k|k-1}H_k^T(H_kP_{k|k-1}H_k^T + R)^{-1},$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - H_k\hat{x}_{k|k-1}),$$

$$P_{k|k} = (I - K_kH_k)P_{k|k-1},$$

where $P_{k|k-1}$ is the a priori error covariance, $K_k$ represents the gain of Kalman filtering, $R$ is the observation error covariance matrix, $\hat{x}_{k|k}$ is the a posteriori state estimate, $\hat{x}_{k|k-1}$ is the a priori state estimate, $P_{k|k}$ is the a posteriori error covariance, and $I$ is the identity matrix. Since the states are estimated using the random walk model, the variance can be very high. Therefore, a smoothing process for the estimates is needed. Our smoothing process [7] is given as:

$$\hat{x}_{k+1} = (1 - c_k)\hat{x}_{k-1} + c_k\hat{x}_{k+1}$$  \hspace{1cm} (6)

where

$$c_k = \frac{C(\hat{x}_{k+1} - \hat{x}_k)^2}{1 + C(\hat{x}_{k+1} - \hat{x}_k)^2}$$  \hspace{1cm} (7)

c_k \in (0, 1)$ is the adaptation parameter and $C$ denotes a positive constant.
The estimated time-varying power spectrum of the lowpass filter output is given as:

$$P_k(f) = \frac{\sigma_v^2}{1 + \sum_{i=1}^{q} \tilde{a}_{k,i} e^{-j\omega_i}}$$  \hspace{1cm} (8)$$

where $\tilde{a}_k$ is the smoothed estimated AR coefficients, $q$ is the order of the time-varying AR model, and $\sigma_v^2$ is the variance of the observation error. Using (8), we can obtain the time-varying power spectrum of the lowpass filter output. The IF is defined as the frequency with the maximum value of the time-varying power spectrum. The estimated beat frequency is defined by

$$\bar{f}_b = \mathcal{E}(\max_f[P_k(f)])$$  \hspace{1cm} (9)$$

where $\mathcal{E}(\cdot)$ is the time average operator in intervals B and C in Fig. 1. The beat frequency is the time averaged value of the estimated IF. The fault distance is calculated from the estimated beat frequency as follows:

$$L = 0.5\bar{f}_b v_p / \xi$$  \hspace{1cm} (10)$$

4 Experiments

The LFMCW radar system for cable fault localization consisted of an arbitrary waveform generator (Tektronix, AWG 5002C), a digital phosphor oscilloscope (Tektronix, DPO 7104), and two directional couplers (Minicircuits, ZFDC-20-5). The fault localization tests were conducted on the 600V-QFR-PN-CMS instruments cable for a nuclear power plant whose length was 31.46 m. The propagation velocity of the cable was $1.5 \times 10^8$ m/s. In the experiments, we set the end of the cable as the impedance discontinuity of the cable. The frequency bandwidth of the incident chirp signal was set as 20 MHz - 36 MHz. Since the attenuation of the instruments cable depends on the frequency, the bandwidth was limited to the low-frequency band. The time duration of the incident chirp signal was 1 $\mu$s. The AR model was determined to be 20-order model. We conducted the fault localization experiment 10 times.
5 Results and conclusion

The estimated time-varying power spectrum is shown in Fig. 2. The lowpass filter output signal, the estimated instantaneous beat frequency, and periodogram are shown in Fig. 3. The averaged beat frequency was 6.604 MHz. Therefore, the estimated fault distance based on the proposed method was 30.9536 m with an error rate of 1.600%. The first peak in the periodogram denotes the beat frequency and its corresponding frequency was 7.08 MHz. The estimated fault distance calculated using periodogram was 33.1875 m with an error rate of 5.490%. Non-stationary characteristics of the lowpass output induces the error of fault localization by using periodogram. Therefore, the proposed method gives a higher range resolution than the conventional methods based on FFT and we can obtain a more accurate fault distance.

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