Adaptive Fault-tolerant Control with Control Allocation for Flight Systems with Severe Actuator Failures and Input Saturation

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Abstract—This paper proposes a novel adaptive fault tolerant control strategy for flight systems which have no sufficient actuator redundancy after severe actuator failures. In addition to distributing the control signals to the remaining actuators based on the effectiveness of actuators, this strategy utilizes model reference adaptive control (MRAC) to compensate for the tracking error caused by severe failures. Furthermore, an improved weighting algorithm and an anti-saturation controller are developed to compensate for the saturation error. Finally, a simulation of the satellite launch vehicle is conducted to demonstrate the effectiveness of the proposed strategy. Compared to the traditional fault tolerant control with control allocation method, the proposed strategy gives better performance.

I. INTRODUCTION

Actuator failures of aircraft significantly reduce reliability performance and even can cause catastrophic accidents. Based on this consideration, designers often add redundant actuators to aircraft, which can provide degrees of freedom to design fault tolerant control (FTC) systems [1]. Zhang presented an overview of historical, current, and future developments on FTC systems in [2]. Control allocation (CA) is an effective method that can manage actuator redundancy, i.e., distribute the virtual control law requirements to the redundant actuators in the best manner while accounting for their constraints [3]. Particularly, when some of the actuators lose effectiveness or become damaged, CA is able to utilize the remaining healthy actuators to ensure that the aircraft performs well. Several popular approaches and applications have been developed for CA. Oppenheimer gave a survey of linear control allocation techniques, and presented the methods in detail in [4]. Bodson evaluated the performance and computational requirements of optimization methods for control allocation in [5]. Härkegård compared control allocation with optimal control for solving actuator redundancy in [6]. Arun Kishore considered CA along with disturbance rejection and gave an algorithm that updated the weighting matrix to deal with actuator limits in [7].

CA is widely used to realize FTC, especially in aerospace systems. Zhou introduced two reconfigurable control allocation schemes and illustrated them with an unmanned aerial vehicle under FTC (solid line) and AFTC (dotted line).

II. PROBLEM STATEMENT

Fig. 2 describes the AFTC strategy. The baseline controller and the control allocation module are designed according to [7], and it offers the initial value for the adaptive fault tolerant controller. The objective of the adaptive fault tolerant controller is to track the reference model with desired performance when the system experiences a severe failure. The control reallocation module is

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able to allocate the virtual input based on the actuator effectiveness matrix $\Delta$. In addition to improving the weighting algorithm, an anti-saturation feedback controller is proposed to stabilize the system when it experiences a severe failure.

### A. Plant Description

This paper considers a linear time-invariant system subject to actuator failures described by

$$
\begin{align*}
\dot{x}(t) &= Ax(t) + B_u \Delta u_a(t) \\
y(t) &= Cx(t)
\end{align*}
$$

where $x(t) \in \mathbb{R}^m$, $u_a(t) \in \mathbb{R}^q$ and $y(t) \in \mathbb{R}^n$ are the state vector, actual control input entering the system and controlled output, respectively. $A, B_u, C$ are constant matrices of appropriate dimensions. $\Delta \in \mathbb{R}^{q \times q}$ is a piece-wise constant uncertain control effectiveness matrix that can be expressed as $\Delta = \text{diag}(\delta_1, \delta_2, \ldots, \delta_q)$, where $0 \leq \delta_i \leq 1$, $i = 1, 2, \ldots, q$ indicates the effectiveness of the $i$th actuator. For example, $\delta_i = 1$ indicates the $i$th actuator is working well; $\delta_i = 0$ means the $i$th actuator has damaged completely; $0 < \delta_i < 1$ implies the $i$th actuator has a failure.

Denoting the saturation error as $\tilde{u}(t) = u_a(t) - u(t)$ and $B_u \Delta = B_f$, the state equation can be modified as

$$
\dot{x}(t) = Ax(t) + B_f \tilde{u}(t)
$$

where $u(t)$ is the control input. The following assumptions on the system are required:

(a) The number of control inputs is greater than the number of control variables, that is, $q > m$.

(b) The matrix $B_f$ can be factorized into a matrix denoted as $B_f \in \mathbb{R}^{m \times n}$ with full column rank and a matrix denoted as $B \in \mathbb{R}^{n \times q}$ with full row rank.

(c) The control input $u(t)$ satisfies

$$
\begin{align*}
\underline{u} \leq u(t) \leq \overline{u}
\end{align*}
$$

where $\overline{u} := [u_{\max_1}, u_{\max_2}, \ldots, u_{\max_q}]^T$ and $\underline{u} := [u_{\min_1}, u_{\min_2}, \ldots, u_{\min_q}]^T$ are the upper bound and lower bound of the actuators respectively.

(d) $(A,B_f)$ is controllable.

Assumption (a) means the aircraft has redundant actuators. Assumption (b) implies the following relations:

$$
B_f \tilde{u}(t) = B_u \overline{u}(t) = B_u \overline{v}(t)
$$

where $\overline{v}(t) \in \mathbb{R}^n$ is defined as the virtual input. Assumption (c) presents the actuator limits of the system.

### B. Control Objective

The control objective is to design an adaptive fault-tolerant feedback controller $\hat{K}(t) \in \mathbb{R}^{n \times m}$ and utilize control reallocation strategy to achieve desired performance when the aircraft experiences a severe failure, and then find a control input $u(t)$ satisfying (3) and (4).

### III. CONTROL REALLOCATION

According to (4), the virtual input is defined as

$$
v(t) = Bu(t)
$$

Different from the traditional control allocation problem, the matrix $B$ incorporates the fault information. Generally, the control reallocation problem can be formulated as the optimization problem [10]:

$$
\min_{u(t)} \ u^T W u_t \quad \text{subject to (4)}
$$

where $W = \text{diag}(w_1, w_2, \ldots, w_q)$ is a diagonal positive definite weighting matrix, and the scalar $w_i$ is the weight of the $i$th actuator.

The solution of problem (6) is [10]:

$$
u(t) = W^{-1} B \hat{u}(t)$$

Equation (7) can be used to distribute the virtual input to the still operating actuators. Unfortunately, the optimal solution is sometimes infeasible due to the actuator limits. To solve this problem, a weighting algorithm is provided in [7]. The scalar $w_i$ is updated when the $i$th actuator exceeds the actuator limits. In order to improve this algorithm, a small positive scalar $0 < \eta \leq 1$ is multiplied by the actuator limits in this paper. With the $\eta$-weighting algorithm, more parameters can be used to update the weighting matrix.

Denoting $\hat{W} = \Theta W \Theta$, where $\Theta = \text{diag}(\theta_1, \theta_2, \ldots, \theta_q)$ is a positive definite matrix and

$$
\theta_i = \begin{cases} 
(1 + \varepsilon) \frac{u_{\max_i}}{u_{\max} - u_{\min}}, & u_i(t) > \eta u_{\max} \\
1, & u_i(t) \leq \eta u_{\max} \\
(1 + \varepsilon) \frac{u_{\min_i}}{u_{\max} - u_{\min}}, & u_i(t) < \eta u_{\min}
\end{cases}
$$

where $\varepsilon$ is a small positive scalar, (7) can be modified as

$$
v(t) = \hat{W}^{-1} B \hat{u}(t)
$$

Unfortunately, this algorithm may become invalid when the aircraft experiences a severe failure. As most actuators experience failures, the saturation errors will be caused in the remaining operating actuators, which may not be eliminated by the $\eta$-weighting algorithm. The larger saturation errors will induce performance degradation and closed-loop instability. To solve this problem, an anti-saturation controller is designed to compensate for the errors when the $\eta$-weighting algorithm is invalid [12].

**Theorem 1.** When a severe failure occurs, the saturation error $\tilde{u}$ can be compensated for by the following anti-saturation controller $v_s$:

$$
v_s = -B \tilde{u}
$$
Proof: Combining (2) with (4), the closed-loop system can be written as
\[ \dot{x} = Ax + B_v v + B_f \hat{u} \\
= Ax + B_v (v_c + v_s) + ... \\
\]
where \( v_c \) is the output of the adaptive fault-tolerant controller, as is shown in Fig. 2. It is clear that substituting (10) into (11) can compensate for the saturation error.

The proof is completed.

IV. CONTROLLER DESIGN

A. Baseline Controller

The baseline controller \( v_{cl} \in \mathbb{R}^n \) is designed as
\[ v_{cl} = K_0 x + K_{ref0} r \]
where \( K_0 \in \mathbb{R}^{n \times m} \) and \( K_{ref0} \in \mathbb{R}^{n \times n} \) denote the state feedback controller and feedforward controller respectively.

The state feedback controller is designed using the \( H_\infty \) method. In order to track the reference input \( r(t) \in \mathbb{R}^n \), the feedforward controller for the nominal system is designed as
\[ K_{ref0} = -K_0 C^T - B_v^T A C^T \]
where \( B_v^T \) and \( C^T \) are the pseudo inverse of \( B_v \) and \( C \). The detailed description can be found in [14].

Note that the baseline controller can stabilize the system and track the reference signal in a fault free condition. The controller need not be reconfigured when an actuator failure occurs during the operation if there is sufficient actuator redundancy. Under this circumstance, combining the baseline controller with CA strategy can achieve satisfactory performance easily. However, when most actuators lose effectiveness or become damaged, i.e., redundant actuators no longer exist, the traditional CA strategy will fail. Thus, an adaptive fault-tolerant controller is proposed in this paper to solve this problem.

B. Adaptive Fault-tolerant Controller

The ideal system with no actuator failures (the effectiveness matrix \( \Delta = I \)) is chosen as the reference system.

Denoting
\[ A + B_v K_0 = A_m \]
\[ B_v K_{ref0} = B_m \]
the reference system can be defined as
\[ \dot{x}_m = A_m x_m + B_m r \]
where \( A_m \in \mathbb{R}^{m \times m} \) is a Hurwitz matrix; \( x_m \in \mathbb{R}^m \) is the desired state. Given a symmetric positive definite matrix \( Q \in \mathbb{R}^{m \times m} \), the following Lyapunov function has a unique solution \( P \in \mathbb{R}^{m \times m} \)[16]:
\[ A_m^T P + P A_m = -Q, \quad P = P^T > 0 \]

The baseline controller guarantees that the reference system has satisfactory stability and dynamic performance, but it may be invalid when a severe failure occurs. Due to the uncertainty of the failure, an adaptive control signal can be used to compensate for the failure. The adaptive fault-tolerant controller is designed as
\[ \dot{v}_c = \hat{K}_{ref} r + \hat{K} x \]
where \( \hat{K}_{ref} \) and \( \hat{K} \) are the estimates of \( K_{ref0} \) and \( K_0 \) respectively.

Denoting the error signals as
\[ \hat{K}_{ref} = K_{ref} - K_{ref0} \]
\[ \hat{K} = K - K_0 \]
and with (13), we can get
\[ \dot{K}_{ref} = -\hat{K} C^T - B_v^T A C^T \]

Combining with (2), (4), (10) and (19), the closed-loop system can be rewritten as
\[ \dot{x} = Ax + B_v (K_{ref} r + \hat{K} x) \]
\[ = Ax + B_v (-\hat{K} C^T - B_v K_{ref0} r + \hat{K} x) \]
\[ = Ax + B_v (-\hat{K} C^T - B_v K_{ref0} r + \hat{K} x) \]
\[ = Ax + B_v (\hat{K}_{ref} r + B_v K_0 x + B_v \hat{K} x) \]
\[ = A_m x + B_m r + B_v \hat{K} (x - C^T r) \]

Denoting \( e = x - x_m \), combining with (17) and (18) yields the error dynamics
\[ \dot{e} = A_m e + B_v \hat{K} (x - C^T r) \]

The error dynamic (21) associates the parameter error \( \hat{K} \) with the tracking error \( e \). Thus, the following theorem can be used to generate the estimated parameter \( \hat{K} \).

Theorem 2. For the control system (1), the adaptive controller (17) with the gain adaption law
\[ \dot{\hat{K}} = -\Gamma_k B_v^T P e (x - C^T r)^T \]
where \( \Gamma_k \in \mathbb{R}^{n \times n} \) is a positive definite matrix, guarantee that all closed-loop signals remain bounded and that the tracking error \( e(t) = x(t) - x_m(t) \) converges to 0 as \( t \to \infty \).

Proof: In order to analyze the closed-loop stability and tracking performance, define the following candidate Lyapunov function:
\[ V = e^T P e + \text{trace} (\hat{K}^T \Gamma_k^{-1} \hat{K}) \]
where \( P \) is given by (16), and trace(\cdot) is the trace of a matrix.

Then,
\[ \dot{V} = (e^T P e + e^T \dot{P} e) + 2 \text{trace} (\hat{K}^T \Gamma_k^{-1} \dot{\hat{K}}) \]
\[ = e^T (A_m^T P + P A_m) e + 2 \text{trace} [\hat{K}^T B_v^T P e (x - C^T r)^T + \hat{K}^T \Gamma_k^{-1} \dot{\hat{K}}] \]

As \( \dot{K} = \hat{K} \), substituting the adaptive law (22) into (24) gives
\[ \dot{V} = -e^T Q e \leq 0 \]

Since \( V(t) \) is a positive definite function and \( \dot{V}(t) \leq 0 \), it is easy to obtain \( V(t) \in L_\infty \), which indicates that \( e(t), K(t) \in L_\infty \). Therefore, all the closed-loop signals are
bounded, and $e(t) \in L_2$. According to (21) and the boundedness of the closed-loop signal, we can get $\dot{e}(t) \in L_\infty$, and thus $\lim_{t \to \infty} e(t) = 0$. Due to $e(t) \in L_2 \cap L_\infty$, $\dot{e}(t) \in L_\infty$, and $x, r \in L_\infty$, all signals and estimated parameters are bounded, realizing $\lim_{t \to \infty} [x(t) - x_m(t)] = 0$.

V. APPLICATION EXAMPLE

Satellite launch vehicles (SLVs) play an important role in placing artificial satellites and space stations into earth orbit, and they require accurate positioning of the payload in the desired orbit, even in adverse conditions [15]. Thus, it is necessary to design a control system for SLVs to meet the desired requirements. In this paper, a SLV model is simulated to demonstrate the effectiveness of the proposed algorithm. The physical system of SLVs chosen in this paper has eight actuators to control four thrusters, thus it has enough redundant actuators. This paper focuses on guaranteeing the performance of SLVs without any significant degradation when a severe failure occurs, i.e., when most actuators lose effectiveness or become damaged. Compared to the traditional CA strategy, the proposed MRAC scheme associated with CA has a significant advantage.

The state space model of SLV is given by (1). The state includes the pitch $\theta$, pitch rate $\dot{\theta}$, yaw $\psi$, yaw rate $\dot{\psi}$, roll $\phi$, and roll rate $\dot{\phi}$, and they are controlled by eight actuators.

$$
x = [\theta \ \dot{\theta} \ \psi \ \dot{\psi} \ \phi \ \dot{\phi}]^T
$$

$$
y = [\theta \ \psi \ \phi]^T
$$

$$
u = [u_1 \ u_2 \ u_3 \ u_4 \ u_5 \ u_6 \ u_7 \ u_8]^T
$$

The control signals $u_i \in [-15, 15] \times \frac{\pi}{180}$, $i = 1, \ldots, 4$ are mainly used to control the actuators of straçpons, and the control inputs $u_i \in [-15, 15] \times \frac{\pi}{180}$, $i = 5, \ldots, 8$ are main engine trusters.

The system matrices are given in the Appendix, and the reference inputs are described as:

$$
r_i(t) = \begin{cases} 
4 \text{deg}, & 5s \leq t < 20s \\
0, & \text{else}
\end{cases}, \quad i = 1, 2, 3.
$$

A. Case 1

The ideal system in (1) with $\Delta = I$ is used to verify the advantages of the $\eta$-weighting algorithm. The simulation results of the weighting algorithm with $\text{diag}(\eta_1, \eta_2, \ldots, \eta_8) = I$ and the $\eta$-weighting algorithm with $\text{diag}(\eta_1, \eta_2, \ldots, \eta_8) = \text{diag}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ are shown in Fig. 3 and Fig. 4.

The two algorithms have the same $\varepsilon$ and the same initial value of $W$. The $\eta$-weighting algorithm can enforce all control inputs into actuator limits, whereas the original weighting algorithm can not due to $u_7$. The simulation results of $u_7$ are depicted in Fig. 3. The variation of $W$ indicates the points where there was a chance of saturation with prior value of $W$, as shown in Fig. 4. Obviously, $W$ in the $\eta$-weighting algorithm updates more frequently than that in the weighting algorithm, and thus the $\eta$-weighting algorithm improves the efficiency.

Fig. 3. Case 1: control input $u_7$ of the weighting algorithm (WA) and the $\eta$-weighting algorithm ($\eta$-WA).

B. Case 2

The simulation results of the novel AFTC method and the traditional FTC method as shown in Fig. 1 are compared in this case. The output tracking errors and control inputs are shown in Fig. 5 and Fig. 6. Assume that $u_1$, $u_2$, $u_3$ lose 30% effectiveness and $u_4$, $u_5$, $u_6$ are totally damaged at the beginning of the flight. Under this circumstance, the $\eta$-weighting algorithm becomes invalid since $u_7$ has exceeded the actuator limits. The anti-saturation controller is designed according to (10). As shown in Fig. 5, the traditional method exhibits large tracking error in pitch angle. By contrast, the tracking errors of the AFTC method are significantly reduced by using the gain adaption law (22).

Fig. 4. Case 1: trace(W) of the weighting algorithm (WA) and the $\eta$-weighting algorithm ($\eta$-WA).
VI. CONCLUSIONS

In this paper we addressed some open problems in fault tolerant control (FTC) of systems with severe actuator failures. We presented a solution to such a problem that there is no sufficient actuator redundancy after the failures. A modified FTC method, which combines control allocation with model reference adaptive control, was developed and shown to guarantee the stability and tracking performance of the system. The practical limitations such as actuator saturation was also taken into consideration in this paper. An improved weighting algorithm and an anti-saturation controller were developed to compensate for the saturation error. Finally, simulation of the SLV model demonstrated that the proposed method gave better tracking error performance. Future work will focus on extension to the case with time-varying actuator failures.

APPENDIX

The system matrices of the SLV model are

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0.7066 & 0 & 1.87e-5 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
2.71e-5 & 0 & 0.4379 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
5.71e-4 & 0 & 5.46e-4 & 0 & 0 & 0
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
B_u = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
5.7508 & 2.191e-4 & -1.495e-3 \\
0 & 0 & 0 \\
2.207e-4 & 5.173 & 2.007e-3 \\
0 & 0 & 0 \\
-4.65e-3 & 2.17e-2 & 14.15276
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0.2851 & -0.2851 & -0.2851 \\
-0.2851 & 0.2851 & -0.2851 \\
-0.2851 & 0.2851 & -0.2851 \\
0.2851 & 0.2851 & 0.2851 \\
0.0968 & 0 & 0 \\
0 & 0.0968 & 0.0968 \\
-0.0968 & 0 & 0 \\
0 & -0.0968 & 0.0968
\end{bmatrix}
\]

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