Generic gamma correction for accuracy enhancement in fringe-projection profilometry

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Fringe-projection profilometry (FPP) is one of the most commonly used noncontact methods for acquiring the three-dimensional (3D) shape information of objects. In practice, the luminance nonlinearity caused by the gamma effect of a digital projector and a digital camera yields undesired fringe intensity changes, which substantially reduce the measurement accuracy. In this Letter, we present a robust and simple scheme to eliminate the intensity nonlinearity induced by the gamma effect by combining a universal phase-shifting algorithm with a gamma correction method. First, by using three-step and large-step phase-shifting techniques, the gamma value involved in the measurement system can be detected. Then, a gamma pre-encoding process is applied to the system for actual 3D shape measurements. With the proposed technique, high accuracy of measurement can be achieved with the conventional small-step phase-shifting algorithm. The validity of the technique is verified by experiments. © 2010 Optical Society of America

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Fringe-projection profilometry (FPP) is one of the leading methods for acquiring three-dimensional (3D) shape information by projecting structured-light patterns onto the objects of interest [1–3]. Because of their numerous advantages, such as low cost and high flexibility, digital projectors and digital cameras are commonly employed in FPP measurements. Furthermore, to obtain full-field 3D shape information with high accuracies, sinusoidal fringe patterns are usually used. Nevertheless, the luminance nonlinearity caused by the gamma effect of the digital camera and projector brings unwanted intensity changes to the captured fringe patterns, which often lead to nonnegligible errors in 3D shape determination [4]. To overcome the nonlinear luminance problem, many approaches of phase-error compensation have been proposed, and they normally fall into two categories. The first uses transform techniques to extract the fringe phase, such as Fourier transform [5], wavelet transform [6], and Hilbert transform [7]. The transform techniques require complicated computation and cannot retrieve the desired phase map accurately for measurements involving multiple objects or an object with a complex shape. On the other hand, the phase-shifting techniques classified into the second category are usually suitable for measuring objects with complex shapes; however, the phase-shifting algorithms generally face a trade-off between the accuracy of phase-error compensation and computation time, as well as complexity [8]. Despite the recent advances in retrieving phase distribution in real time with low aberration [4,10,11], a method that is not only very simple and easy to implement, but is also highly reliable in terms of phase-error elimination for FPP system has not yet been unveiled, to the best of our knowledge. In this Letter, a novel and robust scheme is presented by combining a universal phase-shifting algorithm with a generic gamma correction method, and it provides fast speed, high accuracy, and easy implementation for FPP-based 3D shape measurements.

Mathematically, the existence of nonlinear luminance distortion in the FPP system brings high-order harmonics to the actual fringe patterns. Therefore, the intensity of the captured fringe image can be theoretically expressed as follows:

\[ I'(x, y) = a(x, y) + \sum_{j=1}^{p} b_j \cos(j[\phi(x, y) + \delta]), \quad (1) \]

where \((x, y)\) denotes an arbitrary point in the image, \(a\) is the background intensity, \(b\) is the intensity modulation amplitude, \(\phi\) is the fringe phase, \(\delta\) is the phase-shift amount, and \(p\) is the highest significant harmonic order of the captured fringes. For convenience and simplicity, \((x, y)\) will be omitted from the equations hereafter.

Equation (1) involves \((p + 2)\) unknowns: \(a, b_1, ..., b_p\), and \(\phi\); consequently, \((p + 2)\) phase-shifted fringe patterns are required to solve for these unknowns. Defining a set of variables as \(A_j = b_j \sin(j\phi)\) and \(B_j = b_j \cos(j\phi)\), each of the \((p + 2)\) images can then be described as

\[ I_i' = a + \sum_{j=1}^{p} [B_j \cos(j\delta_i) - A_j \sin(j\delta_i)]. \quad (2) \]

In Eq. (2), \(i = 1, 2, ..., p + 2\) and \(\delta_i = \frac{i-1}{p+2} 2\pi\).

The sum of squared differences between the theoretical pattern \(I_i'\) and actual pattern \(I_i\) is

\[ S = \sum_{i=1}^{p+2} \left( a + \sum_{j=1}^{p} [B_j \cos(j\delta_i) - A_j \sin(j\delta_i)] - I_i \right)^2. \quad (3) \]

To minimize \(S\), the least-square approach requires \(\frac{dS}{d\delta_i} = 0\). This yields

\[ \sum_{i=1}^{p+2} \cos(\delta_i) \left( a + \sum_{j=1}^{p} [B_j \cos(j\delta_i) - A_j \sin(j\delta_i)] - I_i \right) = 0. \quad (4) \]
Considering \( \delta_i = \frac{i-1}{p+2} 2\pi \), it is easy to prove that 
\[ \sum_{i=1}^{p+2} \cos(\delta_i) = 0 \] 
and 
\[ \sum_{i=1}^{p+2} \cos(j\delta_i) = 0 \] 
for any \( j = 2 \) to \( p \), and 
\[ \sum_{i=1}^{p+2} \cos(\delta_i) \sin(\delta_i) = 0 \] 
for any \( j = 1 \) to \( p \). Consequently, Eq. (4) can be simplified as 
\[ \left[ \sum_{i=1}^{p+2} \cos^2(\delta_i) \right] B_1 - \sum_{i=1}^{p+2} \cos(\delta_i) I_i = 0. \] 

This gives 
\[ B_1 = \frac{\sum_{i=1}^{p+2} \cos(\delta_i) I_i}{\sum_{i=1}^{p+2} \cos^2(\delta_i)}. \] 

Similarly, \( A_1 \) can be calculated from \( \frac{\partial S}{\partial A_1} = 0 \):
\[ A_1 = -\frac{\sum_{i=1}^{p+2} \sin(\delta_i) I_i}{\sum_{i=1}^{p+2} \sin^2(\delta_i)}. \]

From Eqs. (6) and (7), and considering \( \sum_{i=1}^{p+2} \cos^2(\delta_i) = \sum_{i=1}^{p+2} \sin^2(\delta_i) \), the phase can be retrieved as
\[ \phi = \tan^{-1} \left( \frac{A_1}{B_1} \right) = \tan^{-1} \left( \frac{-\sum_{i=1}^{p+2} \sin(\delta_i) I_i}{\sum_{i=1}^{p+2} \cos(\delta_i) I_i} \right). \] 

Equation (8) was initially proposed by Surrel [8], and it is rigorously derived here. The equation indicates that a \((p + 2)\) step uniform phase-shifting scheme can be employed to retrieve phase accurately from fringe patterns with nonlinear harmonics up to the \( p \)th order.

Recently, Pan et al. [4] showed that the highest significant harmonic order in a practical FPP image is 5. Accordingly, at least seven images are required.

The universal phase-shifting algorithm with large frames is very simple and effective for extracting correct phase from real FPP images. In fast FPP measurements, however, taking more than three phase-shifted images is typically undesired. To cope with this issue, a generic gamma correction scheme is described as follows.

In theory, the gamma effect of the projector and camera system can be described as
\[ I_{(n)} = I_{(0)}^{\gamma_0}. \]

where \( I_{(n)} \) is the normalized value of intensity \( I \) of the captured image and \( I \) is governed by Eq. (1), \( I_{(0)} \) is the normalized intensity of the computer-generated ideal sinusoidal pattern, and \( \gamma_0 \) is the gamma value of the entire system, including projector and camera. It is noted that, lately, many research efforts have been focused on advancing simpler and more accurate techniques to detect the gamma value [12,13]. Nevertheless, an essential analysis that can detect the gamma quickly and correctly for FPP system remains lacking.

It can be seen from Eq. (9) that applying an appropriate gamma encoding, \( 1/\gamma_p < 1 \), during the generation of ideal image \( I_{(0)} \) may help to attenuate the gamma effect and thus enhance accuracy, even when the number of images is minimum, i.e., three. In this case, \( I_{(n)} = (I_{(0)}^{1/\gamma_p})^{\gamma_0} \); therefore,

\[ I_{(0)} = I_{(n)}^{\gamma_p/\gamma_0}. \] 

In reality, considering the background effect, intensity amplitude, and system complexity, the desired intensity can be determined by the following equation:
\[ I_{(0)} = c_1 I_{(n)}^{\gamma_p/\gamma_n} + c_2 = c_1 I_{(n)}^{\gamma_p/\gamma_n} + c_2, \]

where \( c_0, c_1, c_2, \gamma_n = \frac{\pi}{c_1}, \) and \( \gamma_p \) are system parameters.

When a three-step phase-shifting scheme is employed, the phase can be accurately determined by substituting Eq. (11) and the corresponding \( p = 1 \) (since \( I_{(0)} \) is the ideal sinusoidal pattern) into Eq. (8). Meanwhile, considering that the ratio of \( I \) to \( I_{(n)} \) is a constant, we have
\[ \phi = \tan^{-1} \left\{ \frac{-I_2' \sin \left( \frac{2\pi}{3} \right) + I_3' \sin \left( \frac{4\pi}{3} \right)}{I_1' + I_2' \cos \left( \frac{2\pi}{3} \right) + I_3' \cos \left( \frac{4\pi}{3} \right)} \right\}, \]

where
\[ \gamma' = \gamma_p/\gamma_n + \gamma_p. \]

To detect \( \gamma' \) for the three-step phase-shifting algorithm, the following equation can be used:
\[ \tan^{-1} \left\{ \frac{-I_2' \sin \left( \frac{2\pi}{3} \right) + I_3' \sin \left( \frac{4\pi}{3} \right)}{I_1' + I_2' \cos \left( \frac{2\pi}{3} \right) + I_3' \cos \left( \frac{4\pi}{3} \right)} \right\} - \phi^c = 0, \]

where \( \phi^c \) is the correct phase value, which, in practice, is determined by using the universal large-step phase-shifting algorithm presented previously. It is noted that

Fig. 1. (Color online) Experimental results related to phase error and gamma detection: (a) phase error from multiple-step phase shifting without gamma pre-encoding, (b) linear relationship between \( \gamma' \) and \( \gamma_p \), (c) phase error from three-step phase shifting with various pre-encoding gamma values, and (d) phase error of 300 pixels in an arbitrary line with different phase-shifting schemes.
Eq. (14) is for one pixel only; to solve for \( \gamma' \) from a group of pixels or the entire image to ensure high accuracy, a least-square approach is commonly employed.

Letting \( \gamma' = 1 \) in Eq. (12) can substantially simplify the computation. This can be done by determining \( \gamma_a \) and \( \gamma_b \) from Eq. (10) based on two tests described later:

\[
\gamma_a = \frac{\gamma_1 - \gamma_2}{\gamma_1' - \gamma_2'}, \quad \gamma_b = \frac{\gamma_1' \gamma_2 - \gamma_1 \gamma_2'}{\gamma_1 - \gamma_2}.
\]  

(15)

Once \( \gamma_a \) and \( \gamma_b \) are obtained, setting \( \gamma_p = (1 - \gamma_p)\gamma_a \) will yield \( \gamma' = 1 \). In this way, Eq. (12) becomes identical with the conventional three-step phase-shifting algorithm.

The above approach is summarized as three steps.

1. With a simple-shape object (e.g., a plate) as the target, use a relatively large phase-shifting step (e.g., 20) to determine the correct phase distribution using Eq. (8).
2. With two arbitrary encoding \( \gamma_{p1} \) and \( \gamma_{p2} \) reapplied to the computer-generated fringes, employ Eq. (14) to detect \( \gamma_1' \) and \( \gamma_2' \) and Eq. (15) to determine \( \gamma_a \) and \( \gamma_b \).
3. Apply gamma encoding with \( \gamma_p = (1 - \gamma_p)\gamma_a \) during fringe-pattern generation for all future measurements with the current setup. A conventional phase-shifting algorithm, such as the three-step one, will be used to retrieve phase with nonlinear intensity distortion removed.

An experiment of measuring the 3D shape of a conch shell is conducted. The FPP system is comprised of a regular digital projector and a CMOS digital camera. Figure 1(a) shows the mean phase error (along an arbitrary line in the captured pattern of a background plate) induced by using different numbers of phase-shifting steps without gamma pre-encoding, where the reference phase is extracted by a 20 step phase-shifting measurement. It can be seen that using seven phase-shifting steps can already yield very good results. The system pre-encoding gamma \( \gamma_p \) detected by the approach described previously is 3.0, as shown by the top line in Fig. 1(b). To demonstrate the validity of the linear relationship between \( \gamma' \) and \( \gamma_p \), i.e., Eq. (13), two additional representative results from different setups are included in the figure. The detected \( \gamma_p = 3.0 \) matches very well with the result shown in Fig. 1(c), which illustrates the phase error of using three-step phase-shifting with different pre-encoding gamma values. Figure 1(d) indicates that the three-step phase-shifting algorithm with the proposed gamma pre-encoding approach can achieve accuracy equivalent to or even higher than the one provided by the seven-step phase-shifting algorithm. Despite the similar accuracies, it is important to point out that the difference between the three- and seven-step phase-shifting approaches in practice is enormous, because the three-step scheme can use the projection mechanism of a digital projector to achieve fast-speed measurement (e.g., 30 measurements per second), whereas the seven-step one can hardly achieve five measurements per second.

Figure 2 shows the measurement results obtained with the conventional three-step, seven-step, and the proposed phase-shifting approaches.

In conclusion, when the speed of 3D shape measurement is not a concern, the universal phase-shifting technique with large phase-shifting steps (seven or higher) can be used to enhance the measurement accuracy by automatically eliminating the intensity nonlinearity induced by gamma effect. When fast speed is demanded, the proposed gamma pre-encoding scheme can be employed with the conventional three-step phase-shifting algorithm to achieve high measurement accuracies.

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