On the Autocorrelation Ergodic Properties of Sum-of-Cisoids Rayleigh Fading Channel Simulators

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Abstract—In this paper, we analyze the autocorrelation properties of seven fundamental classes of stochastic sum-of-cisoids (SOC) simulation models for narrowband mobile Rayleigh fading channels. The purpose of this analysis is to determine which classes of SOC models are autocorrelation ergodic (AE), i.e., for which classes of simulation models the time autocorrelation function (TACF) equals the autocorrelation function (ACF) of the ensemble. This information is of practical relevance, since an AE channel simulator allows to accurately emulate the channel’s ACF in a single realization, making it possible to bypass the time-consuming computation of the ensemble average. The obtained results conclusively demonstrate that only the class of SOC models defined by cisoids with constant gains, constant frequencies, and random phases possesses the desired AE property. The findings presented in this paper can be used as guidelines for the design of efficient SOC channel simulators that generate Rayleigh fading waveforms with specified autocorrelation properties in each simulation run.

Keywords—Autocorrelation function, channel simulators, ergodic processes, mobile communications, sum-of-cisoids.

I. INTRODUCTION

Computer simulators have become fundamental tools for the design, test, and optimization of modern mobile communication systems. They provide a powerful, affordable, and reproducible means to assess the system performance. An important component of any computer simulator employed for system analysis is the model chosen to simulate the channel. This is a critical component indeed, as most of the problems affecting the performance of wireless communication systems, e.g., inter-symbol interference and cross-modulation, are caused by the channel [1]. There exist several different approaches to the design of fading channel simulators, such as those described in [2]–[5]. While the underlying methodology may differ, the basic concept of all stochastic channel simulators is the same: Synthesizing a random process that can efficiently be realized on a software or hardware simulation platform under the constraint that its statistical properties resemble those of a non-realizable reference channel model. A typical non-realizable reference channel model is, for example, the Rayleigh channel model under isotropic scattering conditions. A realizable random process constitutes the so-called channel simulation model. To ensure that the channel simulator performs as desired, it is necessary to have a detailed statistical characterization of the simulation model. Of fundamental importance is not only to determine how accurate the simulator is in emulating the reference model’s statistical properties, but also to minimize the running time required by the simulator to produce statistically accurate results. In this latter respect, the information about the simulation model’s AE properties is of utmost relevance. If the simulation model is an AE process, then it is possible to determine the channel’s ACF by a single realization. Otherwise, it is necessary to average across several simulation runs to obtain a reasonable approximation to the ACF of the channel, leading to an increase in the overall simulation run time. We recall that the interest in approximating the channel’s ACF follows from the fact that this statistical function exerts a strong influence on the performance of wireless communication systems [1]. Moreover, in case of the widely accepted multipath Rayleigh fading channel model, the ACF of the underlying complex Gaussian process provides all necessary information to fully characterize the channel [6].

Among the variety of channel simulation models proposed in the literature, those based on Rice’s sum-of-sinusoids principle [7], [8] have widely been in use as a basis for the design of multipath radio channel simulators (e.g., see [9]–[13]). In the conventional SOS channel simulation approach, it is assumed that the inphase and quadrature (IQ) components of the channel’s complex envelope are statistically independent Gaussian processes. Under this consideration, the simulation of the channel’s IQ components is carried out over two uncorrelated SOS simulation models having different sets of parameters—gains, frequencies, phases, and number of sinusoids. This approach has been applied for over four decades to the simulation of a variety of mobile fading channels, ranging from simple single-input single-output (SISO) channels [10] to sophisticated multiple-input multiple-output (MIMO) channels [11]. However, the conventional SOS simulation approach suffers from two serious limitations. On the one hand, it can only be used to simulate fading channels characterized by symmetrical Doppler power spectral densities (DPSDs), leaving aside the more realistic case of channels having asymmetrical DPSDs. On the other hand, the simulation model that results by applying this approach lacks of a physical meaning in the context of the electromagnetic theory.
A solution to the aforementioned limitations is given by a variation of the conventional SOS approach, where a finite sum of complex sinusoids (cisoids) is used to simulate the channel’s complex envelope. Basically, sum-of-cisoids (SOC) models differ from conventional SOS models in the sense that the IQ components of the former models are characterized by the same set of parameters [14]. This feature of SOC models enables the simulation of fading channels having symmetrical and asymmetrical DPSDs [15]. Another important feature of these models is that they have a clear physical meaning, as they are related to the electromagnetic plane wave model.

Despite the significant attention that SOC channel simulation models have attracted, their AE properties have been investigated only partially. In [15, Sec. 3.5], it is shown that a class of SOC models comprising cisoids with constant gains, constant frequencies, and random phases possesses the desired AE property. Basing on the random or deterministic nature of the cisoids’ parameters—gains, frequencies, and phases, we can identify another six basic classes of stochastic SOC simulation models [16]. To the best of the authors’ knowledge, the AE properties of these classes of SOC models have not been studied explicitly. However, it is stated in several papers (e.g., see [17]) that none of these classes of models enables the design of AE channel simulators. Given the significance of this statement, it is very important to provide an explicit proof of its veracity. To close the gap, we present in this paper a comprehensive analysis on the AE properties of seven fundamental classes of stochastic SOC simulation models for mobile Rayleigh fading channels. We conclusively demonstrate that only the class defined by cisoids with constant gains, constant frequencies, and random phases allows for the design of AE simulators. The results herein presented can be used as guidelines to design efficient channel simulators that generate Rayleigh fading waveforms with accurate autocorrelation properties in each simulation run.

The rest of the paper is organized in four sections. In Section II, we discuss the results presented in some relevant papers and highlight the differences with the problem studied herein. In Section III, we provide a brief description of our reference narrowband Rayleigh fading channel model. In Section IV, we analyze the AE properties of the seven classes of stochastic SOC simulation models for mobile Rayleigh fading channels. Finally, we give our conclusions in Section V. Throughout the paper, we will use bold symbols and letters to denote random variables and stochastic processes, whereas constants and deterministic processes will be denoted by normal symbols and letters.

II. Related Work

Even though the AE properties of SOC channel simulation models have not fully been investigated, those of conventional SOS simulators have thoroughly been analyzed in [18]–[20]. In these papers, the authors systematically analyze the autocorrelation and the mean ergodic properties of seven classes of stochastic SOS simulation models for Rayleigh fading channels. They also investigate the first-order stationarity of the envelope of these classes of channel simulators. It is shown in [18]–[20] that an SOS model is an AE process if and only if the sinusoids’ gains and Doppler frequencies are constant quantities, while the phases are random variables. It should be noticed, nonetheless, that the conclusions drawn in these papers cannot be extrapolated to the case of the SOC simulators. The reason is that the investigations performed in [18]–[20] were carried out in the framework of the conventional SOS channel simulation approach, i.e., on the assumption that the IQ components of the channel simulation model are uncorrelated. This consideration does not necessarily hold for an SOC simulator, as the IQ components of this type of simulation models can have any given autocorrelation and cross-correlation properties [17]—it is precisely this feature which enables SOC models to simulate fading channels characterized not only by symmetrical DPSDs but also by asymmetrical DPSDs. It is therefore necessary to extend the analysis presented in [18]–[20] in order to cover the important case of SOC channel simulators.

The results presented in [16] are also worth to be discussed. In that paper, the authors investigate the first-order stationarity of the envelope of seven classes of stochastic SOC models. It is shown there that the envelope of an SOC model having cisoids with random gains, constant frequencies, and constant phases is not a first-order stationary process. One can therefore conclude that this class of SOC models is defined by random processes which are non-AE, since an AE random process is necessarily both first- and second-order stationary [21, Sec. 6.6]. However, nothing can be concluded about the AE properties of the other classes of stochastic SOC models, as these classes comprise (or may comprise, provided that some boundary conditions are fulfilled) only first-order stationary processes [16].

III. The Reference Model

For our investigations, we consider the simulation of a small-scale narrowband SISO Rayleigh fading channel. The channel’s complex envelope is represented in the equivalent baseband by a complex Gaussian random process

\[ \mu(t) = \mu_1(t) + j \mu_Q(t), \quad j \triangleq \sqrt{-1} \tag{1} \]

where \( \mu_1(t) \) and \( \mu_Q(t) \) are stationary real-valued Gaussian processes with mean zero and variance \( \sigma^2_{\mu} / 2 \). An exact statistical characterization of a random process is in general a formidable task that requires knowledge of marginal and multidimensional joint probability density functions (PDFs). However, since the reference model defined in (1) is assumed to be a complex Gaussian process, its statistical properties are completely specified by the ACF \( r_{\mu\mu}(\tau) \triangleq E\{\mu^*(t)\mu(t+\tau)\} \) of \( \mu(t) \). The operator \( E\{\cdot\} \) denotes statistical expectation, while \( \cdot^* \) stands for the complex conjugate operation. Following the correlation model proposed by Clarke in [22] for two-dimensional mobile propagation environments, we can express
the ACF of $\mu(t)$ in integral form as

$$r_{\mu\mu}(\tau) = \sigma_{\mu}^2 \int_{-\pi}^{\pi} p_{\alpha}(\alpha) \exp(j2\pi f_{\text{max}} \cos(\alpha) \tau) d\alpha.$$  (2)

In the equation above, $f_{\text{max}}$ stands for the maximum Doppler shift experienced by the channel’s multipath components, and $p_{\alpha}(\alpha)$ is a circular PDF characterizing the angle-of-arrival (AOA) statistics of the channel.

IV. AUTOCORRELATION ERGODICITY OF SOC SIMULATION MODELS

A. Classification of SOC Simulation Models

It has been shown in several papers (e.g., see [23]–[25]) that the Rayleigh fading channel model characterized by $\mu(t)$ can efficiently be simulated by means of an SOC model comprising a finite number of cisoids. The general structure of an SOC Rayleigh fading channel simulator with $N$ homogeneous1 cisoids is depicted in Fig. 1, where $0 < N < \infty$. Basing on the type of the $N$ homogeneous cisoids, we can distinguish seven fundamental classes of stochastic SOC models and one class of deterministic SOC models. The eight classes, which were originally defined in [16], are listed in Table I. In this section, we will analyze the autocorrelation properties of these classes of SOC models to determine which of them enable the design of AE channel simulators. We point out that the concept of ergodicity does not apply to the Class I simulators, since the nature of this class of models is completely deterministic. However, the information about the time autocorrelation properties of the Class I simulators is fundamental to find out whether or not a given class of stochastic SOC models falls into the category of AE processes.

B. Considerations and Definition of AE Processes

We will perform our analysis under the following considerations:

- All random variables are statistically independent.
- If the cisoids’ phases are random variables, then they are uniformly distributed over $[-\pi, \pi]$.
- If the cisoids’ Doppler frequencies are random variables, then they are obtained from the transformation

$$f_n \triangleq f_{\text{max}} \cos(\alpha_n), \quad n = 1, 2, \ldots, N$$  (3)

where the AOAs $\alpha_n$ are independent and identically distributed random variables, each having a PDF $p_{\alpha}(\alpha)$ identical to that characterizing the reference model’s AOA statistics.

- If the cisoids’ gains are random variables, then they are identically distributed with a mean value $m_c$ and a variance $\sigma_g^2$. Without loss of generality, we assume that $\sigma_g^2 = \sigma_{\mu}^2/N$ holds.

For the sake of clarity, we review briefly the concept of AE processes [21, Sec. 6.6]:

1By homogeneous cisoids, we mean a group of cisoids characterized by the same type of parameters.

**Definition I (AE process):** Let $\tilde{\mu}(t)$ be a random process whose ACF $r_{\tilde{\mu}\tilde{\mu}}(t_1, t_2) \triangleq E\{\tilde{\mu}(t_1)\tilde{\mu}(t_2)\}$ depends only on the time difference $\tau = t_2 - t_1$, so that $r_{\tilde{\mu}\tilde{\mu}}(t_1, t_2) = r_{\tilde{\mu}\tilde{\mu}}(\tau)$, where $r_{\tilde{\mu}\tilde{\mu}}(\tau) \triangleq E\{\tilde{\mu}(t)\tilde{\mu}(t + \tau)\}$. Then, $\tilde{\mu}(t)$ is said to be AE if:

- The TACF $r_{\tilde{\mu}(k)\tilde{\mu}(k)}(\tau) \triangleq \langle \tilde{\mu}(k)(t)\tilde{\mu}(k)(t + \tau) \rangle$ of each sample function $\tilde{\mu}(k)(t)$ of $\tilde{\mu}(t)$ is equal to $r_{\tilde{\mu}\tilde{\mu}}(\tau)$, that is, $r_{\tilde{\mu}(k)\tilde{\mu}(k)}(\tau) = r_{\tilde{\mu}\tilde{\mu}}(\tau)$ $\forall k$. The notation $\langle x(t) \rangle$ stands for the time average of an arbitrary function of time $x(t)$.

C. Analysis of the AE Properties of SOC Channel Simulators

1) Class I Channel Simulators: With reference to Table I, the simulation models of Class I are characterized by a deterministic SOC model

$$\tilde{\mu}(t) = \sum_{n=1}^{N} c_n \exp \{j(2\pi f_n t + \theta_n)\}$$  (4)

where the cisoids’ gains $c_n$, Doppler frequencies $f_n$, and phases $\theta_n$ are constants. To ensure that the DPSD of $\tilde{\mu}(t)$ is band-limited, it is assumed that $f_n \in (-f_{\text{max}}, f_{\text{max}})$, $\forall n$. The TACF $r_{\tilde{\mu}\tilde{\mu}}(\tau) \triangleq \langle \tilde{\mu}(t)\tilde{\mu}(t + \tau) \rangle$ of this class of deterministic SOC models is given as

$$r_{\tilde{\mu}\tilde{\mu}}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \tilde{\mu}(t)\tilde{\mu}(t + \tau)dt.$$  (5)
To find a closed-form expression for $r_{\hat{\mu}}(\tau)$, we assume that

$$f_n \neq f_m, \quad \forall n \neq m$$

$$(6a)$$

$$f_n \neq 0, \quad \forall n.$$  

$$(6b)$$

Thereby, we can write $r_{\hat{\mu}}(\tau)$ as [17]

$$r_{\hat{\mu}}(\tau) = \sum_{n=1}^{N} c_n^2 \exp\{j2\pi f_n \tau\}. \quad (7)$$

We notice that the sample functions of the seven classes of stochastic SOC models can be interpreted by a deterministic process $\mu^{(k)}(t)$ similar to that defined in (4), with the peculiarity that the cisoids’ parameters corresponding to an outcome of a random variable take different values from one simulation run to another. The sample functions $\mu^{(k)}(t)$ of the classes of stochastic SOC models can therefore be thought of as a type of the Class I simulators and their TACF of the classes of stochastic SOC models can be expressed as [14]:

$$\mu^{(k)}(t) = \sum_{n=1}^{N} c_n \exp\{j2\pi f_n t + \theta_n\}. \quad (8)$$

The ACF $r_{\hat{\mu}}(t_1, t_2)$ of this class of SOC models can be expressed as [14]:

$$r_{\hat{\mu}}(t_1, t_2) = \sum_{n=1}^{N} c_n^2 \exp\{j2\pi f_n (t_2 - t_1)\}. \quad (9)$$

This result shows that $r_{\hat{\mu}}(t_1, t_2)$ is time-shift insensitive (TSI), meaning that $r_{\hat{\mu}}(t_1, t_2)$ depends only on the time difference $\tau = t_2 - t_1$, i.e., $r_{\hat{\mu}}(t_1, t_2) = r_{\hat{\mu}}(\tau)$. Hence, we can conclude that the Class II simulation models are AE processes, since $r_{\hat{\mu}}(t_1, t_2)$ is TSI and the criterion $r_{\hat{\mu}}(\tau) = r_{\hat{\mu}}(\tau)$ for all $k$, is fulfilled [cf. (7) and (9)].

3) Class III Channel Simulators: This class of simulators is defined by a set of stochastic processes of the form $\hat{\mu}(t) = \sum_{n=1}^{N} c_n \exp\{j2\pi f_n t + \theta_n\}$. For this class of SOC simulators, we can express $r_{\hat{\mu}}(t_1, t_2)$ in terms of the reference model’s ACF $r_{\mu}(\tau)$ as

$$r_{\hat{\mu}}(t_1, t_2) = \sum_{n=1}^{N} c_n^2 \exp\{j2\pi f_n \tau\} + \frac{1}{\sigma_{\mu}^2} r_{\mu}(\tau) r_{\mu}(t_2) \sum_{n=1}^{N} c_n c_m \exp\{j(\theta_m - \theta_n)\}. \quad (10)$$

Assuming that $c_n = \sigma_{\mu} \sqrt{1/N}, \quad \forall n$, we can rewrite the following equation as

$$r_{\hat{\mu}}(t_1, t_2) = r_{\mu}(\tau) + \frac{1}{N \sigma_{\mu}^2} r_{\mu}^*(t_1) r_{\mu}(t_2) \sum_{n=1}^{N} c_n c_m \exp\{j(\theta_m - \theta_n)\}. \quad (11)$$

It becomes apparent from (10) and (11) that the ACF of the Class III simulators is time-shift sensitive. Thus, since $r_{\hat{\mu}}(t_1, t_2) \neq r_{\hat{\mu}}(\tau)$, we can conclude that SOC simulation models of the Class III are non-AE processes. Nevertheless, we observe that if the boundary condition $\sum_{n=1}^{N} c_n \exp\{j(\theta_m - \theta_n)\} = 0$ is met, then $r_{\hat{\mu}}(t_1, t_2)$ is TSI and equal to $r_{\hat{\mu}}(t_1, t_2) = r_{\mu}(\tau)$. However, $\hat{\mu}(t)$ is a non-AE random process even if the aforementioned boundary condition is satisfied, since $r_{\hat{\mu}}(\tau)$ is clearly different from $r_{\hat{\mu}}(\mu(k))\hat{\mu}(\tau)$ [cf. (2) and (7)].

4) Class IV Channel Simulators: The Class IV simulators are characterized by a stochastic process $\hat{\mu}(t) = \sum_{n=1}^{N} c_n \exp\{j2\pi f_n t + \theta_n\}$. It is straightforward to show that the ACF of the Class IV simulators is TSI and equal to the ACF of the reference model. In mathematical terms, for this class of simulators, we have

$$r_{\hat{\mu}}(t_1, t_2) = r_{\hat{\mu}}(\tau) = r_{\mu}(\tau). \quad (12)$$

Again, $\hat{\mu}(t)$ is a non-AE process, since the ACF defined in (2) is different from $r_{\hat{\mu}}(\mu(k))\hat{\mu}(\tau)$.

5) Class V Channel Simulators: This class of simulators is defined by the set of stochastic SOC models of the form $\hat{\mu}(t) = \sum_{n=1}^{N} c_n \exp\{j2\pi f_n t + \theta_n\}$. In this case, the ACF of $\hat{\mu}(t)$ can be written as

$$r_{\hat{\mu}}(t_1, t_2) = \sum_{n=1}^{N} \left(\frac{N}{2} \sigma_{\mu}^2 + m_n^2\right) \exp\{j2\pi f_n \tau\}$$

$$+ \sum_{m=1}^{N} m_c^2 \exp\{j2\pi (f_m t_2 - f_n t_1)\} \times \exp\{j(\theta_m - \theta_n)\}. \quad (13)$$

It is clear from the previous equation that if $m_c = 0$, then the ACF of the Class V simulators is time-shift sensitive, implying that $\hat{\mu}(t)$ is a non-AE process. However, if the mean value of the random gains $c_n$ is equal to zero, then

$$r_{\hat{\mu}}(t_1, t_2) = r_{\hat{\mu}}(\tau) = \sigma_{\mu}^2 \sum_{n=1}^{N} \exp\{j2\pi f_n \tau\}. \quad (14)$$

Despite the fact that the ACF of $\hat{\mu}(t)$ depends only on the time difference $\tau = t_2 - t_1$ if $m_c = 0$, this random process continues to be non-AE. This is clear, since the TACF $r_{\hat{\mu}}(\mu(k))\hat{\mu}(\tau)$ of the k-th sample function $\hat{\mu}(t)$ of $\hat{\mu}(t)$ is a realization-dependent function that changes from one simulation run to another. Notice that $r_{\hat{\mu}}^{\mu(k)}\hat{\mu}(\tau) = \sum_{n=1}^{N} (c_n^{(k)})^2 \exp\{j2\pi f_n \tau\}$, where $c_n^{(k)}$ is the outcome of the random gain $c_n$ associated with $\hat{\mu}(k)$. Since the condition $r_{\hat{\mu}}^{\mu(k)}\hat{\mu}(\tau) = r_{\hat{\mu}}(\tau)$ for all $k$ is not fulfilled, we have that the Class V of SOC simulators is defined by a set of non-AE processes even if $m_c = 0$. 
6) Class VI Channel Simulators: The Class VI simulators are characterized by a stochastic process \( \hat{\mu}(t) = \sum_{n=1}^{N} c_n \exp \{ j(2\pi f_n t + \theta_n) \} \). For this class of simulators, we have

\[
r_{\hat{\mu}\hat{\mu}}(t_1, t_2) = r_{\hat{\mu}\hat{\mu}}(\tau) = \sigma_\mu^2 + m_\mu^2 \sum_{n=1}^{N} \exp \{ j2\pi f_n \tau \}. \quad (15)
\]

We can conclude that the Class VI of SOC simulators is defined by a set of non-AE processes due to the reasons stated at the end of Subsection IV-C5.

7) Class VII Channel Simulators: This class of simulators is defined by the set of stochastic SOC models of the form \( \hat{\mu}(t) = \sum_{n=1}^{N} c_n \exp \{ j(2\pi f_n t + \theta_n) \} \). The ACF of this class of simulators is equal to

\[
r_{\hat{\mu}\hat{\mu}}(t_1, t_2) = \sum_{n=1}^{N} \left[ \frac{(\sigma_\mu^2 + m_\mu^2)}{\sigma_\mu^2} r_{\mu\mu}(\tau) + r_{\mu\mu}^*(t_1) r_{\mu\mu}(t_2) \right] \times \sum_{m=1, m \neq n}^{N} m_{\mu}^2 \exp \{ j(\theta_m - \theta_n) \}. \quad (16)
\]

From the previous equation, it follows that the ACF of the Class VII simulators is TSI provided that \( m_c = 0 \), as in the case of the Class V simulators. If this condition is fulfilled, and if \( \sigma_\mu^2 = \sigma_\mu^2 / N \), then \( r_{\hat{\mu}\hat{\mu}}(t_1, t_2) \) reduces to

\[
r_{\hat{\mu}\hat{\mu}}(t_1, t_2) = r_{\hat{\mu}\hat{\mu}}(\tau) = r_{\mu\mu}(\tau). \quad (17)
\]

In either case, \( \hat{\mu}(t) \) is a non-AE process, since \( r_{\hat{\mu}\hat{\mu}}(t_1, t_2) \neq r_{\hat{\mu}\hat{\mu}}(\tau) \) if \( m_c \neq 0 \) and \( r_{\hat{\mu}\hat{\mu}}(\tau) \neq r_{\hat{\mu}\hat{\mu}}(\tau) \) if \( m_c = 0 \).

8) Class VIII Channel Simulators: Simulation models of the Class VIII are characterized by a random process \( \hat{\mu}(t) = \sum_{n=1}^{N} c_n \exp \{ j(2\pi f_n t + \theta_n) \} \). For this class of SOC models, one can easily verify that

\[
r_{\hat{\mu}\hat{\mu}}(t_1, t_2) = r_{\hat{\mu}\hat{\mu}}(\tau) = \frac{N(\sigma_\mu^2 + m_\mu^2)}{\sigma_\mu^2} r_{\mu\mu}(\tau). \quad (18)
\]

Again, \( r_{\hat{\mu}\hat{\mu}}(\tau) = r_{\hat{\mu}\hat{\mu}}(\tau) \) holds if \( m_c = 0 \) and \( \sigma_\mu^2 = \sigma_\mu^2 / N \). It is worth pointing out that in contrast to the Class VII simulators, the ACF of the Class VIII simulators is TSI even if \( m_c \neq 0 \). However, the simulation models of the Class VIII prove to be non-AE processes, since the equality \( r_{\hat{\mu}\hat{\mu}}(\tau) = r_{\hat{\mu}\hat{\mu}}(\tau) \) does not hold.

D. Summary of Results

From the results presented in the previous subsection, it follows that an SOC channel simulation model is an AE process if and only if the cisoids’ gains and Doppler frequencies are constant quantities and the phases are random variables. The results presented herein for SOC models and those obtained in [18]–[20] for SOS models show that the AE properties of the classes of stochastic SOC models and SOS models are identical. However, their autocorrelation properties are not exactly the same. In fact, in case of the SOC models of the Class V we have found that the ACF of \( \hat{\mu}(t) \) is TSI if the mean value of the cisoids’ gains is equal to zero. In contrast, it is shown in [20, Eq. (4)] that the ACF of the Class V SOS models is always time-shift sensitive. The results obtained in this paper and those given in [18]–[20] are summarized in Table II.

<table>
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<tr>
<th>CLASS</th>
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<th>FREQUENCIES</th>
<th>PHASES</th>
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a) If the boundary condition \( \sum_{m=1}^{N} \sum_{m=1, m \neq n}^{N} \exp \{ j(\theta_m - \theta_n) \} = 0 \) is satisfied.

b) If the mean value of the random gains is equal to zero.

c) If the density of the random Doppler frequencies is even and the condition \( \sum_{n=1}^{N} \cos(2\theta_n) = 0 \) is met.

V. Conclusions and Future Work

In this paper, we provided a comprehensive analysis of the AE properties of seven classes of stochastic SOC simulation models for mobile Rayleigh fading channels. We conclusively demonstrate that only the class defined by SOC models having cisoids with constant gains, constant Doppler frequencies, and random phases enables the design of AE channel simulators. A comparison with the results obtained in other papers for the AE properties of conventional SOS models shows that the stochastic SOC models and the SOS models have identical AE properties. However, the autocorrelation functions and their properties are completely different for these two types of simulation models.

The AE property of the Class II simulators holds provided that the lengths of the generated waveforms (sample functions) are infinite. However, this condition cannot be fulfilled in practice. Hence, to determine if the aforementioned property holds under realistic simulation conditions, it is necessary to revisit the analysis presented in this paper by considering waveforms of finite length. We leave this important research problem open for future research.

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