Late-time acceleration in a brane with curvature effects

Mariam Bouhmadi-López[1*] and Paulo Vargas Moniz[†]

[*]*Centro Multidisciplinar de Astrofísica, Departamento de Física, Instituto Superior Técnico, Av. Rovisco Pais 1,1049-001 Lisboa, Portugal*
[†]*Departamento de Física, Universidade da Beira Interior, Rua Marquês d'Avila e Bolama, 6201-001 Covilhã, Portugal*

Abstract. In this paper we investigate if the phantom-like regime in the LDGP model can be enlarged by the inclusion of a Gauss-Bonnet (GB) term into the bulk. However, we show that the opposite occurs: the GB effect seems instead to induce a breakdown of the phantom-like behaviour at an even smaller redshift.

Keywords: Dark energy theory, cosmology with extradimensions
PACS: 04.50-h, 04.20.Dw,98.80.-k,98.80.Es

INTRODUCTION

One of the most astonishing discoveries of the last years has been the discovery that the universe has started recently to accelerate. A possibility to describe such an acceleration is within modified theories of gravity like the ΛDGP which, in addition, is able to mimic a phantom-like behaviour without invoking any real phantom matter on the brane or in the bulk.

More precisely, the ΛDGP scenario is a 5D brane-world model with infrared modifications to general relativity caused by an induced gravity term on the brane [1, 2, 3, 4]. The model is based on the normal branch of the Dvali-Gabadadze-Porrati proposal (DGP) [5], being the brane filled with cold dark matter (CDM) and a cosmological constant which drives the late-time acceleration of the brane. The phantom-like behaviour is a consequence of the extra-dimension which screens the brane cosmological constant and it is based in mapping the brane evolution to that of an equivalent 4D general relativistic phantom energy model [1, 2]. More precisely, the basis of this mimicry is an effective energy density (in the 4D general relativistic picture) corresponding to the cosmological constant corrected by the curvature effect due to the induced gravity term on the brane. This effective energy density grows as the brane expands and therefore effectively it behaves as a phantom fluid; i.e. $w_{\rm eff} < -1$, where $w_{\rm eff}$ corresponds to the ratio between the effective energy density and the effective pressure[2]

The effective description of the phantom behaviour in the ΛDGP model breaks down

---

[1] email:mariam.bouhmadi@fisica.ist.utl.pt
[2] Other brane proposals aiming to produce such a mimicry are based on a bulk filled with matter and/or on an energy exchange between the brane and the bulk, therefore modifying the effective equation of state of dark energy on the brane. [6]

at a finite redshift; i.e. the effective energy density vanishes and becomes negative over a certain redshift. When the effective energy density vanishes, the effective equation of state blows up. Given that the phantom-like behaviour results from (*i*) induced gravity effects on the brane causing curvature corrections and (*ii*) describing the brane model as a 4D relativistic phantom energy setup, could the break down of the phantom-like behaviour be eliminated by considering further curvature effects on the brane-world scenario? This is the main question we address in this paper. We will model such additional and new curvature effects through a Gauss-Bonnet term (GB) in the bulk [7, 8].

THE MODEL

We consider the ΛDGP brane, i.e. the normal DGP branch filled with CDM and a cosmological constant, with GB corrections in the bulk. Then the Friedmann equation reads [8]

$$E^2(z) = \Omega_m(1+z)^3 + \Omega_\Lambda - 2\sqrt{\Omega_{r_c}}\left[1+\Omega_\alpha E^2(z)\right]E(z), \qquad (1)$$

where we have assumed a mirror symmetry across the brane, $E(z) = H/H_0$ and

$$\Omega_m = \frac{\kappa_4^2 \rho_{m_0}}{3H_0^2}, \ \ \Omega_\Lambda = \frac{\kappa_4^2 \Lambda}{3H_0^2}, \ \ \Omega_{r_c} = \frac{1}{4r_c^2 H_0^2}, \qquad (2)$$

are the usual convenient dimensionless parameters while the new parameter Ω_α is defined as

$$\Omega_\alpha = \frac{8}{3}\alpha H_0^2. \qquad (3)$$

Finally, r_c is the crossover scale in the DGP model and α is the GB parameter in the bulk action.

Evaluating the Friedmann equation (1) at $z = 0$ gives a constraint on the cosmological parameters of the model

$$\Omega_m + \Omega_\Lambda = 1 + 2\sqrt{\Omega_{r_c}}\left(1+\Omega_\alpha\right). \qquad (4)$$

This constraint implies that the region $\Omega_m + \Omega_\Lambda < 1$ is unphysical. Moreover, although the brane is spatially flat, the previous constraint can be interpreted in the sense that our model constitutes a mimic of a closed FLRW universe. Furthermore, by imposing that the universe is currently accelerating, we obtain another constraint on the set of cosmological parameters Ω_m, Ω_{r_c} and Ω_α, which reads

$$3\Omega_m < 2 + 2(1+\Omega_\alpha)\sqrt{\Omega_{r_c}}. \qquad (5)$$

On the other hand, it can be shown that the brane never super-accelerates; i.e. the Hubble rate decreases as the brane accelerates.

PHANTOM-LIKE BEHAVIOUR ON THE BRANE

The phantom-like behaviour on the brane is based in defining a corresponding effective energy density $\rho_{\rm eff}$ and an effective equation of state with parameter $w_{\rm eff}$. More precisely, the effective description is inspired in writing down the modified Friedmann equation of the brane as the usual relativistic Friedmann equation so that $H^2 = \frac{\kappa_4^2}{3}(\rho_m + \rho_{\rm eff})$ where $\rho_{\rm eff}$ reads

$$\rho_{\rm eff} = \frac{3H_0^2}{\kappa_4^2}\left[\Omega_\Lambda - 2\sqrt{\Omega_{r_c}}(1+\Omega_\alpha E^2(z))E(z)\right]. \quad (6)$$

This effective energy density corresponds to a balance between the cosmological constant and geometrical effects encoded on the Hubble parameter. On the other hand, gravity leakage at late-time screens the cosmological constant like in the ΛDGP scenario [1, 2]. This phantom-like behaviour is obtained without any matter violating the null energy condition and without any super-acceleration on the brane.

As in the ΛDGP model, it is possible to define an effective equation of state or parameter $w_{\rm eff}$ associated to the effective energy density as

$$\dot{\rho}_{\rm eff} + 3H(1+w_{\rm eff})\rho_{\rm eff} = 0. \quad (7)$$

This effective equation of state is defined in analogy with the standard relativistic case. Then using Eq. (6), we obtain

$$1 + w_{\rm eff} = \frac{2}{3}\frac{\dot{H}/H_0^2\sqrt{\Omega_{r_c}}\left(1+3\Omega_\alpha E^2(z)\right)}{E(z)\left[\Omega_\Lambda - 2\sqrt{\Omega_{r_c}}\left(1+\Omega_\alpha E^2(z)\right)E(z)\right]}.$$

Because the brane never super-accelerates, i.e. $\dot{H} < 0$, we can then conclude that $\rho_{\rm eff}$ mimics the behaviour of a phantom energy component on the brane: I.e. $1 + w_{\rm eff} < 0$, as long as the effective energy density $\rho_{\rm eff}$ is positive. However, there is always a redshift z_b at which $\rho_{\rm eff}$ vanishes and therefore the phantom mimicry ceases to be valid. The above behaviour raises the following possibility: Can the phantom behaviour break down in our model at a redshift z_b , such that $z_b > \tilde{z}_b$, where $\tilde{z}_b$ is the redshift at which the phantom mimicry is no longer valid in the ΛDGP model? The answer will depend on the cosmological parameters that characterise both models, three in the ΛDGP scenario namely ($\Omega_{\tilde{m}}$,$\Omega_{\tilde{\Lambda}}$,$\Omega_{\tilde{r}_c}$) and four in the model we are analysing namely (Ω_m,Ω_Λ,Ω_{r_c},Ω_α).

We consider that only one of the parameters (Ω_Λ, Ω_m, Ω_{r_c}) is fixed in the ΛDGP-GB brane. We further assume that the given value of the fixed parameter is very close to that obtained by constraining the ΛDGP model with observational data ($H(z)$, CMB shift parameter and SNIa data)[9]

$$\Omega_m = 0.26 \pm 0.05, \quad \Omega_{r_c} \leq 0.05. \quad (8)$$

Our results are presented in figure 1: The phantom-like behaviour in the ΛDGP-GB brane breaks down sooner (smaller redshift) than in the ΛDGP brane.

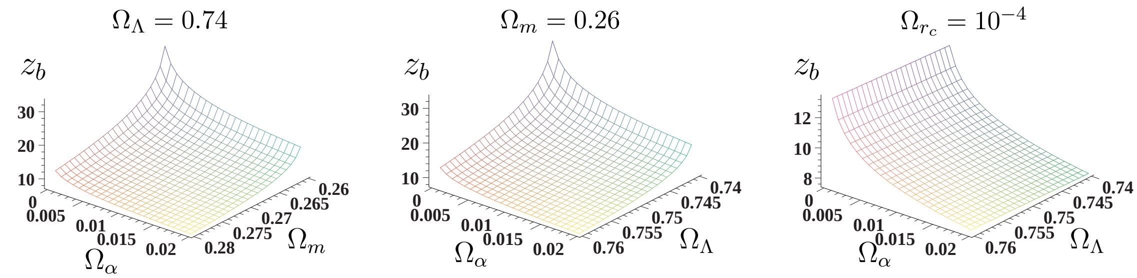

FIGURE 1. Plot of the redshift z_b at which the phantom-like behaviour in the ΛDGP-GB model breaks down. When $\Omega_\alpha = 0$, we obtain the redshift $\tilde{z}_b$ at which the phantom-like behaviour in the ΛDGP scenario breaks down. As can be noticed $z_b < \tilde{z}_b$.

CONCLUSIONS

We have analysed the ΛDGP-GB model which corresponds to a 5D brane-world model where the bulk is a 5D Minkowski space-time. The model contains a GB term in the bulk and an induced gravity term on the brane. Our analysis was performed for the normal branch which we have assumed to be filled by CDM and a cosmological constant, the latter driving the late-time acceleration of the brane. We have shown how the brane accelerates at late-times.

On the other hand, we have shown that there is a phantom-like behaviour on the brane without resorting to any real phantom matter on the brane or the bulk. However, the phantom-like behaviour in the ΛDGP-GB brane breaks down sooner (smaller redshift) than in the ΛDGP brane. Namely, the ΛDGP has a regular phantom-like behaviour for $[0, \tilde{z}_b)$ whereas the phantom-like behaviour in the ΛDGP-GB model is regular only for $[0, z_b)$, with $z_b < \tilde{z}_b$. Thus, we conclude that the GB term does extend the regime of validity of the phantom mimicry in the ΛDGP model.

ACKNOWLEDGMENTS

MBL is supported by the Portuguese Agency Fundação para a Ciência e Tecnologia through the fellowship SFRH/BPD/26542/2006. This research work was supported by the grant FEDER-POCI/P/FIS/57547/2004.

REFERENCES

1. V. Sahni and Y. Shtanov, JCAP **0311**, 014 (2003) [arXiv:astro-ph/0202346].
2. A. Lue and G. D. Starkman, Phys. Rev. D **70**, 101501 (2004) [arXiv:astro-ph/0408246].
3. R. Lazkoz, R. Maartens and E. Majerotto, Phys. Rev. D **74**, 083510 (2006) [arXiv:astro-ph/0605701].
4. M. Bouhmadi-López and R. Lazkoz, Phys. Lett. B **654**, 51 (2007) [arXiv:0706.3896 [astro-ph]].
5. G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485**, 208 (2000) [arXiv:hep-th/0005016].
6. M. Bouhmadi-López, Nucl. Phys. B **797**, 78 (2008) [arXiv:astro-ph/0512124]; M. Bouhmadi-López and A. Ferrera JCAP **0810**, 011 (2008) [arXiv:0807.4678 [hep-th]]; ibid arXiv:0811.4069 [hep-th].
7. G. Kofinas, R. Maartens and E. Papantonopoulos, JHEP **0310**, 066 (2003) [arXiv:hep-th/0307138].
8. M. Bouhmadi-López and P. Vargas Moniz Phys. Rev. D **78**, 084019 (2008) [arXiv:0804.4484 [gr-qc]].
9. R. Lazkoz and E. Majerotto, JCAP **0707**, 015 (2007) [arXiv:0704.2606 [astro-ph]].