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The 1986 CODATA Recommended Values of the Fundamental Physical Constants

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Abstract

This paper gives the values of the basic constants and conversion factors of physics and chemistry resulting from the 1986 least-squares adjustment of the fundamental physical constants as recently published by the CODATA Task Group on Fundamental Constants and as recommended for international use by CODATA. The new, 1986 CODATA set of recommended values replaces its predecessor published by the Task Group and recommended for international use by CODATA in 1973.

Key words: CODATA; conversion factors; fundamental physical constants; least-squares adjustments; recommended values; Task Group on Fundamental Constants.

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CODATA[1] (Committee on Data for Science and Technology) has recently published a report of the CODATA Task Group on Fundamental Constants prepared by the authors [1][2] under the auspices and guidance of the Task Group. The report summarizes the 1986 least-squares adjustment of the fundamental physical constants and gives a set of self-consistent values for the basic constants and conversion factors of physics and chemistry derived from that adjustment. Recommended for international use by CODATA, this 1986 set of values is reprinted here for the convenience of the many readers of the *Journal of Research of the National Bureau of Standards* and to assist in its dissemination throughout the scientific and technological communities. The 1986 CODATA set entirely replaces its immediate predecessor, that recommended for international use by CODATA in 1973. This set was based on the 1973 least-squares adjustment of the fundamental physical constants which was also carried out by the authors under the auspices and guidance of the Task Group [2][3].

As in previous least-squares adjustments of the constants [3][4][5], the data for the 1986 adjustment were divided into two groups: auxiliary constants and stochastic input data. Examples of the 1986 auxiliary constants are the speed of light in vacuum $c \equiv 299\,792\,458$ m/s; the permittivity of vacuum $\mu_0 \equiv 4\pi \times 10^{-7}\,\mathrm{N/A^2}$; the Rydberg constant for infinite mass R_∞; and the quantity $E \equiv 483\,594.0 \times 10^9$ Hz/V which is equal numerically to the value of the Josephson frequency-voltage ratio $2e/h$ (e is the elementary charge and h is the Planck constant) adopted in 1972 by the Consultative Committee on Electricity of the International Committee of Weights and Measures for defining laboratory representations of the volt [6][7]. Quantities in this category are either defined constants such as c, μ_0, and E with no uncertainty, or constants such as R_∞ with assigned uncertainties sufficiently small in comparison with the uncertainties assigned the stochastic input data with which they are associated in the adjustment that they can be taken as exact (i.e., their values are not subject to adjustment in contrast to the stochastic data). In the 1986 adjustment the uncertainty[3] of each auxiliary constant was no greater than 0.02 parts-per-million or ppm. In contrast, the uncertainties assigned the 38 items of stochastic input data considered in the 1986 adjustment were in the range 0.065 to 9.7 ppm. (The 38 items were of 12 distinct types with the number of items of each type ranging from one to six.) Examples of such data are measurements of the proton gyromagnetic ratio γ_p' (uncertainty in the range 0.24 to 5.4 ppm), the molar volume of silicon M(Si)/ρ(Si) (1.15 ppm), and the quantized Hall resistance $R_H = h/e^2$ (0.12 to 0.22 ppm).

Because new results which can influence a least-squares adjustment of the constants are reported continually, it is always difficult to choose an optimal time at which to carry out a new adjustment and to revise the recommended values of the constants. In the present case, all data available up to 1 January 1986 were considered for inclusion, with the recognition that any additional changes to the 1973 recommended values that might result by taking into account more recent data would be much less than the changes resulting from the data available prior to that date.

Each of the 38 items of stochastic data are expressed (using the auxiliary constants as necessary) in terms of five quantities that serve as the ``unknowns'' or variables of the 1986 adjustment. These are α^{-1}, the inverse fine-structure constant: K_V, a dimensionless quantity relating the SI (International System of Units) volt V to the unit of voltage V_{76-BI} maintained at the International Bureau of Weights and Measures (BIPM) using a value of the Josephson frequency-voltage ratio equal numerically to E: V$_{76-BI} = K_V$ V, and thus $2e/h = E/K_V$; K_Ω, a dimensionless quantity relating the SI ohm to the BIPM as-maintained unit of resistance as it existed on 1 January 1985, Ω_{BI85}, based on the mean resistance of a particular group of wire-wound precision resistors: $\Omega_{BI85} = K_\Omega\ \Omega$; d_{220}, the (220) lattice spacing of a perfect crystal of pure silicon at 22.5 °C in vacuum; and μ_μ/μ_p, the ratio of the magnetic moment of the muon to that of the proton. ``Best'' values in the least- squares sense for these five quantities, with their variances and covariances, are thus the immediate output of the adjustment.

After a thorough analysis using a number of least-squares algorithms, the initial group of 38 items of stochastic input data was reduced to 22 items by deleting those that were either highly inconsistent with the remaining data or had assigned uncertainties so large that they carried negligible weight. The adjusted values of the five unknowns, and hence all the other 1986 recommended values that were subsequently derived from them (with the aid of the auxiliary constants), are therefore based on a least-squares adjustment with 17 degrees of freedom.

The 1986 adjustment represents a major advance over its 1973 counterpart; the uncertainties of the recommended values have been reduced by roughly an order of magnitude due to the enormous advances made throughout the precision measurement-fundamental constants field in the last dozen years. This can be seen from the following comparison of the 1973 and 1986 recommended values for the inverse fine-structure constant α^{-1}, the elementary charge e, the Planck constant h, the electron mass m_e, the Avogadro constant N_A, the proton electron mass ratio m_p/m_e, the Faraday constant F, and the Josephson frequency-voltage ratio $2e/h$:

Quantity	Uncertainty of Recommended value in ppm 1973	1986	Change in 1973 Recommended value in ppm resulting from 1986 adjustment
α^{-1}	0.82	0.045	− 0.37
e	2.9	0.30	− 7.4
h	5.4	0.60	−15.2
m_e	5.1	0.59	−15.8
N_A	5.1	0.59	+15.2
m_p/m_e	0.38	0.020	+ 0.64
F	2.8	0.30	+ 7.8
$2e/h$	2.6	0.30	+ 7.8

It is also clear from this comparison that unexpectedly large changes have occurred in the 1973 recommended values of a number of these constants (i.e., a change which is large relative to the uncertainty assigned the 1973 value). These changes are a direct consequence of the 7.8 ppm decrease from 1973 to 1986 in the quantity K_V and the high correlation between K_V and the calculated values of e, h, m_e, N_A, and F. Since $2\ e/h = E/K_V$, the 1986 value of K_V also implies that the value of the Josephson frequency-voltage ratio adopted by the Consultative Committee on Electricity in 1972, which was believed to be consistent with the SI value and which most national standards laboratories adopted to define and maintain their laboratory unit of voltage, is actually 7.8 ppm smaller than the SI value. This unsatisfactory situation should be rectified in the near future [8][9].

The large change in K_V and hence in many other quantities between 1973 and 1986 would have been avoided if two determinations of F which seemed to be discrepant with the remaining data had not been deleted in the 1973 adjustment. In retrospect, the disagreement was comparatively mild. In view of this experience it is important to recognize that there are no similar disagreements in the 1986 adjustment; the measurements which were deleted were so discrepant that they obviously could not be correct, or of such low weight that if retained the adjusted values of the five unknowns would change negligibly. Thus, it is unlikely that any alternate evaluation of the data considered in the 1986 least- squares adjustment could lead to significant changes in the 1986 recommended values. Moreover, the quality of the 1986 data and its redundancy would seem to preclude future changes in the 1986 recommended values relative to their uncertainties comparable to the changes which occurred in the 1973 values.

The 1986 recommended values of the fundamental physical constants are given in five tables. Table 1 is an abbreviated list containing the quantities which should be of greatest interest to most users. Table 2 is a much more complete compilation.

Table 1. Summary of the 1986 recommended values of the fundamental physical constants. An abbreviated list of the fundamental constants of physics and chemistry based on a least-squares adjustment with 17 degrees of freedom. The digits in parentheses are the one-standard-deviation uncertainty in the last digits of the given value. Since the uncertainties of many of these entries are correlated, the full covariance matrix must be used in evaluating the uncertainties of quantities computed from them.

Table 1 or **GIF version (external viewer required)**
Non-Si units used with SI or **GIF version**

Table 2. The 1986 recommended values of the fundamental physical constants. This list of the fundamental

constants of physics and chemistry is based on a least-squares adjustment with 17 degrees of freedom. The digits in parentheses are the one-standard- deviation uncertainty in the last digits of the given value. Since the uncertainties of many of these entries are correlated, the full covariance matrix must be used in evaluating the uncertainties of quantities computed from them.

Universal Constants or **GIF version**
Electromagnetic Constants or **GIF version**
Atomic Constants or **GIF version**
Electron or **GIF version**
Muon or **GIF version**
Proton or **GIF version**
Neutron or **GIF version**
Deuteron or **GIF version**
Physico-Chemical Constants or **GIF version**

Table 3 is a list of related ``maintained units and standard values,'' while table 4 contains a number of scientifically, technologically, and metrologically useful energy conversion factors. Finally, table 5$_4$ is an extended covariance matrix containing the variances, covariances, and correlation coefficients of the unknowns and a number of different constants (included for convenience) from which the like quantities for other constants may be readily calculated. Such a matrix is necessary, of course, because the variables in a least-squares adjustment are statistically correlated. Thus, with the exception of quantities which depend only on auxiliary constants, the uncertainty associated with a quantity calculated from other constants in general can be found only with the use of the full covariance matrix.

Table 3. Maintained units and standard values. A summary of ``maintained'' units and ``standard'' values and their relationship to SI units, based on a least-squares adjustment with 17 degrees of freedom. The digits in parentheses are the one-standard-deviation uncertainty in the last digits of the given value. Since the uncertainties of many of these entries are correlated, the full covariance matrix must be used in evaluating the uncertainties of quantities computed from them.

Standard Values or **GIF version**
"As-Maintained" Electrical Units or **GIF version**
X-Ray Standards or **GIF version**

Table 4. Energy conversion factors. To use this table note that all entries on the same line are equal; the unit at the top of a column applies to all of the values beneath it. **Example**: $1\ \mathrm{eV} = 806\,554.10\ \mathrm{m}^{-1}$.

Energy Conversion Factors for J, kg, m-1, Hz or **GIF version**
Energy Conversion Factors for K, eV, u, hartree or **GIF version**

Table 5. Expanded covariance and correlation coefficient matrix for the 1986 recommended set of fundamental physical constants. The elements of the covariance matrix appear on and above the major diagonal in (parts in $10^9)^2$; correlation coefficients appear in *italics* below the diagonal. The values are given to as many as six digits only as a matter of consistency. The correlation coefficient between m_e and N_A appears as -1.000 in this table because the auxiliary constants were considered to be exact in carrying out the least-squares adjustment. When the uncertainties of m_p/m_e and M_p are properly taken into account, the correlation coefficient is -0.999 and the variances of m_e and N_A are slightly increased.

Expanded covariance and correlation coefficient matrix or **GIF version**

To use table 5, note that the covariance between two quantities Q_k and Q_s which are functions of a common set of variables $x_i\ (i = 1, \ldots, N)$ is given by

$$v_{ks} = \sum_{i,j=1}^{N} \frac{\partial Q_k}{\partial x_i} \frac{\partial Q_s}{\partial x_j} v_{ij} \qquad (1)$$

where v_{ij} is the covariance of x_i and x_j. In this general form, the units of v_{ij} are the product of the units of x_i and x_j and the units of v_{ks} are the product of the units of Q_k and Q_s. For most cases of interest involving the fundamental constants, the variables x_i may be taken to be the fractional change in the physical quantity from some fiducial value, and the quantities Q can be expressed as powers of physical constants Z_j according to

$$Q_k = q \prod_{j=1}^{N} Z_j^{Y_{kj}} \quad , \qquad (2)$$

where q is a numerical factor. If the variances and covariances are then expressed in relative units, eq (1) becomes,

$$v_{ks} = \sum_{i,j=1}^{N} Y_{ki} Y_{sj} v_{ij} \quad , \qquad (3)$$

where the v_{ij} are to be expressed for example, in (parts in $10^9)^2$. Equation (3) is the basis for the expansion of the covariance matrix to include e, h, m_e, N_A, and F.

In terms of correlation coefficients defined by $r_{ij} \equiv v_{ij}(v_{ii}v_{jj})^{-1/2} \equiv v_{ij}/\epsilon_i\epsilon_j$, where ϵ_i is the standard deviation ($\epsilon_i^2 = v_{ii}$), we may write, from eq (3),

$$\epsilon_k^2 = \sum_{i=1}^{N} Y_{ki}^2 \epsilon_i^2 + 2 \sum_{j<i}^{N} Y_{ki} Y_{kj} r_{ij} \epsilon_i \epsilon_j \quad , \qquad (4)$$

where the standard deviations are to be expressed in relative units.

As an example of the use of table 5, consider the calculation of the uncertainty of the Bohr magneton $\mu_B = e\hbar/2m_e(\hbar = h/2\pi)$. In terms of the variables of the 1986 adjustment this ratio is given by

$$\mu_B = [2\pi\, \mu_0\, R_\infty\, E]^{-1}\ (\alpha^{-1})^{-3}\ K_V \quad , \qquad (5)$$

where the quantities in brackets are auxiliary constants taken to be exact. Using eq (3)$_5$ and letting α^{-1} correspond to $i = 1$ and K_V to $i = 2$ gives

$$\epsilon^2_{\mu_B} = Y_1^2\, v_{11} + 2Y_1Y_2v_{12} + Y_2^2 v_{22} \quad . \qquad (6)$$

Comparing eq (5) with eq (2) yields $Y_1 = -3$ and $Y_2 = 1$. Thus eq (6) and table 5 lead to

$$\epsilon^2_{\mu_B} = [9(1997) - 6(-1062) + 1(87\,988)] \times (10^{-9})^2 \qquad (7)$$

or $\epsilon_{\mu_B} = 0.335$ ppm. An alternate approach is to evaluate $e\hbar/2m_e$ directly from table 5; then e corresponds to $i = 5$, h to $i = 6$, and m_e to $i = 7$ with $Y_5 = Y_6 = 1$ and $Y_7 = -1$. Then

$$\epsilon^2_{\mu_B} \;=\; Y_5^2 v_{55} + 2Y_5Y_6v_{56} + Y_6^2 v_{66} + 2Y_5Y_7v_{57} + 2Y_6Y_7v_{67} + Y_7^2 v_{77} \quad ,$$

which leads to

$$\begin{aligned}\epsilon^2_{\mu_B} \;=\;& [1(92\,109) + 2(181\,159) + 1(358\,197) - 2(175\,042) - 2(349\,956 \\ & +1(349\,702)] \times (10^{-9})^2 \quad , \qquad (8b)\end{aligned}$$

which also yields $\epsilon_{\mu_B} = 0.335$ ppm.

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