Multiple-Access Capabilities of Amplify-and-Forward Relaying

Abdurrahman Alfitouri and Khairi Ashour Hamdi
SSchool of Electrical and Electronic Engineering, The University of Manchester, Manchester, UK

Abstract—This paper is concerned with the performance of a gateway connecting a group of independent users to their destinations and employ the random slotted-Aloha type protocol. The users are isolated from their destinations and the only way to communicate with their destinations is through the common gateway which acts as a blind amplify and forward relay. Users are randomly distributed around the gateway and experience a Rayleigh or Nakagami-m fading channels. This paper develops a new accurate models for the multiple access interference at both the gateway and the destinations taking into account, in addition to the random number of the active users and their locations, the effects of fading and thermal noise at both the relay and the destinations. This leads to the derivation of new analytic expressions for the overall spectral efficiency which can be used to estimate the throughput of the aloha based gateways in Rayleigh and Nakagami-m fading channels and to study the impact of the different system parameters into their efficiency. The accuracy of the new mathematical results is confirmed by Monte Carlo simulation.

Index Terms—Amplify and Forward, ALOHA, multiple-access capabilities, moment generation function, Nakagami fading, Rayleigh fading

I. INTRODUCTION

Multiple access techniques are used to allow a large number of users to share the allocated frequency in the most efficient way. The aim of this work shows the capabilities of multiple access technique AF relaying, taking into account the random number and locations of the active users. This work considers slot-ALOHA protocol, where a number of users are isolated and the only way to communicate with their destinations through a gateway. This model is used widely in underwater acoustic sensor networks which have many applications such as pollution monitoring, undersea oilfields and military applications [1], [2]. However, in acoustic networks, there are factors needs to be taken into account such as speed of the signal, propagation delay and frequency band [3].

Another application of this scenario is existed in satellite communications, where a satellite relaying the Uplink carrier into a down link [4]. Moreover, this idea became more popular in cooperative wireless communication systems, where a relay working as a connection between the source and the destination, when the channel between them can not be reliable due to some problems such as shadowing, deep fade, and large distance [5], [6].

Amplify-and-forward repeater enhanced random access in single cell wireless communications have been reported in [7], where the author investigate the impact of amplify and forward repeaters to enhance slot-ALOHA based random access throughout in single cell. However, the author consider more than one relay in this model, instead of one common relay. In addition, he represent the users by one mobile station, which make this model interference free.

The sum rate of multi-pair AF relaying with very large antenna arrays have been proposed in [8], where the each user has only single antenna, while the relay is equipped with very large antenna. However, this model consider multi antenna at relay, as well as, the random number and location of the users did not taken into account. The end-to-end performance of a two-hops system with non-regenerative relays over flat Rayleigh-fading channels have been proposed in [9], where the outage probability formulas for noise limited systems are derived. However, the authors considered fixed gain relay instead of channel state information (CSI) assisted relays, as well as one point communicate with another point which means interference free.

In this paper, we derive a simple mathematical expression for spectral efficiency, where the gain of the gateway considered as variable channel state information (CSI) assisted gateway. We find the overall spectral efficiency of this model, by find the expectation in equation (8). This kind of averaging requires at least (2K+2) folds numerical integrations, to find this expectation by direct approach. We investigate a simple useful Lemma to evaluate this averaging, where the result includes a single integral that can be evaluated easily. We used the moment generating function (MGF) which considered as the powerful technique for digital communication systems to simplify analysis performance of such system [10], [11]. Our results include two integrations instead of (2K+2) averaging which can be found directly.

Exact-form expressions for the spectral efficiency of these models are derived. Furthermore, simulations are provided to validate our analysis. The results of the simulation show that the spectral efficiency of the proposed system effects by factors such as users number, users status, and path loss exponent.

The remainder of this paper is organized as follow. In Section II the system model is described briefly. The Raleigh fading channels analysis and exact-form are presented in Section III. In Section IV, analysis for Nakagami fading channels considered. In section V, results are presented and discussed. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

The system model under consideration has K random number of active users, distributed uniformly in the service area around the gateway. The service area is considered as a circular with radios of D. In this model, we will represent K by binomial random variable with probability \( P_i(K = i) = \binom{M}{i} \rho^i (1 - \rho)^{M-i}, \) where \( M \) is the total number of users, and \( \rho \) probability user is (idle/busy), which takes a value from 0 ≤ ρ ≤ 1. each user has random location, which represented by random variable with uniform distribution \((2\pi/D^2)\). The received composite signal at the relay is

\[ y_r = \sum_{k=1}^{K} \sqrt{P_k} \sigma_k h_k r_k^{\frac{\alpha}{2}} + n_g \quad (1) \]
where $p_k$ is the transmitted power by user $k$, $x_k$ is the transmitted symbol with unit power, $h_k$ is the complex channel gain between user $k$ and the gateway, $r_k$ is the distance between user $k$ and gateway, $\alpha$ path loss exponent, and $n_g$ is the additive white Gaussian noise (AWGN) of the gateway. The gateway will receive $y_i$, and amplify it with gain of $G$, where according to channel state information (CSI) assisted AF relaying, to satisfy the peak power concentration, the gain of the gateway is given as

$$G = \left(\sum_{k=1}^{K} \frac{P_k}{p_k^2 \alpha} + N_0\right)^{-1/2}$$

where $P_g$ is the power of the gateway. In the second hop, the gateway sends the signal that received from the users. The received signal at the destination from $i$th user can be expressed as

$$y_i = g_iG\left(\sqrt{p_i}x_ih_i^{\frac{-\alpha}{2}} + n_G\right) + \left(\sum_{k \neq i} \sqrt{p_k}h_kx_k^{\frac{-\alpha}{2}}\right) + n_D$$

where $g_i$ is the complex channel gain between the gateway and the destination $i$, whereas $n_D$ is the AWGN at the destination. The signal-to-interference plus noise ratio (SINR) can be written as

$$\text{SINR}_i = \frac{p_i|g_i|^2|g_i|^2r_i^{-\alpha}}{|g_i|^2\left(N_0 + \sum_{k \neq i} P_k|h_k|^2r_k^{-\alpha}\right) + N_0/G^2}$$

By substituting of the gain of the gateway in (2) into $\text{SINR}_i$ in (4) we get

$$\text{SINR}_i = \frac{p_i|g_i|^2}{|g_i|^2\left(N_0 + \sum_{k \neq i} P_k|h_k|^2r_k^{-\alpha}\right) + N_0/G^2}$$

The final form of $\text{SINR}_i$ can be expressed as

$$\text{SINR}_i = \frac{\sum_{k \neq i} P_k|h_k|^2r_k^{-\alpha}}{\sum_{k \neq i} P_k|h_k|^2r_k^{-\alpha} + N_0/G^2}$$

The overall spectral efficiency of the two phase gateway system is expressed as

$$\bar{R} = \frac{1}{2} E\left\{K\text{log}_2(1 + \text{SINR}_k)\right\}$$

where $[\text{log}_2(1 + \text{SINR}_k)]$ is the instantaneous spectral efficiency of the users. We assume that the users are independent and identically distributed, the overall spectral efficiency of this model can be written as

$$\bar{R} = \frac{1}{2} E\left\{K[\text{log}_2(1 + \text{SINR}_1)]\right\}$$

where $\text{SINR}_1$, is the SINR of the first user, the factor $\frac{1}{2}$ comes from the fact that the communications between the source and the destination is performed in two phases [12], [13].

The overall spectral efficiency of our proposed model can be found from equation (8), where we need to find the expectation of this equation with respect to the following random variables: $h = \{h_1, h_2, \ldots, h_K\}, \alpha = \{r_1^{-\alpha}, r_2^{-\alpha}, \ldots, r_K^{-\alpha}\}$, and $K$, which equal to the set of $(2K + 2)$ random variables.

The direct approach to compute this expectation is difficult to obtain in general, as it may require to compute $(2K+2)$ folds convolution integral to get the distribution of the $\text{SINR}_1$. As result, in this paper we investigate a simple useful Lemma 1 to evaluate the averaging in equation (8), where the result includes a single integral that can be evaluated easily.

**Lemma 1:** In the special case of two random variables $U$ and $V$ are independent then

$$\mathbb{E}\left[\ln\left(1 + \frac{UV}{1 + U + V}\right)\right] = \int_0^\infty \frac{1}{z} \left(1 - M_U(z)\right) \left(1 - M_V(z)\right) e^{-z} dz$$

where $M_U(z) = \mathbb{E}[e^{-zU}]$ and $M_V(z) = \mathbb{E}[e^{-zV}]$ are the moment generation functions of $U$ and $V$, respectively.

**Proof:** the proof is given in the Appendix

From (9), the overall spectral efficiency expression can be given by

$$\bar{R} = \frac{1}{2} \text{log}_2 e \left[ K \int_0^\infty \frac{1}{z} \left(1 - M_U(z)\right) \left(1 - M_V(z)\right) e^{-z} dz \right]$$

In this paper, we will apply two different scenarios. Firstly, we will model the complex channels gain between the gateway and the source and the destination as Rayleigh fading channel. Secondly, we will model these complex channels gain as Nakagami-m fading channel. We will evaluate the overall spectral efficiency of the proposed model by apply the equation (10) for different two scenarios. We will show the mathematical analysis of each scenario individually.

### III. CASE 1: RAYLEIGH FADING CHANNELS

In first scenario, All channels are assumed to be subject to independent and identically distributed (i.i.d) complex Gaussian fading with zero mean and unit variance, e.g., $h, g_i \sim CN(0,1)$. As a result, the magnitude of channels, $h$ and $g_i$, follow a Rayleigh distribution. Therefore, the probability density function (PDF) of power channel gains, $|h|^2$ and $|g_i|^2$, is an exponential distribution. To find the spectral efficiency, by calculating the expected value in (10), we need to find $M_U(z)$ and $M_V(z)$. Let $U = \frac{\sum_{k \neq i} P_k|h_k|^2r_k^{-\alpha}}{\sum_{k \neq i} P_k|h_k|^2r_k^{-\alpha} + 1}$ and $V = \frac{P_0|g_i|^2}{N_0}$. Also, we assume equal transmit power for all users. This assumption does not affect our analytical methodologies. To normalize the received signal to noise ratio (SNR), $SNR = \frac{N_0}{\sum_{k \neq i} P_k|h_k|^2r_k^{-\alpha}}$ is taken as a reference at the edge of the serves area. Hence

$$U = \frac{|h_1|^2r_1^{-\alpha}}{\sum_{k \neq i} |h_k|^2r_k^{-\alpha} + \frac{1}{SNR}}$$

To find $M_U(z)$, we need the PDF of $U$, which is not easy to find directly. Therefore, $M_U(z)$ can be evaluated with the help of Pr ($U > u$)

$$\text{Pr}(U > u) = \text{Pr}\left(\frac{|h_1|^2r_1^{-\alpha}}{\sum_{k \neq i} P_k|h_k|^2r_k^{-\alpha} + 1 + \alpha} > u\right)$$

where $\alpha = \frac{1}{SNR}$, (12) can be rewritten as

$$\text{Pr}(U > u) = \text{Pr}\left(|h_1|^2 > u \left(\frac{\sum_{k \neq i} P_k|h_k|^2r_k^{-\alpha} + \alpha}{N_0}\right)\right)$$

(13)
(13) has more than one random variables, we condition (13) on \( |h|^2, z^{-\alpha}, K \), therefore
\[
\Pr (U > u \mid |h|^2, z^{-\alpha}, K) = \Pr \left( |h|^2 > u \left( \sum_{k \neq i} |h_k|^2 a_k^{\alpha} + a \right) \mid |h|^2, z^{-\alpha}, K \right)
\]
\[
= \frac{\int_{|h|^2}^{\infty} e^{-u (\sum_{k \neq i} |h_k|^2 a_k^{\alpha} + a)} d|h|^2}{\int_{0}^{\infty} e^{-u (\sum_{k \neq i} |h_k|^2 a_k^{\alpha} + a)} d|h|^2}
\]
\[
(K)^{-1} e^{-u \rho M}
\]
\[
(17)
\]
\[
\text{which given by}
\]
\[
\log_{2} e \int_{0}^{\infty} \frac{1}{z} \left( E \left[ K \right] - E \left[ KM_U \left( z \right) \right] \right) \left( 1 - V \left( z \right) \right) e^{-z} dz
\]
\[
(K)^{-1} b^{M-1}
\]
\[
(22)
\]
\[
\text{This leads to the following tractable explicit expression for the average in (10) for the overall spectral efficiency of this model, which is shown in (18) at the top of this page, where}
\]
\[
\text{the probability density function}
\]
\[
\log_{2} e \int_{0}^{\infty} \frac{1}{z} \left( \rho M - E \left[ KM_U \left( z \right) \right] \right) \left( 1 - V \left( z \right) \right) e^{-z} dz
\]
\[
(23)
\]
\[
\text{IV. CASE 2: NAKAGAMI FADING CHANNELS}
\]
\[
\text{In second scenario, the magnitude of complex channels gain, } h \text{ and } g_t, \text{ follows Nakagami Fading Channels, with two}
\]
\[
\text{parameters: a shape parameter } m \text{ and a second parameter controlling spread, } \Omega. \text{ Therefore, the probability density function (PDF) of power channel gains, } |h|^2 \text{ and } |g_t|^2, \text{ is a gamma distribution. The Nakagami fading represents a wide range of multipath channels through change the } m \text{ fading parameter.}
\]
\[
\text{For example, when } m = \frac{1}{2}, \text{ it represents one sided Gaussian distribution. As well as a special case of Nakagami-m fading, when } m = 1, \text{ which is Rayleigh fading distribution. Moreover, Rician distribution can also be closely approximated, when } m > 1. \text{ It is good to mention that Nakagami-m distribution fit for urban radio multipath environments and indoor applications.}
\]
\[
\text{For } M_U \left( z \right), \text{ where in our model } U = \frac{|h|^2 \sum_{k \neq i} |h_k|^2 a_k^{\alpha} + a^\alpha}{\sum_{k \neq i} |h_k|^2 a_k^{\alpha} + a^\alpha}, \text{ it can be written as } U = \frac{x}{y + \alpha}, \text{ while } x = |h|^2, y = r_i^\alpha \sum_{k \neq i} |h_k|^2 a_k^{\alpha} \text{ and } b = \frac{a^\alpha}{\sum_{k \neq i} |h_k|^2 a_k^{\alpha}}.
\]
\[
\text{According to [10, eq. (7)], the } E \left[ g \left( \frac{x}{y + \alpha} \right) \right] \text{ can be found as}
\]
\[
E \left[ g \left( \frac{x}{y + \alpha} \right) \right] = g (0) + \int_{0}^{\infty} g_m (s) M_y (ms) e^{-smb} ds
\]
\[
(25)
\]
\[
M_y (ms) = E \left[ e^{-smb} \right] \text{ is the MGF of } y \text{ and can be written as}
\]
\[
M_y (ms) = E \left[ e^{-smb} \right] \text{ can be written as}
\]
\[
K \prod_{k \neq i} E \left[ e^{-smb} \right]
\]
\[
(26)
\]
\[ R = \log_2(\frac{1}{z} z M) - \left( \sum_{i=0}^{\infty} \frac{(zSNR)^i}{i!} \left( D_e^2 \rho d\alpha \right)^{\frac{M-1}{2}} \right) \left( 1 - \frac{1}{1 + zSNR} \right) e^{-z} dz \]

Taking the expected value in (26), where \( |h_k|^2 \), \( r_k^{-\alpha} \) and \( r_k^{-\alpha} \) stand for the power of complex channels gain (gamma distribution), and random location of useful signal and interference (uniform distribution), respectively. Therefore, \( M_0 (m) \) can be written as (27), which shown on the top of next page, where \( 2F_1(1; \frac{2}{\alpha}; \frac{2 + \alpha}{\alpha} - \frac{D_e^2}{ur^2}) \) is the confluent hypergeometric function.

Back to (25), where \( g(s) = \exp(-s) \). This leads to \( g(0) = 1 \) and \( g_m(s) = -m \cdot F_1(1 - m; 2; s) e^{-s} \), this yields to (29), which shown on the top of next page. We assume that \( z = \frac{s}{m_u}n \) and \( s = \frac{dau}{m} M_u(z) \), becomes as (30) which shown on the top of next page.

Similarly, the second MGF in (10), i.e. \( M_V(z) \), is found as

\[ M_V(z) = \mathbb{E}[e^{-zV}] = \int_0^\infty e^{-zSNR} \frac{1}{\Gamma(m)} V^{m-1} e^{-\frac{V}{mSNR} + 1} dV = \frac{1}{\left( \frac{z}{mSNR} + 1 \right)^m} \]

By substituting (30) and (28) into (10), and applying the same methodology which used in Rayleigh fading case to find the expected value of random number of active users \( \mathbb{E}[K] \), we can find the overall spectral efficiency, which is shown in (31), at the top of the sixth page.

Even though, the result in (31) includes three integrations for the overall spectral efficiency of this model, in the case of Nakagami-m fading channel. It still easier than the direct method to find the overall spectral efficiency. This result reflects that, our mathematical analysis and the useful Lemma1 valid for different types of channel fading, and can be used in different system models.

V. NUMERICAL AND SIMULATION RESULTS

In this section, the spectral efficiency achieved by the multiple-access of AF gateway which evaluated by Monte-Carlo simulations, and compared to the derived asymptotic results. The channels fading has Rayleigh and Nakagami distributions. However, Different figures of spectral efficiency are plotted corresponding to various user number, SNR values, path loss exponent, and user status. The status factor (\( \rho \)), path loss exponent (\( \alpha \)), and SNR has taken different values and user number set as \( M = 10 \) users, whereas the radius of the cell (\( D \)) set as 1000m.

A. Rayleigh Fading Channel

In Fig. 1, the spectral efficiency is plotted versus the number of users for different values of \( \rho \). From this figure, it is clear that as the number of users increases the spectral efficiency increases correspondingly until it maintains a stable level, where the spectral efficiency around 0.38 bits/s/Hz. In general, when \( \rho = 1 \), the better performance is achieved compared to the case when \( \rho = 0.7 \), which better than \( \rho = 0.4 \). This is due to the fact that the sources of the useful signal in the first case is larger than that of the latter two which leads to achieving better SNR value.

Fig. 2 illustrates the spectral efficiency as a function of the SNR, when the user status \( \rho = 1 \), 0.5 and 0.1. It is clearly noticeable that, in noise limited region or low SNR region, as SNR is increased, the system performance improves dramatically. Moreover, the spectral efficiency when \( \rho = 1 \) and 0.5 is higher than when \( \rho = 0.1 \) at low SNR. However, at high SNR, the spectral efficiency for \( \rho = 0.1 \) exceeds the spectral efficiency of \( \rho = 1 \) and 0.5 by 0.2 bits/s/Hz. This can be interpreted that, in interference limited systems, the noise contribution in the schemes becomes minimal as (\( \frac{1}{SNR} \rightarrow 0 \)), which means as \( \rho \) decrease, the interference of the system is decres as well.

B. Nakagami Fading Channel

In the case of the Nakagami channel, the results show a similar trend to that of Rayleigh fading channel, i.e. spectral efficiency increases as the number of users increases. In Fig. 3, when \( m = 5 \), the spectral efficiency increases until it maintains a stable level when the spectral efficiency achieved 0.45 bits/s/Hz. In general, the performance of three cases (\( \rho = 1, 0.7 \) and 0.4) was the same as the Rayleigh fading channel case as a result of the same reasons outlined earlier.

In case of high SNR, which illustrated in Fig. 4, the initial value of spectral efficiency is high and it increases dramatically as \( \rho \) increases. Moreover, the effect of the noise is limited, which help to improve the system performance. This increases continue until it get the maximum value when \( \rho = 0.2 \), as \( \rho \) increases, the random number of users is increases, which mean the interference is increases as well, as results the spectral efficiency start decrease until it get the stable level. The effect...
\[ M_y (ms \mid K) = \int_0^D \left( \frac{2 \left( \frac{m}{s(\pi)} \right)^m}{2 + m\alpha} \right)^{-K/2} e^{-\frac{2r_i D}{2m\alpha}} \, dr_i \] (27)

\[ E \left[ \exp \left( \frac{x}{y + b} \mid K \right) \right] = 1 - m \int_0^\infty F_1(1 - m; 2; s) e^{-s} \times \int_0^D \left( \frac{2 \left( \frac{m}{s(\pi)} \right)^m}{2 + m\alpha} \right)^{-K/2} e^{-\frac{2r_i D}{2m\alpha}} \, dr_i, ds \] (29)

\[ M_u (z \mid K) = 1 - m \int_0^\infty F_1(1 - m; 2; \frac{zU}{m}) \times \int_0^D \left( \frac{2 \left( \frac{m}{u(\pi)} \right)^m}{2 + m\alpha} \right)^{-K/2} e^{-\frac{2r_i D}{2m\alpha}} \, dr_i, du \] (30)

![Fig. 2: The Spectral Efficiency vs SNR when \( \rho = 1, 0.5, 0.1 \) and \( M = 10 \) users.](image1)

![Fig. 3: The Spectral Efficiency vs M when SNR = 1dB, \( \rho = 0.4, 0.7, 1 \).](image2)

of SNR is positive on performance of the system, where higher SNR leads to better performance.

VI. CONCLUSIONS.

In this paper, we have analyzed the performance of a gateway connecting a group of independent users to their destinations and employ the random slotted-Aloha type protocol. The users are randomly distributed around the gateway and experience a Rayleigh or Nakagami-m fading channels. We develop a new accurate models for the multiple access interference at both the gateway and the destinations taking into account, in addition to the random number of the active users and their locations, the effects of fading and thermal noise at both the relay and the destinations. We validated our results accuracy with Mont Carlo simulations. We examined the impact of increasing number of users on the system performance, we found that as the number of users increase, the spectral efficiency increase as well, and then, it maintaining a stable level. In addition, we showed the effect of important factors in this model on the system performance, SNR, user status.

APPENDIX

PROOF OF Lemma 1

In this appendix, we aims to prove the useful Lemma 1, which presented in equation (9), where two independent random variables \( U \) and \( V \) in this equation are considered. The right hand side of (9) can simplify as

\[ \left( 1 + \frac{UV}{1 + U + V} \right) = \frac{1 + U + V + UV}{1 + U + V} = \frac{(1 + U)(1 + V)}{1 + U + V} \] (32)

By apply the rules of ln for the two sides of equation (32),
After some algebra to the (35), it can be written as
\[
\ln \left( 1 + \frac{UV}{U + V} \right) = \int_0^\infty \frac{1}{z} \left( 1 - e^{-zU} \right) e^{zV} dz
\] (36)

Which can be expressed as
\[
\ln \left( 1 + \frac{UV}{U + V} \right) = \int_0^\infty \frac{1}{z} \left( 1 - e^{-zU} \right) \left( 1 - e^{-zV} \right) e^{z} dz
\] (37)

This leads to the desired result in equation (9).

REFERENCES


vol. 1, 2000, pp. 7–12 vol.1.

user cooperation diversity,” in Int. Symp. Inf. Theory (ISIT), Aug 1998, 
pp. 156–.

[7] Y. Xue, “Amplify-and-forward repeater enhanced random access in 
single-cell wireless communications,” in IEEE Int. Symp. Personal, 

pair amplify-and-forward relaying with very large antenna arrays,” in Int. 

systems with relays over rayleigh-fading channels,” IEEE Trans. Wireless 


relay selection on the capacity of communications systems with outdated 
Conf. (WCNC), Apr. 2013, pp. 3720–3725.

performance limits and space-time signal design,” IEEE J. Select. Areas 

\[
\mathbb{R} = \frac{1}{2} (\log_2 e) \int_0^\infty \int_0^1 \rho^M \left( \rho M - \int_0^\infty \int_0^D \rho M \cdot F_1(1-m; 2; \frac{2U}{m}) \right) dx
\]

\[
\times \left( 1 - \rho + \rho \left( \frac{2}{m} \right) \int_0^\infty \int_0^D \rho M \cdot F_1(1-m; 2; \frac{2U}{m}) \right) dz
\]

\[
\times e^{-z\left( \frac{2r_i}{D^2} \right)} e^{zU} \int \frac{1 + \left( \frac{1}{m} SNR + 1 \right)^m}{\left( \frac{1}{m} SNR + 1 \right)^m} \frac{2r_i}{D^2} dz \] (31)