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I. INTRODUCTION

In wireless relay networks, multiple relays can help a source to send its information symbols to a destination. Since the relays cooperatively form a virtual array of transmit antennas, such a cooperation can significantly improve the transmission reliability of the source signals [1]. Should the relays know both the backward, i.e., source-to-relay (S → R) and forward, i.e., relay-to-destination (R → D) channels, they can beam their retransmitted signals such that the received signals at the destination are coherently constructed. This cooperative strategy, referred to as distributed beamforming, was investigated in [2]–[5]. In particular, reference [2] considered the problem of controlling the transmitted power at each relay in order to maximize the signal-to-noise ratio (SNR) at the destination. Reference [3] studied a distributed relay strategy for wireless sensor networks to obtain a certain target SNR at the destination, whereas reference [4] investigated a similar problem with the objective of minimizing the sum of relay powers, referred to as “sum relay power” hereafter. More recently, distributed beamforming with second-order statistics of the channel state information was examined in [5]. However, it is noted that most of the early works in distributed beamforming only consider the system with one source and one destination.

The focus of this paper is to study the optimal distributed beamforming designs in a multiuser multi-relay network using amplify-and-forward (AF) protocol. Under a similar system setup, distributed beamforming in a multiuser system was studied in [6], where multiple pairs of source and destination were simultaneously assisted by multiple relays on the same channel. The optimization problem was formulated as a non-convex quadratically constrained quadratic program (QCQP). Through convex relaxation techniques, the problem can be efficiently solved [6].

To avoid inter-user interference at the destinations, this work assumes that all source-destination pairs operate in orthogonal channels. With full channel state information (CSI) at the relays, the distributed beamforming designs are then optimized to minimize the sum relay power with guaranteed Quality of Service (QoS) in terms of SNR at the destinations. Considered are optimization problems with and without per-relay power constraints. Since the two optimization problems might be cast as convex second-order conic programs (SOCPs), they can be solved effectively by any conic software package. However, as the required conic package is not always readily available, the approach may not be suitable in real-time communications. To overcome this difficulty, the paper also proposes simple and fast numerical algorithms to solve the two problems under consideration.

Notations: Superscripts (·)ᵀ, (·)*, and (·)ᴴ stand for transpose, complex conjugate, and complex conjugate transpose operations, respectively; x* denotes the optimal value of the variable x; CN(0, σ²) denotes a circularly symmetric complex Gaussian random variable with variance σ².

II. SYSTEM MODEL

We consider a R-relay network with N pairs of source-destination users (SN, DN, n = 1, . . . , N). All relays are assumed to work in a half-duplex mode, i.e., they cannot receive and transmit at the same time. Assume that there is no direct link between any source and destination and the communication between the two terminals of each user is assisted by all the relays, and implemented in two transmission stages. In the first stage, each user’s source broadcasts its signals to all the relays. For the nth user, given sₙ as the source signal, the received signals at the relays are given as rₙ = fₙsₙ + zₙ ∈ Cₚ×1, where fₙ = [fₙ,1, . . . , fₙ,R]ᵀ, and fₙ,i is the channel from the nth source to the ith relay; zₙ,i represents the AWGN at the relays, whose components are i.i.d. CN(0, σ²ᵣ). At the ith relay, the received signal for the nth user is processed by a complex beamforming weight wₙ,i, which is to be designed. Let wₙ = [wₙ,1, . . . , wₙ,R]ᵀ be the vector of the beamforming weights for the nth user. Also define Wₙ = diag(wₙ). Accordingly, by applying the AF protocol
In the second stage of transmission, all the relays simultaneously transmit to the $n$th user’s destination. Similar to the first stage, the transmission to each user’s destination is carried out over orthogonal channels to avoid inter-user interference. Let $g_n = [g_{1,n}, ..., g_{R,n}]^T$ represent the channels from $R$ relays to the $n$th destination. The received signal at the $n$th destination is written as

$$y_n = g_n^T t_n + z_{dn} = g_n^T W_n f_n s_n + g_n^T W_n z_{rn} + z_{dn},$$

where $z_{dn} \sim \mathcal{CN}(0, \sigma^2_D)$ represents the AWGN at the destination. Define $h_n^* = [h_{1,n}^*, ..., h_{R,n}^*]^T = f_n \circ g_n$, where $\circ$ represents the component-wise Hadamard product. As a result, $h_n^*$ models the effective channel from source-$n$ to destination-$n$ through all the relays, excluding the beamforming factors. Then, one has $g_n^T W_n f_n = h_n^R w_n$. Let $\sigma_n^2 = \mathbb{E}[|s_n|^2]$ be the average transmitted power of the $n$th source. Then, the SNR at the $n$th destination is given by

$$\text{SNR}_n = \frac{\sigma_n^2 |h_n^R w_n|^2}{\sigma_R^2 |G_n^R w_n|^2 + \sigma_D^2},$$

where $G_n = \text{diag}(\|g_{1,n}\|^2, ..., \|g_{R,n}\|^2)$.

Let $p_n$ be the total relay power allocated for the $n$th user. It can be computed as $p_n = \mathbb{E}[\|t_n\|^2] = w_n^H D_n w_n$, where $D_n$ is a $R \times R$ diagonal matrix, with the $ith$ diagonal element $[D_n]_{ii} = \sigma_n^2 |f_{i,n}|^2 + \sigma_R^2$. On the other hand, the transmitted power at the $n$th relay is $P_n = \sum_{i=1}^N \mathbb{E}[|t_{i,n}|^2] = \sum_{i=1}^N w_n^H D_i E_i w_n$, where $E_i$ is a $R \times R$ matrix whose elements are zero, except the $(i,i)$-element, which is $[E_i]_{ii} = 1$. The total transmitted power of all the relays is therefore given by

$$P_{\text{relay}} = \sum_{i=1}^R P_i = \sum_{n=1}^N p_n = \sum_{n=1}^N w_n^H D_n w_n.$$ 

### III. SUM POWER MINIMIZATION

This section considers the optimal design of the beamforming vectors to minimize the sum power at the relays given a set of target SNRs at the destinations. The optimization problem is formulated as follows:

$$\begin{align*}
\text{minimize} & \quad \sum_{n=1}^N p_n \\
\text{subject to} & \quad \text{SNR}_n \geq \gamma_n, \quad \forall n,
\end{align*}$$

where $\gamma_n$ is the target SNR at the $n$th destination. Obviously, this optimization problem can be performed separately through $N$ smaller optimization problems, each corresponds to one user. That is

$$\begin{align*}
\text{minimize} & \quad w_n^H D_n w_n \\
\text{subject to} & \quad \frac{\sigma_n^2 |h_n^R w_n|^2}{\sigma_R^2 |G_n^R w_n|^2 + \sigma_D^2} \geq \gamma_n, \quad \forall n.
\end{align*}$$

There are several approaches to solve the above optimization problem. The first approach, proposed in [5], is to establish $w_n$ in the direction of $D_n^{-1/2} x_n$, where $x_n$ is the principal eigenvector of $D_n^{-1/2} \left( \sigma_n^2 h_n^R h_n^H - \gamma_n \sigma_R^2 G_n \right) D_n^{-1/2}$. Another approach is to cast the SNR constraint as a second-order conic constraint [4]:

$$\sqrt{\frac{\sigma_n^2}{\gamma_n} h_n^R w_n} \geq \left\| \frac{\sigma_R^2 |G_n^R w_n|^2 + \sigma_D^2}{\sigma_D^2} w_n \right\|,$$

then solve the optimization problem as an SOCP. Here, we consider an alternative approach to solve problem (5) by optimizing the relay power factor $p_n$ directly, instead of dealing with the beamforming vector. The proposed approach, which is independent of external software package, also motivates a simple iterative fixed point algorithm. Besides, we note that the proposed algorithm is useful in solving the sum relay power minimization with per-relay power constraints in Section IV and the inverse problem to maximize the SNR in [7].

First, problem (5) can also be recast as

$$\begin{align*}
\text{minimize} & \quad p_n \\
\text{subject to} & \quad \sum_{n=1}^N \frac{\sigma_n^2 |h_n^R w_n|^2}{\sigma_R^2 |G_n^R w_n|^2 + \sigma_D^2} \geq \gamma_n, \\
& \quad w_n^H D_n w_n = p_n.
\end{align*}$$

Second, the following lemma establishes the relationship between the optimal beamforming vector $w_n$ and the allocated relay power $p_n$.

**Lemma 1.** (From [8]) Given $p_n$ as the relay power allocated for user-$n$, the optimal beamforming weights at the relays to maximize the SNR of user-$n$ are

$$w_{n,i} = \frac{\sqrt{\delta_{n,i}} f_{n,i} g_{n,i}^*}{p_n \sigma_R^2 |g_{n,i}|^2 + \sigma_D^2 (\sigma_n^2 |f_{n,i}|^2 + \sigma_R^2)},$$

where the normalization factor $\delta_{n,i}$ is

$$\delta_{n,i} = \frac{p_n}{\sum_{i=1}^R |f_{n,i} g_{n,i}|^2 (\sigma_n^2 |f_{n,i}|^2 + \sigma_R^2)^{1/2}}.$$ 

The corresponding maximum SNR is

$$\text{SNR}_n(p_n) = \sum_{i=1}^R \frac{p_n \sigma_n^2 |f_{n,i}|^2 |g_{n,i}|^2}{p_n \sigma_R^2 |g_{n,i}|^2 + \sigma_D^2 (\sigma_n^2 |f_{n,i}|^2 + \sigma_R^2)}.$$ 

**Proof:** The proof of this lemma is based on the Rayleigh-Ritz theorem [9], and is given in [8].

By applying Lemma 1, one can optimize the power allocation $p_n$ for user-$n$, then determine the optimal beamforming vector accordingly. For notational simplicity, let

$$a_{n,i} = \frac{\sigma_n^2 |f_{n,i}|^2 |g_{n,i}|^2}{\sigma_R^2 |g_{n,i}|^2 + \sigma_D^2 (\sigma_n^2 |f_{n,i}|^2 + \sigma_R^2)}, \quad b_{n,i} = \frac{\sigma_D^2 (\sigma_n^2 |f_{n,i}|^2 + \sigma_R^2)}{\sigma_R^2 |g_{n,i}|^2}.$$ 

Then, the achievable SNR can be written as

$$\text{SNR}_n(p_n) = \sum_{i=1}^R \frac{a_{n,i} p_n}{b_{n,i} + p_n},$$

which means that the achievable SNR at the $n$th destination solely depends on the relaying power $p_n$.  

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The optimization problem is now restated as

\[
\begin{align*}
\text{minimize} & \quad p_n \\
\text{subject to} & \quad \sum_{i=1}^{R} \frac{a_{n,i} p_n}{b_{n,i} + p_n} \geq \gamma_n.
\end{align*}
\]  

(12)

Like the original problem in (5), the problem in (12) is also convex, which then can be solved efficiently. In addition, the structure of the restated problem also reveals several interesting properties of the problem, including its feasibility and solution. Since \(p_n / (b_{n,i} + p_n) < 1\), one has \(\text{SNR}_n = \sum_{i=1}^{R} \frac{a_{n,i} p_n}{b_{n,i} + p_n} < \sum_{i=1}^{R} a_{n,i}\). Thus, if the target SNR \(\gamma_n \geq \sum_{i=1}^{R} a_{n,i}\), the problem will be infeasible.

Now, suppose that the target SNR is set such that the problem is feasible. Since \(\sum_{i=1}^{R} \frac{a_{n,i} p_n}{b_{n,i} + p_n}\) is a monotonically increasing function, the constraint \(\sum_{i=1}^{R} \frac{a_{n,i} p_n}{b_{n,i} + p_n} \geq \gamma_n\) must be met with equality at optimum. Thus, the unique solution of

\[
\sum_{i=1}^{R} \frac{a_{n,i} p_n}{b_{n,i} + p_n} = \gamma_n
\]

(13)

is also the optimal solution to (12).

It is then of interest to find a simple and fast numerical algorithm to solve the \(R\)-th polynomial in (13). The structure in (13) motivates a simple iterative fixed point algorithm to find the optimal \(p_n^*\). By rearranging (13), one has the following simple iteration:

\[
p_n^{(t+1)} = \frac{\gamma_n}{\sum_{i=1}^{R} \frac{a_{n,i}}{b_{n,i} + p_n^{(t)}}}
\]

(14)

If (13) is feasible, then the above iteration will converge from any initial point \(p_n^{(0)} \geq 0\). The convergence analysis of the fixed point iteration is based on the standard function approach introduced in [10]. Denote \(f_n(p_n^{(t)}) = \gamma_n / \left(\sum_{i=1}^{R} \frac{a_{n,i}}{b_{n,i} + p_n^{(t)}}\right)\).

Then the fixed point iteration \(p_n^{(t+1)} = f_n(p_n^{(t)})\) will converge to a unique fixed point \(p_n^*\) if the function \(f_n(p_n)\) has the following properties [10]:

1) **Positivity:** \(f_n(p_n) > 0\) for all \(p_n > 0\).
2) **Monotonicity:** if \(p_n > p_n'\), then \(f_n(p_n) > f_n(p_n')\).
3) **Scalability:** if \(\alpha > 1\), then \(\alpha f_n(p_n) > f_n(\alpha p_n)\).

It is easy to verify that all these three properties are satisfied by the function \(f_n(p_n)\). Thus, the fixed point iteration (14) will surely converge if (13) is feasible. Numerical results show that the proposed algorithm converges in a few iterations.

IV. SUM POWER MINIMIZATION WITH PER-RELAY POWER CONSTRAINTS

In the previous section, sum relay power minimization with guaranteed QoS at the destinations was considered. No restrictions on the individual power at each relay were imposed. However, in practical relay communications, each relay is equipped with its own amplifier and usually has its own power limit. This section considers an approach to uniformly minimize the margin \(P_i / P_i^{\text{max}}\) over all the relays, where \(P_i^{\text{max}}\) denotes the maximum transmitted power of the \(i\)th relay. The problem is stated as follows:

\[
\begin{align*}
\text{minimize} \quad & \alpha \\
\text{subject to} & \quad \text{SNR}_n \geq \gamma_n, \quad \forall n \\
& \quad P_i \leq \alpha P_i^{\text{max}}, \quad \forall i.
\end{align*}
\]

(15)

The approach of minimizing the power consumption margin was first investigated for the multiuser beamforming downlink problem in point-to-point communications [11]. When applied to relay networks, the idea is to serve all the users, while maintaining the balance in power consumption at the relays. The problem is equivalent to

\[
\begin{align*}
\text{minimize} \quad & \alpha \sum_{i=1}^{R} P_i^{\text{max}} \\
\text{subject to} & \quad \sigma^2 \frac{|h_n^H w_n|^2}{\sigma^2 R ^{1/2} |D_n E_i w_n|^2 + \sigma^2_D} \geq \gamma_n, \quad \forall n \\
& \quad \sum_{n=1}^{N} w_n^H D_n E_i w_n \leq \alpha P_i^{\text{max}}, \quad \forall i,
\end{align*}
\]

(16)

which can be interpreted as a sum relay power minimization problem with per-relay power constraint awareness. It is noted that by including the per-relay power constraints, the sum relay power minimization problem in (16) is not the same as the one in (5). At the optimal solution of (16), it might happen that \(\alpha > 1\), i.e., at least one of the per-relay power constraints is violated. As a result, it is infeasible to find the beamforming vectors that meet both the QoS constraints and the per-relay power constraints. In such a case, an inverse problem, which tries to maximize the SNR under per-relay power constraints, may be of interest. This inverse problem is investigated in [7].

It is also noted that the optimization problem stated in (16) is not readily convex. However, as the SNR constraints can be recast as a SOC constraint as in (6), the problem can be transformed into a convex one. Furthermore, by following a similar technique as in Proposition 1 of [11], it is easy to verify that the Lagrangians of the convex and nonconvex forms are the same. As strong duality holds for a convex optimization problem [12], strong duality also holds for problem (16). This means that the optimal value of problem (16) can be found by its dual problem. In the next section, the dual problem of (16) is investigated in detail. The solution of the dual problem will then reveal both the structure of the original problem’s solution and the algorithm to solve it.

A. Beamforming Duality

The Lagrangian of problem (16) is established as

\[
\begin{align*}
\mathcal{L}(\alpha, w_n, \lambda, \mu) = & \quad \alpha \sum_{i=1}^{R} P_i^{\text{max}} + \sum_{i=1}^{R} P_i \left(\sum_{n=1}^{N} w_n^H D_n E_i w_n - \alpha P_i^{\text{max}}\right) \\
& \quad - \sum_{n=1}^{N} \lambda_n \left(\frac{\sigma^2}{\gamma_n} |h_n^H w_n|^2 - \sigma^2 R ^{1/2} |D_n E_i w_n|^2 - \sigma^2_D\right). 
\end{align*}
\]

(17)
Let $Q = \text{diag}(\mu_1, \ldots, \mu_R)$ and $P = \text{diag}(P_1^{\text{max}}, \ldots, P_R^{\text{max}})$. Rearranging the Lagrangian $\mathcal{L}(\alpha, w_n, \lambda, \mu)$ in (17), one has

$$\mathcal{L}(\alpha, w_n, \lambda, Q) = \sum_{n=1}^{N} \lambda_n \sigma_D^2 + \sum_{n=1}^{N} \mathcal{L}_n(w_n, \lambda_n, Q) - \alpha \left( \text{tr}(QP) - \text{tr}(P) \right),$$

where

$$\mathcal{L}_n(w_n, \lambda_n, Q) = w_n^H \left( D_n Q - \frac{\lambda_n \sigma_D^2}{\gamma_n} h_n h_n^H + \lambda_n \sigma_0^2 G_n \right) w_n,$$

which only depends on $w_n$, $\lambda_n$, and $Q$. The dual function of (18) is established as

$$g(Q, \lambda) = \sum_{n=1}^{N} \lambda_n \sigma_D^2 + \min_{w_n} \mathcal{L}(w_n, \lambda_n, Q) - \inf_{\alpha} \left\{ \alpha \left( \text{tr}(QP) - \text{tr}(P) \right) \right\}.$$

It is clear that if $D_n Q - \frac{\lambda_n \sigma_D^2}{\gamma_n} h_n h_n^H + \lambda_n \sigma_0^2 G_n$ is not a positive semidefinite matrix, there exists $w_n$ to make $\mathcal{L}_n$ unbounded below. Similarly, if $\text{tr}(QP) - \text{tr}(P) > 0$, it is possible to find $\alpha > 0$ to make $\mathcal{L}$ unbounded below. Thus, the dual problem is stated as

$$\begin{align*}
\max_{Q} & \quad \lambda \sum_{n=1}^{N} \lambda_n \sigma_D^2 \\
\text{subject to} & \quad D_n Q + \lambda_n \sigma_0^2 G_n \succeq \frac{\lambda_n \sigma_D^2}{\gamma_n} h_n h_n^H, \forall n \ 	ext{tr}(QP) \leq \text{tr}(P), Q \text{ is diagonal, } Q \succeq 0.
\end{align*}$$

In the next section, an interpretation via a virtual single-input multiple-output (SIMO) uplink channel shows that the dual problem (19) is equivalent to the following minimax problem:

$$\begin{align*}
\min_{Q} & \quad \lambda \sum_{n=1}^{N} \lambda_n \sigma_D^2 \\
\text{subject to} & \quad \frac{\lambda_n \sigma_0^2}{\gamma_n} |h_n^H \hat{w}_n|^2 \geq \gamma_n, \forall n \ 	ext{tr}(QP) \leq \text{tr}(P), Q \text{ is diagonal, } Q \succeq 0,
\end{align*}$$

where $\hat{w}_n$ is interpreted as the receive beamforming vector of the virtual uplink channel for user-$n$.

B. An Interpretation via a Virtual Uplink Channel

Consider a virtual SIMO uplink channel where a single-antenna transmitter with power $\hat{p}_n$ wants to communicate with an $R$-antenna receiver. The channel is modeled as $\sigma_S h_n \in \mathbb{C}^{1 \times R}$. The effective additive Gaussian noise at the receiver has the following covariance: $\sigma_D^2 D_n Q + \hat{p}_n \sigma_0^2 G_n$. One can interpret $\sigma_D^2 D_n Q$ as the added noise at the receiver and $\hat{p}_n \sigma_0^2 G_n$ as the noise induced by the transmitter, which depends on the transmitted power $\hat{p}_n$. Then, it is of interest to find the optimal combiner at the receiver and the minimal transmitted power $\hat{p}_n$ at the transmitter to obtain a certain target SNR of the virtual uplink channel’s receiving end.

Let $\hat{w}_n$ be the receive beamforming vector. The SNR at the receiver can be expressed as

$$\text{SNR}_n = \frac{\hat{p}_n \sigma_0^2}{\sigma_D^2 |\hat{w}_n|^2}$$

To maximize the above SNR, using the Rayleigh-Ritz theorem [9], the optimal receive beamformer is

$$\hat{w}_n = \left( \sigma_D^2 D_n Q + \hat{p}_n \sigma_0^2 G_n \right)^{-1} h_n.$$

Given a specific value of the transmit power $\hat{p}_n$, the weight of the optimal combiner at the $i$th receive antenna only depends on the channel connected to itself, and is given by

$$\hat{w}_{n,i} = \frac{f_{n,i} g_{n,i}}{(\sigma_S^2 |f_{n,i}|^2 + \sigma_D^2 \sigma_0^2 \mu_n + \hat{p}_n \sigma_0^2 g_{n,i}^2)^{1/2}}.$$

With the optimal combiner, the constraint on the SNR at the receiver, $\text{SNR}_n \geq \gamma_n$, is now equivalent to

$$\hat{p}_n \sigma_0^2 \gamma_n \left( \sigma_D^2 D_n Q + \hat{p}_n \sigma_0^2 G_n \right)^{-1} h_n \geq \gamma_n.$$

The next task is to determine the minimal uplink transmitted power $\hat{p}_n$. This problem is stated as

$$\begin{align*}
\min_{\hat{p}_n} & \quad \hat{p}_n \gamma_n \\
\text{subject to} & \quad \hat{p}_n \sigma_0^2 \gamma_n \left( \sigma_D^2 D_n Q + \hat{p}_n \sigma_0^2 G_n \right)^{-1} h_n \geq \gamma_n.
\end{align*}$$

Obviously, if the inequality is reversed, the optimization problem can be restated as a maximization problem as follows:

$$\begin{align*}
\max_{\hat{p}_n} & \quad \hat{p}_n \\
\text{subject to} & \quad \hat{p}_n \sigma_0^2 \gamma_n \left( \sigma_D^2 D_n Q + \hat{p}_n \sigma_0^2 G_n \right)^{-1} h_n \leq \gamma_n.
\end{align*}$$

Observe that the constraint above is equivalent to $\sigma_D^2 D_n Q + \hat{p}_n \sigma_0^2 G_n \succeq \frac{\hat{p}_n \sigma_0^2}{\gamma_n} h_n h_n^H$ (from Lemma 1 in [11]). Furthermore, identify $\hat{p}_n = \lambda_n \sigma_D^2$, then the minimax problem in (20) must be equivalent to the dual problem (19) of the distributed beamforming problem. This allows us to solve the distributed beamforming problem with per-relay power constraint by solving the minimax problem (20).

C. Numerical Algorithm

The minimax problem (20) can be solved by iteratively solving $N$ inner minimization problems on $(\hat{w}_n, \lambda_n)$ and the outer maximization problem on $Q$. For the outer problem, we compute the maximization

$$\begin{align*}
\max_{Q} & \quad f(Q) \\
\text{subject to} & \quad \text{tr}(QP) \leq \text{tr}(P), Q \text{ is diagonal, } Q \succeq 0.
\end{align*}$$

where $f(Q) = \min_{\lambda} \sum_{n=1}^{N} \lambda_n \sigma_D^2$. For the inner problems, with a fixed $Q$, the optimal combiner $\hat{w}_n$ for the user-$n$ is given by (22), and the optimal power factor $\lambda_n$ is obtained by solving problem (25). Note that problem (25) is equivalent to

$$\begin{align*}
\min_{\hat{p}_n} & \quad \hat{p}_n \\
\text{subject to} & \quad \sum_{i=1}^{R} \hat{p}_n a_{n,i} + \hat{p}_n \geq \gamma_n.
\end{align*}$$
Thus, the optimal value $\hat{\rho}_n^*$ can be obtained from the simple fixed point iteration, as presented in Section III. Moreover, with $\lambda_n = \hat{\rho}_n^*/\sigma_D^2$, the fixed point iteration

$$\lambda_n^{(t+1)} = \frac{\gamma_n}{\sigma_D^2 \sum_{i=1}^R \frac{a_n}{b_n + \mu_n + \sigma_D^2 \lambda_n}}$$

(29)

will surely converge to the optimal value $\lambda_n^*$. Having known the optimal combiner of the virtual uplink channel $w_n$ and its power factor $\lambda_n \sigma_D^2$, the optimal distributed beamformer $w_n$ for user-$n$ can be determined by exploiting the relationship between the two beamformers in the next lemma.

Lemma 2. The optimal distributed beamforming vector $w_n$ in the multiuser beamforming problem is a scaled version of $\hat{w}_n$, i.e., $w_n = \sqrt{\gamma_n} \hat{w}_n$.

Proof: From Karush-Kuhn-Tucker (KKT) condition [12], the gradient of Lagrangian $L_n(Q, \lambda_n, w_n)$ vanishes at the optimum of $w_n$, i.e.,

$$\frac{\partial L_n}{\partial w_n^*} = (D_n Q - \lambda_n \sigma_D^2 h_n^\ast h_n + \lambda_n \sigma_R^2 G_n) w_n = 0.$$

Thus,

$$w_n = (D_n Q + \lambda_n \sigma_D^2 G_n)^{-1} \frac{\lambda_n \sigma_D^2}{\gamma_n} h_n^\ast h_n^\ast w_n$$

$$= \frac{\lambda_n \sigma_D^2}{\gamma_n} h_n^\ast h_n^\ast w_n$$

which suggests $\sqrt{\gamma_n} = (\lambda_n \sigma_D^2 \sigma_S^2 / \gamma_n) h_n^\ast h_n^\ast w_n$. The next step is to determine the value $\zeta_n$ that is independent of $w_n$. As the SNR constraint in (16) is met with equality at optimum, i.e., $(\sigma_S^2 / \gamma_n |h_n^\ast w_n|^2 = \sigma_R^2 w_n^\ast G_n w_n + \sigma_D^2$. Substituting $w_n = \sqrt{\gamma_n} \hat{w}_n$ into the SNR constraint, one has $(\zeta_n \sigma_S^2 / \gamma_n) |h_n^\ast w_n|^2 = \zeta_n \sigma_R^2 w_n^\ast G_n w_n + \sigma_D^2$. Therefore,

$$\zeta_n = \frac{\sigma_S^2 |h_n^\ast w_n|^2 - \gamma_n \sigma_D^2}{\sigma_R^2 w_n^\ast G_n w_n}.$$  

(30)

We now return to the maximization problem of $f(Q)$ in (27). This problem can be computed by the subgradient projection method, as presented next.

Lemma 3. The function $f(Q)$ is concave in $Q$, and its subgradient is given by $\text{diag} \left( \sum_{n=1}^N w_n w_n^\ast D_n \right)$, where $w_n$ is the optimal distributed beamforming vector obtained from Lemma 2.

Proof: The proof of this lemma is similar to the proof of Proposition 3 in [11] for the point-to-point multiuser downlink beamforming problem.

Having derived the subgradient of $f(Q)$, $Q$ is then updated by applying the Euclidean projection $P_{S_Q}$ of the subgradient of $f(Q)$ on the constraint set $S_Q = \{ Q : tr(Q P) \leq tr(P), Q \succeq 0 \}$, i.e.,

$$Q^{(t+1)} = P_{S_Q} \left\{ Q^{(t)} + a_t \text{diag} \left( \sum_{n=1}^N w_n w_n^\ast D_n \right) \right\},$$

(31)

where $a_t$ is an appropriate step size. This subgradient projection method is guaranteed to converge to the global optimum of $f(Q)$ [12]. We now summarize the iterative algorithm to solve the distributed beamforming problem with per-relay constraints with the property of distributed implementation as follows.

1) Initialize $Q^{(t)}$. Set $t = 1$.
2) Repeat: fix $Q^{(t)}$, then the relays transmit $Q^{(t)}$ to every destination. Each destination then solves the fixed point iteration in (29) to determine the required power $\lambda_n \sigma_D^2$ for its corresponding virtual uplink channel. The optimal receive beamformer $\hat{w}_n$ and the scaling factor $\zeta_n$ are then determined by the $n$th destination.
3) The $n$th destination broadcasts $\lambda_n$ and $\zeta_n$ back to the relays. The $n$th relay calculates the beamforming coefficients $w_{n,1}, w_{n,2}, \ldots, w_{n,N}$, with local information pertaining to the relay as

$$w_{n,i} = \sqrt{\gamma_n} \sigma_S^2 |f_{n,i}|^2 + \sigma_R^2 |S_n^2 G_{n,i}^\ast|^2 |G_{n,i} w_n + \sigma_D^2$$

(31).
4) The relays cooperates with each other to update $Q^{(t)}$ as in (31).
5) Set $t = t + 1$ and return to Step 2 until convergence.

V. Numerical Results

In this section, we present the numerical results on the power consumptions at the relays of a multiuser multi-relay network with and without per-relay power constraints. Also presented are the convergence plots of the proposed iterative algorithms. The network being considered is equipped with 4 relays. The number of users to be served by the networks is 3. The source power is set at 10 (10 dB) for all the users’ sources. The noise variances $\sigma_R^2$ and $\sigma_D^2$ are set at unity. Flat Rayleigh fading is assumed in all the channels, where each $S \rightarrow R$ and $R \rightarrow D$ channel coefficients are assumed to be i.i.d $CN(0,1)$. When the per-relay power constraints are imposed, the maximum per-relay power is set at 10. The target SNR $\gamma_n$ is set at 5 (7 dB) for all the destinations.

Fig. 1 illustrates the power consumptions at the relays for 50 different channel realizations. At each channel realization, the sum relay power, the highest relay power level of the 4 relays, and the difference between the highest and lowest relay power levels of the 4 relays are plotted and compared between the two relaying strategies: with and without per-relay power constraints. As can be seen from the figure, imposing the per-relay power constraints does increase the sum relay power, compared with the optimal strategy that does not impose the constraints. However, the chief advantage of applying the per-relay power constraints is that it balances the power consumption at the relays and does not overuse any of them. Consequently, the highest relay power level of the 4 relays with the per-relay power constraints is always smaller than that without the constraints. In addition, all the relays transmit at the same power level almost all the time when the constraints are applied; whereas the difference between the highest and lowest power levels are quite significant without the constraints.

The convergence of the proposed algorithms is illustrated in Figs. 2 and 3. Fig. 2 plots the evolution of the sum relay
power allocated for user-1, \( p_1 \), and the corresponding SNR after each iteration by the iterative fixed point algorithm (14). It can be seen that the algorithm converges very quickly after only a few iterations to the optimal \( p_1^* \) from various arbitrary starting points, while the corresponding SNR also converges to its target value \( \gamma_1 = 5 \). Fig. 3 displays the convergence of the proposed iterative algorithm in Section IV-C in finding the optimal distributed beamformers \( \mathbf{w}_n^* \) with per-relay power constraints. The step-size \( \alpha_t = 1/t \) is used for the subgradient update of the iterative algorithm. The summation \( \sum_{n=1}^N \| \mathbf{w}_n - \mathbf{w}_n^* \| \), which is the norm residue of the beamformers, plotted after each iteration clearly shows the convergence of the proposed algorithm. Numerous simulations also show that the proposed algorithm converges in a small fraction of the running time required by the \textit{cvx} package [13].

![Fig. 1. Power consumptions at the relays over 50 channel realizations with different power constraints: with per-relay power constraints (solid lines), without per-relay power constraints (“dash-dot” lines).](image1)

![Fig. 2. Convergence of the iterative fixed point algorithm (14) with different starting points and the achievable SNR at user-1’s destination after each iteration.](image2)

![Fig. 3. Convergence of the proposed algorithm in finding the optimal distributed beamformers with per-relay power constraints.](image3)

VI. CONCLUSIONS

This paper has studied the optimal distributed beamforming design in a multiuser multi-relay network to minimize the total relay power with guaranteed QoS at the destinations. We considered two optimization problems, with or without per-relay power constraints. Although these two optimization problems are convex SOCP and can be effectively solved by any conic software package, the paper also explored simple and fast iterative algorithms to efficiently solve them.

REFERENCES


