Energy Consumption Analysis of Wireless Networks using Stochastic Deployment Models

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Abstract—This paper aims to discuss the influence of adjustable base station (BS) power parameters, such as power consumed during active mode and sleep modes, on the overall energy consumption of a network and highlight potential energy savings that can be achieved by the introduction of sleep modes.

I. INTRODUCTION

A perpetual expansion of cellular networks is required to meet an increasingly exacting user demand, which lends itself to estimating its environmental impact. It’s been shown that the carbon footprint of cellular networks is estimated to almost triple by 2020 [1]. A key to stunting the carbon footprint of future cellular networks is minimizing the overall energy consumption of a network. With this in mind, two basic paths have been pursued in the literature thus far: hardware improvements, and the introduction of sleep modes. Improving hardware is the most palpable attempt at saving energy in a network. When macro BSs are considered, the power amplifier is the biggest source of energy inefficiency. Therefore, considerable effort has been put into finding methods for reducing the energy consumption of power amplifiers (e.g. [2], [3]).

Although improvements in hardware are essential for the overall success of efforts toward making the network more energy efficient, energy management strategies that prudently exploit fluctuations in the load (of a network) for energy savings have to be designed to ensure that the energy consumed at any given point in time is minimized. One such energy management strategy is the introduction of sleep modes. Sleep modes, in this context, are defined as modes where a sizable fraction of the hardware in a BS is switched off, and it should not be confused with micro sleep modes where only a small fraction of the hardware is switched off during a short interval (as seen in [2], [3]) – on the order of a few micro seconds. Several propositions for sleep modes have been made in recent literature (e.g. [4]–[6]). Practical solutions that allow BSs to be switched off while maintaining coverage are being proposed in consortia like GreenTouch1. One such idea consists of separating the data and signaling networks [7], where the signaling network ensures coverage while BSs catering to data requirements are placed in sleep mode (when not being utilized). Strategies that demonstrate these gains, in terms of achievable energy savings, are usually validated using lengthy Monte Carlo simulations. In this paper, a theoretic framework based on stochastic geometry is proposed for such an evaluation. To grasp the effects of randomness in deployments, BS locations are considered to be homogeneous Poisson point processes. The use of random deployments also allows conclusions to be drawn about the efficacy of a particular energy conservation strategy, or the lack thereof, by providing information about the average values of a metric of interest for an area under consideration, i.e. the values provided are averaged over all possible cell topologies. The validity of using Poisson process based BS deployments to mimic real world deployments is shown in [8]. This is then used to predict energy savings that can be achieved in a network with macro BSs.

This paper considers the downlink of a wireless network and provides a general expression that relates spatially averaged rate with user and BS densities for a given path loss exponent (Subsection II-A). This can be considered an extension of the general expression derived in [8]. Details of the differences between the expressions derived are highlighted in the upcoming section (Section II). This expression forms a foundation for the analysis of the energy consumption of a network with a fixed user demand and a known user density which varies according to a traffic profile (given in [9]). A linear power model, whose details are elucidated in the

1http://greentouch.org
following, is used to find the average energy consumption of a network through the day. The power model and its utility are given in Section III. The energy saving potential of sleep modes are discussed in Subsection III-B, based on which, conclusions and future work are delineated in Section IV.

II. DOWNLINK SYSTEM MODEL

A downlink interference limited system model is considered where single antenna BSs are arranged according to some homogeneous Poisson point process $\Phi_b$, with intensity $\lambda_b$ (BS density) in the Euclidean plane. The users are considered to be located according to an independent stationary Poisson point process $\Phi_u$, with intensity $\lambda_u$ (user density). Since there are usually a larger number of users than BSs, it is assumed that they are both non-zero and $\lambda_u > \lambda_b$. The noise power is assumed to be zero (i.e. $W = 0$), hence the model is interference limited. The Euclidean plane is assumed to be tessellated around the BS process and the Voronoi cell is defined as:

$$C_{X_b} = \{ y \in \mathbb{R}^2 : SIR_y \geq T \} = \{ y \in \mathbb{R}^2 : L(y, x_i) \geq T (I_{\Phi_b}(y)) \}$$

where $X_b = \{ x_i \}$ is the set of BS locations, $T$ is the threshold, and $SIR_y$ is the Signal-to-Interference Ratio at $y$. $L(y, x_i)$ is received power at point $y$, and $I_{\Phi_b}(y)$ is the interference at $y$.

The received power and the interference are defined as

$$L(y, x_i) = \frac{Ph}{l(|y - x_i|)}; I_{\Phi_b}(y) = \sum_{x_j, j \neq i} L(y, x_j),$$

where $P$ is the transmit power, $h$ is a fading parameter defined as an exponential random variable with mean 1. This implies that the product $Ph$ at any point $y$ (i.e. the power that is received at a given point without taking the pathloss into consideration) can be represented by an exponential random variable of mean $P^{-1}$. Lastly, $l(|y - x_i|)$ is the omni-directional path loss function which can be represented as $l(r) = (Ar)^\beta$, for some $A > 0$ and the path loss exponent $\beta > 2$, where $r$ is the distance between the point $y$ and the BS at $x_i$. The interference received at a point $y$ is also dependent on the number of users connected to a BS, an aspect which has not been included in [8]. The general expression derived in this paper includes this aforementioned aspect, and can therefore be considered an extension of their findings. The average number of users connected to a given BS has been shown to be $E(N) = \lambda_u/\lambda_b$ in [10], where $\lambda_u$ is the user density and $\lambda_b$ is the BS density. Incorporating this fact, results in the product $Ph$ that is exponentially distributed with mean $(\lambda_uP/\lambda_b)^{-1}$. These parameters are used to derive the probability of coverage (i.e. probability that a point $y \in \mathbb{R}^2$ is covered by its nearest BS, $x_i \in X_b$) and the spatially averaged rate of the system for the energy consumption analysis.

A. Probability of Coverage and Spatially Averaged Rate

**Theorem 1.** The probability of coverage for a path loss exponent $\beta > 2$ and a threshold $t$ is:

$$p_c(\lambda_b, \lambda_u, t) = \frac{\beta - 2}{(\beta - 2) + 2\lambda_u t \exp \left( \frac{1 - \lambda_u}{\lambda_b} \right)}.$$

From which the spatially averaged rate for a path loss exponent, $\beta = 4$ can be shown to be:

$$\bar{R}_{\Phi_b}(\lambda_b, \lambda_u) = \int_0^\infty \frac{2y}{[1 + y \tan^{-1}(y)]} [y^\beta + \lambda_b y^\beta] dy.$$

Further simplification to establish a complete analytic relationship gives eqn.(3).

**Proof:** The average rate can be written as

$$\bar{R}_{\Phi_b}(\lambda_b, \lambda_u) = \mathbb{E}^o[\log(1 + SIR_o) > \gamma],$$

where $\mathbb{E}^o$ is the Palm expectation, $\gamma$ is the threshold, and $SIR_o$ is the Signal-to-Interference Ratio at the origin. Note: The origin is always assumed to be at the point that is being considered. The spatially average rate $\overline{R}_{\Phi_b}(\lambda_b, \lambda_u)$ then becomes

$$\mathbb{E}^o [\log(1 + SIR_o) > \gamma] = \mathbb{E}^o[SIR_o > \gamma - 1].$$

Using the Refined Campbell Theorem [11],

$$\bar{R}_{\Phi_b}(\lambda_b, \lambda_u) = \int_{\gamma = 0}^{\infty} \int_{r > 0} \mathbb{P}^o(SIR_o > \gamma - 1) \mathbb{P}(dr).$$

Here, $\mathbb{P}^o$ is the Palm distribution of $SIR_o$ and $\lambda_b$ is the intensity measure of the BS Poisson process. By the Theorem of Slivnyak [11], the spatially average rate is given by

$$\bar{R}_{\Phi_b}(\lambda_b, \lambda_u) = \int_0^\infty \int_0^\infty 2\pi \lambda_b r \exp \left( -\pi \lambda_b r^2 \right) \mathbb{P}(SIR_o > \gamma - 1) ddrd\gamma.$$

From [10], the probability of coverage or the probability that a point $y \in \mathbb{R}^2$ is covered by its nearest transmitter can be defined as

$$p_c(\lambda_b, \lambda_u, t) = \int_0^\infty 2\pi \lambda_b r \exp \left( -\pi \lambda_b r^2 \right) \mathbb{P}(SIR_o > t) dr$$

$$= \int_0^\infty 2\pi \lambda_b r \exp \left( -\pi \lambda_b r^2 \right) \mathcal{L}_{I_{\Phi_b}}(\mu tr^\beta) dr,$$

where $\mathcal{L}_{I_{\Phi_b}}(\mu tr^\beta)$ is the Laplace transform of the interference and $\mu$ is the fading mean, and it’s assumed that the constant ‘A’ in the omni-directional path loss function is equal to 1 for the rest of this proof. The above equation implies that the spatially averaged rate can then be written as

$$\bar{R}_{\Phi_b}(\lambda_b, \lambda_u) = \int_0^\infty p_c(\lambda_b, \lambda_u, (\gamma - 1)) d\gamma.$$

The probability of coverage can be written as

$$p_c(\lambda_b, \lambda_u, t) = \int_0^\infty 2\pi \lambda_b r \exp \left( -\pi \lambda_b r^2 \right) \times$$

$$\exp \left( -2\pi \lambda_b \int_{u > r} (1 - \mathcal{L}_p(\mu tu^\beta/r^\beta)) du \right) dr,$$
where $L_P(\mu a^3/r^3)$ is the Laplace transform of the power received at a point $y$. This implies

\[
p_c(\lambda_b, \lambda_u, t) = \int_0^\infty 2\pi \lambda_b r \exp\left(-\pi \lambda_b r^2 \right) \times 
\exp\left(-2\pi \lambda_b \int_r^\infty \frac{u}{1 + \frac{u}{\lambda_u}} \left(\frac{u}{r^2}\right) du\right) dr.
\]

Therefore, if

\[
\lambda_b (\beta - 2) + 2t \lambda_u \text{Re} \left[ 2F1 \left( 1, \frac{\beta - 2}{\beta}; 2 - \frac{2}{\beta}; -\frac{t \lambda_u}{\lambda_b} \right) \right] > 0
\]

and

\[
2F1 \left( 1, \frac{\beta - 2}{\beta}; 2 - \frac{2}{\beta}; -\frac{t \lambda_u}{\lambda_b} \right) \in \mathbb{R},
\]

the probability a point $y \in \mathbb{R}^2$ is covered by its nearest base station when there are $\frac{\lambda_u}{\lambda_b}$ users connected to any given base station is

\[
p_c(\lambda_b, \lambda_u, t) = \frac{\beta - 2}{(\beta - 2) + 2 \lambda_u t_2 F_1 \left( 1, \frac{\beta - 2}{\beta}; 2 - \frac{2}{\beta}; -\frac{\lambda_u}{\lambda_b} \right)}.
\]

where $2F1 (a, b; c; z)$ is the Gaussian hypergeometric function. Since the values of $\lambda_u, \lambda_b$ are positive, the threshold $t \geq 0$, and $\beta > 2$, it’s easy to show that the above conditions are satisfied.

Substituting the result obtained above in equation (4) gives the spatially averaged rate, $R_{\Phi_b}(\lambda_b, \lambda_u)$

\[
= \int_0^\infty \frac{\beta - 2}{(\beta - 2) + 2 \lambda_u (e^{\gamma - 1}) \gamma_2 F_1 \left( 1, \frac{\beta - 2}{\beta}; 2 - \frac{2}{\beta}; -\frac{\lambda_u}{\lambda_b} \right)} d\gamma.
\]

Substituting $\frac{(e^{\gamma - 1})}{\lambda_b} = \tan^2 (z)$, we get the average rate to be

\[
= \int_0^{\pi/2} \frac{2 (\beta - 2) \tan (z) \sec^2 (z)}{(\beta - 2) + 2 \tan^2 (z) 2F1 \left( 1, \frac{\beta - 2}{\beta}; 2 - \frac{2}{\beta}; -\tan^2 (z) \right)} \frac{1}{\tan^2 (z) + \frac{\lambda_u}{\lambda_b}} dz.
\]

Assuming $\beta = 4$, changing the limits of integration using $\tan (z) = y$, and then applying Euler’s transformation to the hypergeometric results in

\[
\bar{R}_{\Phi_b}(\lambda_b, \lambda_u) = \int_0^\infty \frac{2y}{\left[ 1 + y \tan^{-1}(y) \right] \left[ y^2 + \frac{\lambda_u}{\lambda_b} \right]} dy.
\]

Using the series expansion of the inverse trigonometric function and integrating over the limits gives $R_{\Phi_b}(\lambda_b, \lambda_u)$ to be equal to equation (3).

**B. Analysis of Results**

The theorem provides the spatially averaged rate that can be achieved over a normalized area in bps/Hz. The plot obtained using the theorem above has been shown to concur with the plot obtained for the system model under consideration by Monte Carlo simulations in figures Fig. 1a and Fig. 1b, respectively. These plots indicate the variation in the spatially averaged rate with increasing user and BS densities.

There are, however, a few obvious differences – the reasons for which are explained in the remainder of this section. The

![Fig. 1. Comparison of theoretic results with simulations](image-url)
peak average theoretic rate is slightly lower than the peak average rate obtained by simulations. This is due to the fact that the theoretic case averages over a far greater number of rates (that are close to zero), by considering different cell sizes which are obtained by integrating over thresholds from 0 to $\infty$, whereas during simulations the number of iterations that determine varying cell sizes (though large) are finite. Another factor responsible for this discrepancy is the area of observation: it is considered to be all of $\mathbb{R}^2$ in the theoretic case, whereas, the simulations consider a large (finite) area for generating the points of the Poisson process and observe rates over smaller windows to effectively mimic the theoretic case. Smoothness of the curves is another aspect that differs. The simulated curve is less smooth due to the inability to evaluate an expectation precisely with simulations. Fig. 2. shows a plot of variation in the theoretic spatially averaged rate (equation (3)) versus the ratio of user density and BS density (i.e. $\lambda_u/\lambda_b$).

Note that equation (6) assumes that all BSs use the same transmit power. Considering a homogeneous Poisson point process to represent BSs implies that all BSs in the network are of the same type and we consider them to be macro BSs. The values considered for calculations shown below are taken from [9]. The framework to calculate the average energy consumption of the network per square kilometer over a period of 24 hours is as follows:

1) For a given user demand $D$ Mbps, a daily traffic/load profile $\eta(t)$, and maximum rate density per unit area ($U$ Mbps/km$^2$); the user density (users/km$^2$) at a given time of day can be found by:

$$\lambda_u = \frac{U\eta(t)}{D} \tag{7}$$

2) The average rate that is provided by the system $\bar{R}$ is always assumed to meet or exceed user demands, i.e. $\bar{R} \geq D$. The spatially averaged rate $\bar{R}_{\Phi_b}(\lambda_b, \lambda_u)$ derived above has the units of bps/Hz and for a given bandwidth $B$, it can be given as:

$$\bar{R}_{\Phi_b}(\lambda_b, \lambda_u) = \frac{\bar{R}}{B} \tag{8}$$

3) The user to BS density ratio that can satisfy the average rate obtained using equation (8) is found using Fig. 2.

4) The number of BSs required to provide the average rate in equation (8), is found by dividing the user density found in equation (7) by the ratio of user to BS densities (i.e. the value of $\lambda_u/\lambda_b$ from the x-axis of Fig. 2).

5) The BS density found in the step above is then used in equation (6) to obtain the power density required to satisfy a particular user demand over the entire area.

6) Inferences about the overall energy consumption of the network and the effectiveness of sleep modes (when deployed) are drawn from comparison of power densities obtained under different load constraints.

Fig. 2. Spatially averaged rate Vs. ($\frac{\lambda_u}{\lambda_b}$)

The figure (Fig. 2.) is utilized in the analysis of the energy consumption of a network using the power model described in the section below.

### III. ENERGY CONSUMPTION MODEL

The power consumption model of a BS, as in [9], is assumed to be:

$$P_c = \begin{cases} \Delta_P P_{T_c} + P_0 & \text{if the BS is active} \\ P_S & \text{if the BS is in sleep mode} \end{cases} \tag{5}$$

where $P_{T_c}$ is the transmit power, $P_0$ is the power consumed by the BS at the lowest possible output power, and $\Delta_P$ is the slope of the load dependent power consumption. The metric of interest, however, is the power density which is “power consumed per square kilometer” (W/km$^2$). It follows that the power density (without sleep modes) can be defined as:

$$D_P = \lambda_b P_c = \lambda_b (\Delta_P P_{T_c} + P_0) \tag{6}$$

Fig. 3. Daily traffic/ load profile

The values considered for our energy consumption analysis are as follows:

![Daily Traffic Profile](image)
1) User demand, $D = 2$ Mbps; Maximum rate density per unit area for a dense urban deployment, $U = 120$ Mbps/km$^2$; and the load profile $\eta(t)$ is shown in Fig. 3.

2) The average rate provided by the system, $\bar{R}$ is considered to be 2 Mbps unless specified as being otherwise.

3) The effective bandwidth $B = 6.39$ MHz, which is obtained by considering a 10 MHz LTE system (600 sub-carriers, frequency spaced at 15 KHz, and a control overhead of 29%).

4) Slope of the load dependent power consumption, $\Delta P = 5.32$; Transmit power $P_{Tx} = 20$ W; Fixed power consumed, $P_0 = 118.7$ W. The values of sleep mode power $P_S$ are assumed to vary depending on the type of sleep mode considered. For ex: $P_S = 0.5$ W for deep sleep mode. Note: Since there are no practical values available, we use 0.5 W which is the typical power consumption of standby mode in Wi-Fi transmitters.

Note: The values of parameters in step (4) above are obtained from [9].

A. Analyzing the energy consumption of the network

Consider a scenario without sleep modes and the following observations can be made:

\[
\text{Power Saved} = D_{P_{max}} - D_P - (\lambda_{b_{max}} - \lambda_b) P_S \quad (9)
\]

Fig. 4. Daily fluctuations in power density

Fig. 5 shows that the plot of power density through the day follows the daily traffic profile (Fig. 3) very closely, if a linear power model (like the one described above) is used.

Fig. 5 shows the variation in the number of BSs required per square kilometer when the average rate $\bar{R}$ provided by the network increases from 2 Mbps to 5 Mbps in unit increments, while all other values remain unchanged. It’s observed that every (individual) curve in the plot of the BS density through the day (Fig. 5) also varies according to the daily traffic profile (Fig. 3). Another aspect worth examining is the amount of additional power that needs to be provided to increase the average rate ($\bar{R}$) available at any given point by 1 Mbps. Fig. 6 validates our expectations in this regard, and indicates that the amount of power required to increase the average rate ($\bar{R}$) of the network by 1 Mbps increases nonlinearly. For example:

\[
B. \text{ Energy consumption analysis using sleep modes}
\]

In practice, BS deployments are usually designed to be able to satisfy user demands when they are operating under full load (i.e. $\eta(t) = 100\%$). To reflect this scenario, the maximum BS density and the power density which correspond to a full load scenario are found using the procedure expounded above. The number of BSs that can be turned off per square kilometer are found by subtracting the BS density required to satisfy the load during a specified hour of the day from the density required to meet the demands at full load. The power saved due to the introduction of sleep modes is given by:

\[
\text{Power Saved} = D_{P_{max}} - D_P - (\lambda_{b_{max}} - \lambda_b) P_S \quad (9)
\]
where \( D_{P_{\text{max}}} \) is the power density at 100% load, \( \lambda_{b_{\text{max}}} \) is the BS density required at 100% load, \( D_P \) is the power density at a given load, and \( \lambda_b \) is the BS density at the same load.

\[
\begin{align*}
W/sq.km & \quad 100,000 \quad 120,000 \quad 140,000 \quad 160,000 \quad 180,000 \\
20,000 & \quad 60,000 \quad 80,000 \\
0,000 & \quad 20,000 \\
\end{align*}
\]

Power Saved in W/sq.km with \( P_s = 93 \) W

Power Saved in W/sq.km with \( P_s = 0.5 \) W

Almost double deep sleep modes can, on average, for satisfying a user demand of 2 Mbps. This indicates that energy savings can still be achieved as long as \( P_S \) is less than \( P_0 \). It also validates the linear relationship between \( P_S \) and power saved that is apparent from equation (9).

For the traffic profile considered in Fig. 3, it is noted that approximately 40% of the power density consumed daily (an average of about 72 W/km²) can be saved per day) when deep sleep modes are used and approximately 23% of the power density consumed daily (an average of about 42 W/km² per day) can be saved when a sleep mode power of 93W is used, for satisfying a user demand of 2 Mbps. This indicates that deep sleep modes can, on average, almost double the power saved per square kilometer per day.

**IV. CONCLUSION**

For an interference limited downlink system model, analytic expressions for the probability of coverage and the average rate have been found. The expression relates spatially averaged rate with user and BS densities, and has been used to provide valuable insights about the energy consumption of a network. For a linear power model, it can be concluded that the number of BSs required per square kilometer to meet a given user demand and the power density (in W/km²) follow variations in traffic density very closely. Another valuable insight, is the fact that the additional power density required to increase the average rate provided by the network by 1 Mbps increases non-linearly as the initial (preexisting) average rate provided increases. The analytic expressions also allow examination of the utility of sleep modes and help confirm that sleep modes can, indeed, provide significant power savings when the network operates in conditions of less than full load. It also confirms the intuition that, for any traffic profile \( \eta(t) \), deep sleep modes help save more power than other sleep modes. For the traffic profile considered here, deep sleep modes almost double the average power saved when compared with using the sleep mode power of micro sleep modes (i.e. \( P_S = 93 \) W). The energy consumption analysis and an affirmation of benefits of sleep modes provided by this paper, though insightful, are far from presenting a holistic solution to the problem considered. Questions such as the effect of noise on the system, or the energy consumption of a network with different BS types (micro and macro BSs), and comparison of theoretic results (shown here) with simulations and real world models are left unanswered. The effects of utilizing more complicated power models, within the framework described here, are also unknown. An effort to answer some of the questions posed above forms the basis for future work of the authors.

**ACKNOWLEDGMENT**

This work is a part of the GreenTouch consortium.

**REFERENCES**


**Fig. 7.** Power saved through the day