BER Performance of Reed-Solomon Code
Using $M$-ary FSK Modulation in AWGN Channel

Saurabh Mahajan$^{1}$ and Gurpadam Singh$^{2}$

$^{1}$Department of Electronics and Communication, Sri Sai College of Engg. and Tech., Badhani, Ptk., Punjab, India
saurabh_raja1@yahoo.co.in

$^{2}$Department of Electronics and Communication, Beant College of Engg. and Tech., Gurdaspur, Punjab, India
gurpadam@yahoo.com

Abstract
The main objective of this simulation evaluation is to determine the Bit-Error-Rate performance of Reed-Solomon codes in a noisy communication channel. $M$-ary Frequency Shift Keying (M-FSK) modulation technique is used for this simulation evaluation of the coded communication system. The forward error correction (FEC) technique is applied to detect and correct error of information received from the AWGN channel. The message is encoded and decoded using error-correcting Reed-Solomon Codes. For the model based simulation evaluation the MATLAB/SIMULINK is chosen as the investigating tool. Monte Carlo method of simulation is used for plotting the results in between Bit-Error-Rate (BER) and Signal to Noise ratio ($E_b/N_0$). As the length of codeword symbols ($n$) increase at constant code rate $R_c$ and error correcting capability, the BER performance improves is the observation of simulation evaluation. The Bit-Error-Rate performance improves as the redundancy increases by lowering code rate is also finding of this work. The properly chosen error-correction code can significantly improve the BER performance is the key observation of this simulation evaluation.

Keywords: Bit-Error-Rate, $M$-ary Frequency Shift Keying (M-FSK), MATLAB/SIMULINK, Reed-Solomon codes.

1. Introduction

Digital communication became a stuff of grand importance for both the engineering and the mathematical communities as a mathematical model for information theory was introduced by Claude E. Shannon. The Claude E. Shannon gave a formal description in 1948 to make communication more reliable by introduced a valuable theory about the concept of information, including a good measure for the amount of information in a message. By dissecting a communication system into individual components, it is possible to improve certain aspects of the communication limitations. In particular, proper coding scheme can be utilized to improve the reliability of a communication system by reducing the error rate [5]. In addition, suitable modulation techniques can be chosen to improve the utilization of the available bandwidth. The task of information coding theory is to detect and correct errors in the transmitted information (data) through the channel like as AWGN channel. Generally, coding may defined as source coding and channel coding. Source coding involves changing the message source to a suitable code to be transmitted through the channel and an appropriate amount of redundancy is added to these source bits to protect them against the errors in the channel is called channel coding. The purpose of channel coding is to add controlled redundancy into the transmitted or stored information (data) to increase the reliability of transmission and lower transmission requirements. Error-correcting coding has become one essential ingredient for the latest information transmission systems. The most sophisticated codes use block code to improve the performance of data transmission.

The main problem in communication is to achieve reliable and consistent transmission of data from the information source to the destination [5]. The transmission of data may be digital or analog. The transmission of data with the minimum error rate is the objective of any communication system. Digital communication system provides solution to transmit the data with a maximum data rate and a
minimum error rate. In the present and future communication systems, data transmission at high bit rates is essential for many services such as video, high-quality audio, and mobile integrated service digital network (ISDN). To increase the reliability of transmitted or stored data is an immense practical significance and to achieve this practical significance the error control codes are used. The use of forward error correcting codes in digital communication systems is an integral part of ensuring reliable communication [12].

The main objective of this simulation evaluation is to determine the Bit-Error-Rate performance of Reed-Solomon codes using $M$-ary modulation in a noisy communication channel (AWGN). In this communication system model the information is transmitted using $M$-ary modulation technique ($M$-FSK) with the attendance of Additive White Gaussian Noise channel. $M$-ary Frequency Shift Keying modulation is a power proficient modulation method considered for low power applications. The power capability improves as ‘$M$’ increases and undesirably complexity of system is also increased.

2. System Model

Block diagram of the digital RS coded communication system under study is shown in Figure1. The basic functional blocks of a RS coded communication system are as follow: Source of information, RS Encoder, $M$-ary FSK Modulator, AWGN communication channel, $M$-ary FSK Demodulator, RS Decoder and Sink respectively. The binary numbers (0 and 1) are sequence of discrete symbols used in digital communication. The source of information produces digital information i.e., (0 or 1) and transmit it electronically to the sink with the aid of transmitter-receiver pair through communication channel. The source encoder processes the source output to eliminate redundancy, compressing the digital sequence into a more competent symbol for transmission. The channel encoder adds redundancy to the compressed information in order to control the errors offered by channel impairments.

\[ C = B \log_2 (1 + \text{SNR}) \text{ bit/sec}, \]

where, $B$ is the channel bandwidth and SNR is the signal to noise ratio [3]. The noisy communication (AWGN) channel is assumed for transmitting information from source to sink in communication system model.

![Figure1. Block Diagram of System Model](image)
In Multiple Frequency Shift Keying (M-FSK), the change in different frequencies can be quite unexpected because there is no requirement for the phase to be continuous. In practice, \( M \) usually a non-zero power of 2 and the signal set is characterized by Cartesian Coordinates in a way that each of the mutually perpendicular axes represents a sinusoid with a different frequency. For \((n, k)\) RS code, there are a total of \( M = 2^k \) code symbols and ‘\( k \)’ is the number of information bits per symbol [13].

The general expression for FSK modulation for each signal is,

\[
S_i(t) = \sqrt{\frac{2E}{T}} \cos(w_i t + \theta) , \quad 0 \leq t \leq T, \{i = 1, 2, 3 \ldots , M\}
\]  

(2.1)

Therefore, each code symbol out of the encoder can be mapped to one of the \( M \) frequencies in the M-FSK signal set [7]. Also, if \( T \) is taken as the time required to transmit each code symbol or one of the \( M \) frequencies, the transmission rate is given by \( R_b = \frac{k}{T} \) bits/sec and the information rate is given by \( R_i = R_c R_b \) bits/sec, where \( R_c = \frac{k}{n} \) the code rate.

3. Theory of Reed-Solomon Codes

Reed-Solomon codes are frequently used in coded communication systems and data storage systems, to get back data from possible errors that take place throughout transmission. Reed-Solomon codes were invented by Larry S. Reed and Gustave Solomon in 1960, who wrote “Polynomial Codes over Certain Finite Fields” [8]. Reed-Solomon codes are effective for deep fades channel and are considered as a structured sequence that is most widely used in error control codes.

The basic structure of RS code as shown in figure 2 represented that the codeword symbols \((n)\) is unite of two segments information symbols \((k)\) and parity symbols \((2t)\). The information symbols \((k)\) is having message that is to be transmitted and parity symbols \((2t)\) is the redundancy added to message to transmit it from source to destination without error i.e., noise.

![Figure2. Structure of Reed-Solomon Code](image)

Reed-Solomon codes are a subset of BCH codes. Reed-Solomon code is a linear cyclic systematic non-binary block code. Reed-Solomon codes are mainly used for burst error correction. Reed-Solomon code has very high coding rate and low complexity. Reed-Solomon codes are suitable for many applications including storage and transmission. RS code is a block based code and can be represented as RS \((n, k, t)\). The variable \((n)\) is the length of the codeword symbols, \((k)\) is the number of information symbols, \((t)\) is the error-correction capability and each symbol contains \(m\)-bits [14]. The correlation of the symbol size \((m)\), and the codeword symbols \((n)\) is specified as \(n = 2^m - 1\) for \(m\)-bits in one symbol, there exist \(2^m - 1\) distinct symbol in one codeword, excluding the one with all zeros. The error-correction capability \((t)\) or the maximum number of correctable random errors can be calculated by \(t = \frac{(n-k)}{2}\). For example, the maximum number of symbol errors that RS \((31, 27)\) decoder can correct is calculated as \(\frac{31-27}{2} = 2\) symbols for codeword symbols \(n = 31\) and information symbols \(k = 27\). The code is competent of correcting any combination of \(t\) or smaller number errors and can be obtained from following expressions of the error-correction capability, \(t = \frac{(d \min -1)}{2}\).
The code minimum distance can be given by relation \( d_{\text{min}} = n - k + 1 \). The code operates by oversampling a polynomial constructed from the data, thus redundant information is generated. This redundant information allows errors during transmission or storage to be corrected. The quantity and type of errors that can be corrected depends on the characteristics of the Reed-Solomon code. Codes that realize these “optimal” error correction capabilities are called maximum distance separable (MDS) codes. Reed-Solomon codes are by far the leading members, both in number and utility, of the class of MDS codes [10-12].

The RS \((n, k, t)\) code parameters can be represented as follows [7]-[9].

<table>
<thead>
<tr>
<th>Codeword symbols</th>
<th>( n = 2^m - 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Information symbols</td>
<td>( k = 1, 3, \ldots, n - 2 )</td>
</tr>
<tr>
<td>Code minimum distance</td>
<td>( d_{\text{min}} = n - k + 1 ) and ( t = \frac{d_{\text{min}} - 1}{2} = \frac{n-k}{2} )</td>
</tr>
<tr>
<td>The error-correction capability symbols</td>
<td>( t )</td>
</tr>
</tbody>
</table>

### 3.1 Encoding and Decoding Process of RS codes

In order to realize the encoding and decoding principles of RS codes, it is necessary to know the area of finite fields. The enormous mathematical calculations and arithmetical operations involves in the RS encoding and decoding process. Such, arithmetical calculations over a finite field with certain properties are known as Galois fields [2]. Galois Field is a set that contains a finite number of elements and the miscellaneous operations of addition and multiplication for such sets are defined. Symbols from the extension field are used in the construction of Reed-Solomon codes. Besides the binary number 0 and 1, there are additional unique elements in the extension field that denotes the variable \( a \). A finite set of elements, say \( S \) is formed by the element \( \{0, 1, a\} \) and generating additional elements by multiplying the last entry by \( a \) which yields,

\[
S = \{0, a^0, a^1, a^2, a^3, \ldots, a^j\} \tag{3.2}
\]

Thus, to obtain the finite set of elements from \( S \), a condition must be imposed on \( S \) so that it may contain only 2 elements and is closed under multiplication. Therefore, the elements of the finite field \( GF \) are given by,

\[
GF (2^m) = \{0, a^0, a^1, a^2, a^3, \ldots, a^{2m-2}\} \tag{3.3}
\]

A set of polynomials called field generator polynomial is used to define the finite fields of Galois field (GF). It is a polynomial of degree \( m \) which is irreducible, also known as polynomial without any factor. For a Galois field of a particular size, there is sometimes a choice of suitable polynomials [11-12].

Suppose that we want to build a \( (t) \) error-correcting Reed-Solomon code of length \((q-1)\) with symbols in \( GF (q) \). Recall that the nonzero elements in the Galois field \( GF (q) \) can be represented as \((q-1)\) powers of some primitive element \( a \). The Reed-Solomon design criterion is as follows: The generator polynomial \( g(x) \) for a \( (t) \) error-correcting code must have as roots \( 2t \) consecutive powers of \( a \)

\[
g(x) = \prod_{j=1}^{2t} (x - a^j) \tag{3.4}
\]

Valid code polynomials can thus have degrees from \( 2t \) up to \((q-2)\). It follows that the dimension of a code with a degree \( 2t \) generator polynomial is \( k = q - 2t - 1 \). Any valid code polynomial must be a multiple of the generator polynomial. It follows that any valid code polynomial must have as roots the same \( 2t \) consecutive powers of \( a \) that form the roots of \( g(x) \). This provides us with a very convenient means for determining whether a received word is a valid code word. We simply make sure that the corresponding polynomial has the necessary roots [10].
Since, Reed Solomon code is a linear cyclic systematic non-binary block code. In the encoder redundant symbols are generated using a “generator polynomial” and appended to the information symbols \[11\]. In decoder error location and magnitude are calculated using the same “generator polynomial”. Then the correction is applied on the received code. Reed Solomon code has less coding gain as compared to LDPC and turbo codes. But it has very high coding rate and low complexity.

4. Modeling and Simulation Performance Evaluation

Simulation is most widely used in industry and academia for modeling and evaluating the systems. MATLAB is one of the computer based high performance programming language. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in a frequent mathematical notation. A complete RS coded communication system with \(M\)-ary Frequency Shift Keying modulation over AWGN channel has been constructed to evaluate the Bit-Error-Rate performance of Reed-Solomon error correcting code. It is a cyclic symbol error-correcting code and function at the block level rather than the bit level. Such symbols can either be comprised of one bit (binary code) or, of several bits (symbol codes) \[2\]. Reed-Solomon code belongs to a block codes family and is capable of working up to symbols level. The model is constructed using MATLAB/SIMULINK as shown in figure 3. After implementation of communication model by using SIMULINK then their performance is examined using the BERTool of MATLAB. The performance of RS codes can be evaluated from different point of view like Redundancy, Code length, Code Rate and error correction capability.

![Figure 3. Communication System Model with RS Encoder/ RS Decoder over AWGN Channel](image)

4.1 Performance as the Function of Redundancy

The simulation model shown in figure 4 with RS coding is run for following RS\((n, k, t)\) code sets; RS \((31,29,1)\), RS \((31,27,2)\) and RS \((31,23,4)\) respectively. The assumed RS code sets have identical codeword symbols \((n=31)\) whereas the number of information symbols \((k)\) decreases from 29, 27, 23 that apparently resulted in different error correction capability 1, 2 and 4 respectively. Also, the lowering the code rate \((R_c = k/n)\) is from 0.94 to 0.74.
It is observed that the coding gain with high code rate (0.94) is better than the low code rate (0.74). Based on the performance curve, it is observed that as the redundancy increases (that is lowering code rate), the BER performance improved. The BER performance improves as the redundancy \((n-k)\) increases from 2 to 8 symbols thereby lowering the code rate.

4.2 Performance as the Function of Code length (at constant Code Rate)

The communication model runs with following RS codes sets: RS \((15,11)\), RS \((31,23)\), RS \((63,47)\) and RS \((127,95)\) respectively for simulation in AWGN channel. The above RS code sets have varied codeword symbols \((n)\), and data symbols \((k)\) but keeping the almost constant Code Rate \((R_c)\) [6].

<table>
<thead>
<tr>
<th>Codeword Symbol</th>
<th>Assumed RS Code</th>
<th>Code Rate ((R_c=k/n))</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>RS ((15,11))</td>
<td>0.738</td>
</tr>
<tr>
<td>31</td>
<td>RS ((31,23))</td>
<td>0.742</td>
</tr>
<tr>
<td>63</td>
<td>RS ((63,47))</td>
<td>0.746</td>
</tr>
<tr>
<td>127</td>
<td>RS ((127,95))</td>
<td>0.748</td>
</tr>
</tbody>
</table>

The respective code rate used for assumed RS code is given in the table 1. The value of code rate is held nearly constant (i.e., 0.74), while its codeword symbols \((n)\) increases from \(n=15\) symbols to \(n=127\) symbols.

It is observed from the graph below that the error-correcting Reed-Solomon codes become more efficient as its code length increases because the noise effect reduces for larger code lengths. In addition, the noise duration has to represent a relatively small percentage for larger codeword \((n)\) and the received noise supposed to be averaged over a long period of time. This means the optimal large size codeword \((n)\) is a move towards better BER performance. For extensive block length the Reed-Solomon codes are more preferred [4]. On the other hand, very large codeword symbols \((n)\) will increase complexity in the implementation of the transmission system.
4.3 Performance as the Function of Code length (at similar error correcting capability)

The experimental model runs following RS codes sets; RS (31,27), RS (63,59) and RS (127,123) respectively for simulation in AWGN channel at same error correction capability [6].
The values of RS codeword symbols \( n \), and information symbols \( k \) are increasing but keeping the same error correcting capability \( t=2 \). The performance curve shows that as the codeword symbols \( n \) i.e., code length increases from \( n=31 \) to 127, the Bit-Error-Rate (BER) performance improves gradually (see Figure6.)

It is observed that as the codeword symbols \( n \) (i.e., code length) increases from \( n=31 \) to 127, at similar error correction capability \( t \) BER performance improves gradually. It is observed from figure 45 that the larger RS(127,123) code having signal to noise \( (E_b/N_o) \) ratio 3.2dB performs better than the smaller RS(31,27) code having signal to noise \( (E_b/N_o) \) ratio 3.7dB for BER=\( 10^{-2} \) maintaining constant error correcting capability \( t=2 \).There is 0.5dB difference of signal to noise ratio \( (E_b/N_o) \) for larger codeword at similar BER=\( 10^{-2} \).

5. Conclusions

The Bit-Error-Rate (BER) performance of Reed-Solomon code is evaluated in different point of view like Redundancy, Code length, Code Rate and error correction capability. In order to inspect the gap between theoretical and practical sides of communication system, the channel coding is applied to find enhanced system performance. It is observed that a properly chosen RS coding scheme can significantly improve the BER performance. The Bit-Error-Rate performance improves as the redundancy increases by lowering code rate is finding of this work.

The simulation results clearly showed that:

1. the coding gain with high code rate (0.94) is better than the low code rate (0.74).(see Figure 4.)
2. the Bit-Error-Rate performance improves as the redundancy \( (n-k) \) increases from 2 to 8 symbols thereby lowering the code rate. (see Figure 4.)
3. the Bit-Error-Rate (BER) performance improves as the code length \( n \) increases at constant code rate and at the similar error correcting capability (see Figure5. and Figure6. respectively).

The results shown in this paper will be useful in validating the results of simulation studies. RS coding will continue to be used to force communication system performance ever closer to the line drawn by Shannon.

6. References


Authors Profile

Mr. Saurabh Mahajan is doing M.Tech. in Electronics and Communication Engineering from Beant College of Engineering and Technology, Gurdaspur, affiliated to Punjab Technical University Jalandhar. He completed his B.Tech. in Electronics and Communication Engineering in 2002 from IITT College of Engineering and Technology, Pojewal, Nawanshar, affiliated to Punjab Technical University Jalandhar. He is currently working as Assistant Professor in the Department of Electronics and Communication Engineering in Sri Sai college of Engineering and Technology, Badhani (Punjab). He is involved in teaching since 2004 and his major research interest includes Coding Theory, Communication and SCADA Systems.

Mr. Gurpadam Singh received his M.Tech degree from Punjab Technical University, Jalandhar and his B.E. (in Honor) from Punjab University, Chandigarh, India. He is currently working as an Associate Professor in the Department of Electronics and Communication Engineering in Beant College of Engineering and Technology, Gurdaspur (Punjab.). He is having overall teaching experience of 16 years and also guided 5 M.Tech Thesis. He has published more than 15 research papers in various International Journals and Conferences. His major research interests are in Coding Theory, Embedded system design in Digital System Processing and in communication Systems.