Fractal Characteristics for Goose Down Assemblies as Porous Media

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Abstract—In this paper, the fractal characteristics of goose down assembly are described by determining the pore area fractal dimension and the tortuosity fractal dimension. The goose down assembly, filled with mass of down fibers and pores, is loose and ductile. The pores are varying shapes and sizes, and distribute unevenly. Firstly, micro computed tomography is used to observe the internal microstructure of down assembly without destroying the original assembly shape. The derived computed tomography images of down assemblies with various volume fractions are then treated by two-value method. The measurement verifies that the porous structure of down assembly possesses typical fractal characteristic, and the pore area fractal dimensions $D_f$ are determined by the box counting method. According to the capillary principle the tortuous flow path in down assembly is simulated, and the fractal dimension of the tortuous flow path $D_T$ is also calculated by the same way. The results indicate that the pore area fractal dimensions decrease and the tortuosity fractal dimensions increase gradually as the volume fractions $V_f$ of the assembly increase.

Index Terms—goose down assembly, porous media, Micro computed tomography, micropores structure, pore area fractal dimension, tortuosity fractal dimension

I. INTRODUCTION

The goose down fibers is a layer of villus growing on the goose cuticular and covered by the feathers (Fig. 1). As a natural filling material, the goose down fiber has well known for its superior thermal insulating properties, and is widely used as an insulated filling material for winter outerwear and quilters. Down assembly is obtained by ruleless accumulating of a mass of down fibers (Fig. 2), so down assembly can be considered as a kind of representative porous media [1-4]. In this porous media, the porous characterizations play an important role on mass and heat transfer properties of down assembly. In fact, since the microstructures in the porous media are usually disordered and extremely complicated, this makes it difficult to analytically find the physico-chemical properties of the porous media, including the heat transfer property [5,6]. Conventionally, the studies on the heat transfer property of porous media were willing to considering the microstructure even and consecutive. However, such a hypothesis is too simply to reflect the heat transfer processing accurately. In recent years, researchers try to find novel methods and techniques to characterize the pores structure, and the fractal is one of those. So-called fractal is the irregular things with self-similar structure. The self-similarity of fractal is described by fractal dimension. Through the fractal dimension the fractal complexity can be quantified, and thus porous media performances can be described and modeled more precisely.

Katz and Thompson [7, 8] firstly investigated the sandstone pores structure by SEM, and indicated the micropores fractal nature of the porous media. They calculated the pores dimensions were between 2.5 to 2.85. Avnir and Farin [9] discussed the determination of surface fractality, and derived the surface pore fractal dimensions. Chen and Shi [10] developed the soil permeability model based on the calculations of pore area fractal dimension, sizes of particles and pores spectral dimension. Feng and Hao [11] discussed the CT method on the soil macro-pores structures analysis, and proposed the macro-pores fractal model on the soil faults by box-counting method as:

$$D = \lim_{L \to 0} \frac{\log N(L)}{\log L}, \quad (1)$$

![Figure 1. Image of goose down cluster.](image)
where \( N(L) \) is the box numbers, \( L \) is the size of the square box, and \( D \) is the macro-pore area fractal dimension.

For example, if one were to measure the length of a coastline, the length would depend on the size of the measuring stick used. Decreasing the length of the measuring stick leads to a better resolution of the convolutions of the coastline, and as the length of the stick approaches zero, the coastline’s length tends to infinity. This is the fractal nature of the coastline. Since a coastline is so convoluted, it tends to fill space, and its fractal dimension lies somewhere between that of a line (which has a value 1) and a plane (which has a value 2).

The measure of a fractal object \( N(L) \) (namely length, area, or volume) is governed by a scaling relationship of the form

\[
N(L) \sim L^{D_f},
\]

where the “\( \sim \)” should be read as “scales as,” and \( D_f \) is the fractal dimension of the object. As an illustration of the above relationship, consider the microstructure of a metal showing grains of various shapes and sizes. This is a fractal object in a two-dimensional plane. It is observed that the average area, \( N(L) \), of the grains within squares of different sizes \( L \) by \( L \) (defined as the arithmetic mean of only those samples for which the center of the square falls on the grain) scales with the length, \( L \), over a range of lengths, as per the above relationship [21]. The fractal dimension, \( D_f \), calculated as the slope of a log-log plot of \( N(L) \) against \( L \), lies in the range \( 1 < D_f < 2 \). Alternatively, the microstructure can be described in terms of two linear fractal dimensions, each having a value between 0 and 1, along mutually perpendicular directions. The linear fractal dimensions are obtained from a scaling relationship as in (2), where \( N(L) \) denotes the average total length of the intercepts between a measuring line of size \( L \), and the microstructure.

Associated with the (2) is the property of self-similarity, which implies that the \( D_f \) calculated from the relationship in (2) remains constant over a range of length scales, \( L \). Exact fractals like the Sierpinsky gasket, Koch curve, etc., exhibit self-similarity over an infinite range of length scales [20]. In actual applications, self-similarity in a global sense is seldom observed, and the “fractal” description is usually based on a statistical self-similarity, which the objects exhibit in some average sense, over a certain local range of the length scale \( L \), which is of relevance to the problem [22-24]. The fractal dimension calculated based on the local, statistical self-similarity is termed the local fractal dimension to distinguish from the term fractal dimension, which implies global self-similarity at all length scales. The fractal dimensions (local or global) of statistical fractals are usually estimated in the same manner as in the illustration of the microstructure, using a scaling relationship between \( N(L) \) and \( L \) of the form in (2) [22].

The concept of local, statistical self-similarity has been used in many applications ranging from characterization of the microstructure of materials [21] to analysis of the speech waveforms and signals [24, 25]. For example, in the area of speech recognition in a speech reside in the region of short time scales. While the term local fractal dimension is used by some investigators [25], others refer to it simply as the fractal dimension.

### II. Fractal Geometry and Local Fractal Dimension

Fractal is a new subject developed quickly only in recent several years, which reveals the unifications between in-order and out-of-order, and determinability and randomness. The fractal structures have aroused a great deal of interests in different fields [16-19].

Firstly, there are great differences between conventional Euclidean geometry and Fractal geometry. Euclidean geometry describes regular objects like points, curves, surface and cubes using integer dimension 0, 1, 2, and 3, respectively. However, most of the objects in nature, such as the surfaces of mountains, coastlines, microstructure of metals, etc., are disordered and irregular, and they do not follow the Euclidean description due to the scale-dependent measures of length, area and volume. These objects are called fractals, and are described by a nonintegral dimension called the fractal dimension [20].

For example, if one were to measure the length of a coastline, the length would depend on the size of the measuring stick used. Decreasing the length of the measuring stick leads to a better resolution of the convolutions of the coastline, and as the length of the stick approaches zero, the coastline’s length tends to infinity. This is the fractal nature of the coastline. Since a coastline is so convoluted, it tends to fill space, and its fractal dimension lies somewhere between that of a line (which has a value 1) and a plane (which has a value 2).

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### III. Fractal Characteristics of Porous Media

Based on the (2), introduce another important parameter the pore size \( \lambda \), the total numbers of pores, from the smallest diameter \( \lambda_{\text{min}} \) to the largest diameter \( \lambda_{\text{max}} \) can be obtained as

\[
N_f(L \geq \lambda_{\text{min}}) = \left( \frac{\lambda_{\text{max}}}{\lambda_{\text{min}}} \right)^{D_f},
\]
and the differential expression is

\[ -dN = D_r \lambda_{\min}^{D_f} \lambda^{-1} d\lambda, \quad (4) \]

where \( D_r \) is the same fractal dimension as in (2), is the fractal dimension [15]. The negative sign implies that the pore population decrease with the increase of pore size, and \(-dN>0\).

The number of pores \( N \) becomes infinite as \( \lambda \to 0 \), which is one of properties of fractal objects [20]. Dividing (4) by (3) gives

\[ \left( \frac{dN}{N} \right)_r = D_r \lambda_{\min}^{D_f} \lambda^{-1} d\lambda = f(\lambda) d\lambda, \quad (5) \]

where \( f(\lambda) \) is the probability density function, and

\[ f(\lambda) = D_r \lambda_{\min}^{D_f} \lambda^{-1} \lambda_{\max}^{D_f} \lambda^{-1} \lambda_{\max}^{D_f} \lambda^{-1}, \quad (6) \]

As in the probability theory, the probability density function, \( f(\lambda) \), should also satisfy the following relationship:

\[ \int_0^{\infty} f(\lambda) d\lambda = \int_{\lambda_{\min}}^{\lambda_{\max}} f(\lambda) d\lambda = 1 - \frac{\lambda_{\min}}{\lambda_{\max}} D_f = 1, \quad (7) \]

if and only if

\[ \left( \frac{\lambda_{\min}}{\lambda_{\max}} \right)^{D_f} \leq 0. \quad (8) \]

is satisfied. Equation (8) implies that \( \lambda_{\min} \ll \lambda_{\max} \) must be satisfied for fractal analysis of a porous medium [15]; otherwise the porous medium is a non-fractal medium. So the (8) can be considered as the fractal character of the fractal porous media.

A porous medium having various pore sizes can be considered as a bundle of tortuous capillary tubes with variable cross-sectional areas. Let the diameter of a capillary in the medium be \( r \) and its tortuous length along the flow direction be \( L_r(r) \). Due to the tortuous nature of the capillary, \( L_r(r) \geq L_0 \), with \( L_0 \) being the representative length. For a straight capillary, \( L_r(r) = L_0 \). Wheatcraft and Tyler [26] developed a fractal scaling/tortuosity relationship for flow through heterogeneous media, and the scaling relationship is given by:

\[ L_r(r) = L_0^{D_{t(r)}} r^{1-D_{t(r)}}, \quad (9) \]

where \( \epsilon \) is the length scale of measurement. The relationship between the diameter and length of capillaries also exhibits the similar fractal scaling law [5]:

\[ L_r(r) = L_0^{D_{t(r)}} r^{1-D_{t(r)}}, \quad (10) \]

where is the tortuosity fractal dimension, with \( 1 < D_{t(r)} < 2 \), representing the extent of convolutedness of capillary pathways for fluid flow through a medium. Note that \( D_{t(r)} = 1 \) represents a straight capillary path, and a higher value of \( D_{t(r)} \) corresponds to a highly tortuous capillary. The limiting case of \( D_{t(r)} = 2 \) corresponds to a highly tortuous line that fills a plane [26].

IV. BOX-COUNTING METHOD

Both the pore area fractal dimension and the tortuosity fractal dimension can be determined based on the box-counting method [27, 28]. This method is based on the image analysis of cross-sections of samples along a plane normal to the flow direction. The calculation procedures of \( D_{t(r)} \) are

1. \( \lambda \) is selected as the size of square boxes to discretize the cross-section under consideration, then the number, \( N(\lambda) \), of boxes required to completely cover the pore areas is counted.

2. The value of \( \lambda \) is changed to correspond to a new \( N(\lambda) \).

3. repeated the step (1) and (2), the logarithmic plot of the cumulative number of pores \( N(L \geq r) \) versus the pore size \( \lambda \) is then drawn.

4. The pore area fractal dimension can be determined by the value of the slope of the linear fit through data on the plot.

To the tortuosity fractal dimension \( D_{t(r)} \), the tortuous capillary tubes composing the porous media form long convoluted paths. The diameter of the circle represents the pore size. So the \( D_{t(r)} \) can be derived through

(1) \( r \) is selected as the size of square boxes to discretize the flow pathway extracted form the images in terms of capillary principle, then the number, \( L_r(r) \), of boxes required to completely cover the pore pathway is counted.

(2) The value of \( r \) is changed to correspond to a new \( L_r(r) \).

(3) repeated the step (1) and (2), the logarithmic plot of the cumulative number of boxes \( L_r(L \geq r) \) versus the size \( r \) is then drawn.

(4) The fractal dimension for the tortuous flow path can be determined by the value of the slope of the linear fit through data on the plot.

V. A CASE STUDY

A. Image Capturing by Micro-CT

The cross-section images of down assemblies are captured by SkyScan-1172 high-resolution desk-top micro-CT system (Micro Technology Hong Kong Ltd.) operated at 100 kV. Fig. 3 shows the cross-section images of the down assemblies sample with three types of volume fraction \((V_f)\) in terms of the heat transfer properties from down experimental evidence [29]. In the figures, the black and white regions are pores and fibers respectively.
B. Image Processing

The images center is selected to input into the computer. The images are then disposed based on wave filter and noise reducing. Threshold can be determined from histograms, as seen in the Fig. 4. Then the gray level images are transferred into binary images by adjusting threshold, as seen in the Fig. 5.

C. Evaluation of Pore Area Fractal Dimension $D_f$

Based on the binary images as shown in Fig. 5, the certain size image is cut and data processed in the computer. The software, Matlab, were used to record $N(\lambda)$ of the pores. Fig. 6(a), (b) and (c) are the logarithmic plots of the cumulative pores numbers versus pore sizes, for the three porous samples with $V_f = 0.001162$, $V_f = 0.001960$ and $V_f = 0.005659$, respectively. It can be seen that the number of the cumulative pores decreases as the pore size increases. The data follow a linear relationship on the logarithmic scale, and this confirms the statistical fractal nature of the microstructures of the down assembly porous media. From the slopes of these straight lines we can determine the fractal dimensions $D_f = 1.9952$ for the sample with $V_f = 0.001162$ (Fig. 6(a)), $D_f = 1.9948$ for the sample with $V_f = 0.001960$ (Fig. 6(b)), and $D_f = 1.9773$ for the sample with $V_f = 0.005659$ (Fig. 6(c)). The pore area fractal dimension decreases as the volume fraction of down assembly porous media increase.
D. Evaluation of Tortuosity Fractal Dimension $D_T$

Based on the Fig. 5, the equivalent circle radius and barycentric coordinates of the pores are obtained. The curve plotted with the abscissa of the equivalent circle barycentric coordinates as the ordinate and the multiple of the fibers diameter as the abscissa, is a possible tortuous pathway in the down assembly. Fig. 7 respectively presents a random flow pathways in the samples with $V_f=0.001162$, $V_f=0.001960$ and $V_f=0.005659$, respectively, shown in Fig. 5. The same software that was used for finding $D_f$ based on the box-counting method is now applied to find the tortuosity fractal dimension $D_T$ for the flow pathway. The values of $D_T$ are the averaged values of the tortuosity fractal dimension for ten extracted pathways like Fig. 7(a), (b) or (c). Fig. 8 shows the logarithmic plots of $\ln L_\tau(\lambda)$ versus $\ln \lambda$ for the random flow pathways of the samples with $V_f=0.001162$, $V_f=0.001960$ and $V_f=0.005659$, respectively. From the slopes of these straight lines we can determine the tortuosity fractal dimensions $D_T=1.0420$ for the pathway of the sample with $V_f=0.001162$, $D_T=1.0682$ for the pathway of the sample with $V_f=0.001960$, and $D_T=1.1082$ for the pathway of the sample with $V_f=0.005659$. The tortuosity fractal dimension increases as the volume fraction of down assembly porous media increase.
VI CONCLUSIONS

Micro-CT can be applied to observe and derive the microstructure of the goose down assembly without affecting the geometry of the loose goose down assemblies. Using the local fractal theory, the statistical micropores fractal nature of the goose down assemblies as porous media is demonstrated. And the pore area fractal dimensions and the tortuosity fractal dimensions of the assemblies with three typical volume fractions 0.001162, 0.001960 and 0.005659 are determined by the box-counting method. The pore area fractal dimension, between 1.97~1.996, decreases as the volume fraction of down assembly porous media increase. The tortuosity fractal dimension, between 1.04~1.11, increases as the volume fraction of down assembly porous media increase. This demonstrates that as the fibers density increases the micropores among fibers reduce, and the tortuosity and circuitry of the flow pathway improve.

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REFERENCES


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Dr. Gao won third-class prize of Technology Progress Award by China National Textile And Apparel Council in 2003, candidate of Excellent Young Teacher selected by Colleges of Shanghai in China in 2008, and third-class prize of Jiangsu provincial Scientific and Technological Advances Award in 2009.