Generalized CRLB for DA and NDA Synchronization of UWB Signals with Clock Offset

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Abstract—In this paper the Cramér-Rao lower bound (CRLB) of an ultra-wideband (UWB) pulse amplitude modulated (PAM) signal with time hopping (TH) code is derived for the practical case where there is an initial unknown relative clock offset between the transmitter and receiver. However, it is assumed that there is a priori information about the probability density function of clock offset. CRLB expressions are developed for both the data-aided (DA) case with an assumed deterministic pilot for synchronization and for the non data-aided (NDA) case based on blind synchronization. It is shown that the influence of the unknown clock offset on the synchronization performance becomes negligible as the number of multipath components increases emphasizing an advantage in UWB links where the multipath is typically rich. Another observation made is that as the SNR decreases, the difference in the CRLB for the NDA and DA cases increases emphasizing the significance of a pilot. Having a priori information helps to achieve a lower CRLB for the timing or clock offset estimation. This improvement is significant in the case of NDA. Finally at very low SNR CRLB of the clock offset converges to the variance of clock offset based on only a priori information.

Index Terms—Cramér-Rao lower bound, data-aided, non data-aided, channel estimation, clock frequency offset, ultra-wideband communications.

I. INTRODUCTION

ULTRA-WIDEBAND (UWB) technology utilizing subnanosecond monocycles has attracted significant interest from both academia and industry [1], [2]. This is primarily due to its potential high data rate performance over short ranges. However, the large bandwidth of UWB results in stringent synchronization requirements [3]. In [4], the performance of maximum likelihood (ML) channel estimation was assessed based on simulation. In [5] and [6], the authors derived the CRLB for the data aided (DA) and non data aided (NDA) for the ML channel estimator. As shown in [6] the CRLB is surprisingly insensitive to whether the synchronization is based on DA or NDA estimators for moderate to high signal-to-noise (SNR) ratios. In [7] the authors have the same approach as authors in [5] and [6] but included the effect of different pulse shapes for UWB monocycles demonstrating that the CRLB of synchronization parameters is primarily a function of the power spectral density (PSD) of the UWB signal and relatively insensitive to the pulse shape. In the cold-start scenario, the receiver and transmitter clocks are not initially frequency locked which complicates the acquisition and synchronization operations especially for the case of a non-stationary channel. The problem of frame frequency estimation for UWB communications was investigated in [9] and an estimator for frame frequency was proposed by transmitting periodic pilot pulses, however the corresponding CRLB for jointly unknown parameters was not considered. Hence the objective of this paper is to establish the CRLB for the jointly unknown synchronization parameters quantifying the influence of the unknown clock offset. Consequently the paper extends the results of [6] and [7] with clock offset for both DA and NDA cases. A comprehensive exposition is contained in [12]. However, the influence of having a priori knowledge of channel parameters as represented by so-called generalized CRLB (G-CRLB) [13], [14] was not considered in [12] and therefore is investigated here.

This paper is organized as following. In section II system model is presented. Sections III and IV develop the CRLB for the DA and NDA cases respectively with clock offset. Discussions of numerical calculations of the derived CRLB expressions are given in section V with overall conclusions provided in section VI.

II. SYSTEM MODEL

The conventional description of a generic UWB PAM modulated signal is given by [2] as

$$s(t) = \sum_{m=0}^{M-1} b_m \cdot p(t - m\cdot NT)$$  \hspace{1cm} (1)

where $$p(t) = \sum_{n=0}^{N-1} g(t - nT - c_nT_c)$$ is the symbol waveform consisting of \( N \) UWB monocycle pulses denoted by \( g(t) \).
which are typically approximations of second order Gaussian waveforms of duration $T_g$. In this paper binary PAM data modulation is assumed such that $b_m \in \{\pm 1\}$. The analysis can be extended to include multilevel PAM or more generic IR (impulse radio) modulations. However, as the CRLB for the jointly unknown synchronization parameters is rather insensitive to the particular pulse modulation used, binary PAM was deemed to be sufficient. In (1), $T_f$ is the frame period, $T_c$ is the chip period which is assumed to be a sub-multiple of $T_f$, i.e. $T_c = T_f/Q$ where $Q$ is an integer, and $\{c_s\}_{s=0}^{N-1}$ is the time-hopping sequence whose elements are integer values chosen randomly in the range $\{0,1,\ldots,N_{TH}-1\}$ where $N_{TH} < Q$.

In accounting for the clock difference between the receiver and the transmitter it is more convenient to assume that the receiver clock has no offset while the transmitter clock is offset by a relative frequency error denoted as $\Delta$. Hence the actual transmitted signal, denoted by $s_c(t)$, is given as $s_c(t) = s((1+\Delta)t) = s(\kappa t)$ where $\kappa = 1 + \Delta$. The transmitted signal $s_c(t)$ is compressed if ($\kappa > 1$) or dilated if ($\kappa < 1$). In this paper it is assumed that $\Delta$ has a Gaussian distribution with zero mean and variance of $\sigma_\Delta^2$ typically $\sigma_\Delta < 10$ ppm (part per million). After transmission over the UWB channel the received signal can be stated as [6]

$$r(t) = s_c(t) * h(t) + w(t) = \sum_{l=1}^{L_c} \gamma_l s_c(t - \tau_l) + w(t) \tag{2}$$

where $L_c$ is the number of multipath components assumed, and $\tau_l$ and $\gamma_l$ are the delay and the gain of $l$th multipath component. In this paper multiple-access interference and thermal noise are assumed as white Gaussian noise represented by $w(t)$ with one sided PSD of $N_0$. Throughout the paper we refer to $E_p = \int_0^{NT_f} p^2(t)dt$ as symbol waveform energy or energy per symbol and $\Gamma = E_p / N_0$ as the SNR.

From (1), the received signal consisted of $M$ sequential symbols with a support of $T = MNT_f / \kappa$. The parameters to be jointly estimated are the channel parameters and the clock frequency offset or $\kappa$ which is expressed by the parameter vector $\theta = [\tau_1, \ldots, \tau_L, \gamma_1, \ldots, \gamma_L, \kappa]$. The CRLB of the estimate of $\theta$ is given by the inverse of the Fisher information matrix (FIM) denoted by $I$ which its $ij$th element for the case of without a priori information is expressed as [8]

$$I_\theta = E_\theta \left\{ -\frac{\partial^2 \ln \Lambda(r; \theta)}{\partial \theta_i \partial \theta_j} \right\} \tag{3}$$

where $\theta_n$ represents the $n$th element of the vector $\theta$ and $\Lambda(r; \theta)$ represents the likelihood function of the received signal given in (2) which is a function of unknown but deterministic vector parameter $\theta$. Assuming a priori information on some parameters of vector $\theta$ causes an extra term to be added to (3) as follows [13], [14]

$$I_\theta^{(r)} = I_\theta + I_\phi = E_\theta \left\{ -\frac{\partial^2 \ln \Lambda(r; \theta)}{\partial \theta_i \partial \theta_j} \right\} + E_{\theta_0} \left\{ -\frac{\partial^2 \ln f_\theta(\theta)}{\partial \theta_i \partial \theta_j} \right\} \tag{4}$$

$E_{\theta_0}$ denotes that expectation with respect to noise and $\theta$ but $E_\theta$ represents expectation with respect to $\theta$. The probability density function of a priori information regarding $\theta$ is denoted by $f_\theta(\theta)$. The superscripts “D” and “P” in (4) stand for data and a priori information [14]. The function $\Lambda(r; \theta)$ denotes the conditional likelihood function given $\theta$.

Consider the DA case such that $s(t)$ in (1) is deterministic. Define $\hat{s}(t; \hat{\theta}, \theta)$ as the received signal (excluding the Gaussian channel noise) which is also deterministic for a given parameter vector $\theta$ and the data vector $\hat{b}$. The log likelihood function for this DA case, denoted as $\Lambda_{DA}(r; \hat{b}, \theta)$, is then [8]

$$\ln \Lambda_{DA}(r; \hat{b}, \theta) = -\frac{1}{N_0} \int_r \left[ r(t) - \hat{s}(t; \hat{b}, \theta) \right]^2 dt \tag{5}$$

For the NDA case $\hat{b}$ is random such that it is necessary to consider the averaged likelihood function formed by taking the expectation of (5) with respect to $\hat{b}$ which after doing some mathematical operations it is obtained that [12]

$$\ln \Lambda_{NDA}(r; \theta) = C - \frac{1}{N_0} \int_r [\hat{s}(t; \theta, \theta) - \hat{s}(t; \theta, \theta)] dt + \sum_{m=1}^{M} \ln \cosh(\alpha_m) \tag{6}$$

where $C$ denotes the collection of terms that are independent of $\theta$, $\alpha_m = \frac{2}{N_0} \int_r r(t) p(\kappa(t - \tau_l) - mNT_f) dt$ and $\alpha_m = \sum_{l=1}^{L_c} \gamma_l \alpha_m$. The parameter $T_m$ represents the timing interval for the $m$th data symbol which is
\[
[mNT_s / \kappa, (m+1)NT_s / \kappa].
\]

The FIM given \( \theta \) or \( I^{(0)} \) will be calculated based on (5) and (6) for both DA and NDA in the following sections. However, what remains is to calculate the expectation over \( \theta \) which is evaluated numerically. \( I^{(n)} \) in (4) for a Gaussian distribution of clock offset is simply \((\sigma_n^2)^{-1}\) for the \((2L_c+1) \times (2L_c+1)\) element corresponding to clock offset and zero for the other elements since the only available a priori information is clock offset distribution. Hereafter we drop the superscript “D” to simplify the notation.

III. CRLB OF PARAMETER ESTIMATION FOR THE DA CASE

In the DA case, \( r(t) \) is a Gaussian random process with a mean of \( \bar{s}(t; \bar{b}, \theta) \) and a covariance that is independent of \( \theta \) such that (3) can be expressed as [8]

\[
I_q = \frac{2}{N_0} \int_\tau \frac{\partial \bar{s}(t; \bar{b}, \theta)}{\partial \theta_i} \frac{\partial \bar{s}(t; \bar{b}, \theta)}{\partial \theta_j} dt
\]  

(7)

Using (2) in (7) the components of the FIM are given as

\[
I_{\tau, \gamma} = \frac{2\Gamma}{\kappa} R_{\gamma} (\tau_y)
\]  

(8a)

\[
I_{\tau, \gamma} = 2\Gamma \gamma_i \gamma_j R_{\gamma, \gamma} (\tau_y)
\]  

(8b)

\[
I_{\tau, \gamma} = -2\Gamma \gamma_i R_{\gamma} (\tau_y)
\]  

(8c)

\[
I_{\gamma, \gamma} = \frac{2\Gamma}{\kappa} \sum_{i=1}^{L_c} \gamma_i R_{\gamma, \gamma} (\tau_y)
\]  

(8d)

\[
I_{\gamma, \gamma} = \frac{2\Gamma}{\kappa^2} \sum_{i=1}^{L_c} \gamma_i R_{\gamma, \gamma} (\tau_y)
\]  

(8e)

\[
I_{\gamma, \gamma} = \frac{2\Gamma}{\kappa^3} \sum_{m=1}^{L_c} \sum_{n=1}^{L_c} \gamma_m \gamma_n R_{\gamma, \gamma, \gamma} (\tau_{mn})
\]  

(8f)

where

\[
R_{\gamma} (\tau) \triangleq \left(1 / E_p \right) \int_\tau s(t) s(t+\tau) dt
\]  

(9)

\[
R_{\gamma, \gamma} (\tau) \triangleq \left(1 / E_p \right) \int_\tau s'(t) s'(t+\tau) dt
\]  

(10)

\[
R_{\gamma, \gamma} (\tau) \triangleq \left(1 / E_p \right) \int_\tau s'(t) s(t+\tau) dt
\]  

(11)

\[
R_{\gamma, \gamma} (\tau) \triangleq \left(1 / E_p \right) \int_\tau s'(t) s'(t+\tau) dt
\]  

(12)

\[
R_{\gamma, \gamma} (\tau) \triangleq \left(1 / E_p \right) \int_\tau s'(t) s(t+\tau) dt
\]  

(13)

\[
R_{\gamma, \gamma} (\tau) \triangleq \left(1 / E_p \right) \int_\tau s'(t) s'(t+\tau) dt
\]  

(14)

\[
\tau_{ij} \triangleq \kappa (\tau_i - \tau_j)
\]  

(15)

As in [6] a simplifying approximation is made that there is no inter-pulse interference (IPI) for the multipath channel which implies that the multipath components of successive pulses can be intermixed but are non-overlapping. Hence the elements of the FIM of (8) simplifies to [12]

\[
I_{\tau, \gamma} = \frac{2\Gamma}{\kappa} R_{\gamma} (0) \delta_{ij}
\]  

(16a)

\[
I_{\tau, \gamma} = 2\Gamma \gamma_i R_{\gamma, \gamma} (0) \delta_{ij}
\]  

(16b)

\[
I_{\tau, \gamma} = -2\Gamma \gamma_i R_{\gamma} (0) \delta_{ij}
\]  

(16c)

\[
I_{\gamma, \gamma} = \frac{2\Gamma}{\kappa^2} R_{\gamma, \gamma} (0)
\]  

(16d)

\[
I_{\gamma, \gamma} = \frac{2\Gamma}{\kappa^3} \sum_{m=1}^{L_c} \sum_{n=1}^{L_c} \gamma_m \gamma_n R_{\gamma, \gamma, \gamma} (0)
\]  

(16f)

where \( \delta_{ij} \) is Kronecker function which is 1 when \( i = j \) and 0 otherwise. Assuming that \( g(t) \) is symmetric or anti-symmetric about some point in time then \( R_{\gamma} (0) = 0 \).

Consequently from (16c) \( I_{\tau, \gamma} = 0 \) indicating that there is no interaction between estimation of the delay and gain of multipath components. This implies that the unknown multipath delay does not inflate the variance of the estimate of the multipath gain and the other way around. Equations (16d) to (16f) indicate that the elements of row and column corresponding to the parameter \( \kappa \) are nonzero. Hence the unknown clock offset parameter will inflate the variance of the estimation of the other parameters. To see this, the FIM is written as

\[
I_{DA} = \begin{bmatrix}
I_{\tau} & 0 & \tilde{I}_{\gamma, \gamma} \\
0 & I_{\gamma} & \tilde{I}_{\gamma, \gamma} \\
\tilde{I}_{\gamma, \gamma}^T & \tilde{I}_{\gamma, \gamma}^T & I_{\gamma, \gamma}^T
\end{bmatrix}
\]  

(17)

where \( I_{\tau} = \text{Diag} \left( I_{\tau, \tau, \tau}, ..., I_{\tau, \tau, \tau} \right) \), \( I_{\gamma} = \text{Diag} \left( I_{\gamma, \gamma, \gamma}, ..., I_{\gamma, \gamma, \gamma} \right) \), \( \tilde{I}_{\gamma, \gamma} = \left[ I_{\gamma, \gamma} \right]^T \) and \( \tilde{I}_{\gamma, \gamma} = \left[ I_{\gamma, \gamma} \right]^T \). It can be shown that [12]

\[
CRLB_{DA} (\hat{\kappa} | \theta) = \frac{1}{I_{\gamma, \gamma} - \sum_{i=1}^{L_c} \left( I_{\tau, \tau} + I_{\gamma, \gamma} \right) I_{\tau, \tau}}
\]  

(18a)

\[
CRLB_{DA} (\hat{\tau} | \theta) = \frac{1}{I_{\tau, \tau}} + \frac{I_{\gamma, \gamma} CRLB_{DA} (\hat{\Delta} | \theta)}{I_{\tau, \tau}}
\]  

(18b)
\[ CRLB_{DA} (\hat{\tau} | \theta) = \frac{1}{I_{\tau;\tau}} \left( 1 + I_{\tau;\tau}^{2} CRLB_{DA} (\hat{\Delta} | \theta) \right) \]  

\[ \text{where } CRLB_{DA} (\hat{\Delta} | \theta) \text{ denotes the CRLB of the variance of the estimate of the argument evaluated at } \theta. \text{ In (18a) the term } I_{\tau;\tau}^{2} CRLB (\hat{\Delta} | \theta) / I_{\tau;\tau} \geq 0 \text{ such that the unknown clock offset inflates the CRLB of the estimate of the multipath delay. Also the term } I_{\tau;\tau} CRLB_{DA} (\hat{\Delta} | \theta) / I_{\tau;\tau} \geq 0 \text{ showing that the unknown clock offset inflates the variance of the estimate of the multipath delays. Note that when } CRLB (\hat{\Delta} | \theta) = 0, \text{ CRLB}_{\tau i} (\hat{\tau} | \theta) = (I_{\tau;\tau})^{-1} \text{ and } CRLB_{\tau i} (\hat{\tau} | \theta) = (I_{\tau;\tau})^{-1} \text{ which implies that when there is no unknown clock offset the CRLB of the variance of jointly estimating the multipath delay and gain is the same as when these parameters are estimated independently.} \]

\[ E \left\{ \frac{\partial^{2} \ln \Lambda (r | \theta)}{\partial \gamma_{i} \partial \gamma_{j}} \right\} \begin{cases} \frac{2M \Gamma \gamma_{i} \gamma_{j} - \frac{1}{\gamma_{i} \gamma_{j}} \sum_{n=0}^{M-1} G_{in} }{1 - \frac{1}{\gamma_{i} \gamma_{j}} \sum_{m=0}^{M-1} F_{jm} } & i = j \\ \frac{-\sum_{n=0}^{M-1} H_{in}}{1 - \sum_{m=0}^{M-1} (I_{in} + J_{in})} & i \neq j \end{cases} \]  

\[ \text{where } F_{jm} = E \left\{ \gamma_{j} \gamma_{m} \alpha_{mj} \sech^{2} (\alpha_{m}) \right\} \text{ and } G_{in} = E \left\{ \gamma_{i} \alpha_{ni} \sech^{2} (\alpha_{n}) \right\}. \text{ The parameter } \alpha_{ni} \text{ denotes the first order derivative of } \alpha_{ni} \text{ with respect to } \tau_{i}. \text{ Also it can be obtained that} \]

\[ E \left\{ \frac{\partial^{2} \ln \Lambda (r | \theta)}{\partial \tau_{i} \partial \tau_{j}} \right\} \begin{cases} \frac{-\sum_{n=0}^{M-1} H_{in}}{1 - \sum_{m=0}^{M-1} (I_{in} + J_{in})} & i \neq j \\ \end{cases} \]

\[ \text{where } H_{in} = E \left\{ \gamma_{i} \gamma_{m} \alpha_{ni} \alpha_{mj} \sech^{2} (\alpha_{m}) \right\}, \]

\[ I_{in} = E \left\{ \gamma_{i} \alpha_{ni} \sech^{2} (\alpha_{n}) \right\} \text{ and } J_{in} = E \left\{ \gamma_{i} \alpha_{ni} \tanh (\alpha_{n}) \right\}. \text{ The parameter } \alpha_{ni} \text{ represents the second order derivative of } \alpha_{ni} \text{ with respect to } \tau_{i}. \text{ It can be shown that [12]} \]

\[ E \left\{ \frac{\partial^{2} \ln \Lambda (r | \theta)}{\partial \gamma_{i} \partial \gamma_{j}} \right\} \begin{cases} \frac{-1}{\gamma_{i} \gamma_{j}} \sum_{n=0}^{M-1} (K_{in} + L_{in}) = 0 & i = j \\ \frac{-1}{\gamma_{j}} \sum_{m=0}^{M-1} M_{jm} = 0 & i \neq j \end{cases} \]

\[ \text{where } K_{in} = E \left\{ \gamma_{i} \alpha_{ni} \tanh (\alpha_{n}) \right\}, \]

\[ L_{in} = E \left\{ \gamma_{i} \gamma_{i} \alpha_{ni} \alpha_{mi} \sech^{2} (\alpha_{m}) \right\} \text{ and } M_{jm} = E \left\{ \gamma_{j} \gamma_{j} \alpha_{mj} \alpha_{ni} \sech^{2} (\alpha_{n}) \right\}. \text{ Also we can get} \]

\[ E \left\{ \frac{\partial^{2} \ln \Lambda (r | \theta)}{\partial \tau_{i} \partial \tau_{j}} \right\} = \frac{-2 \gamma_{i} \gamma_{j} \Gamma}{\tau_{i} \tau_{j}} - \frac{1}{\gamma_{i}} \sum_{n=0}^{M-1} O_{in} \]  

\[ \text{where } O_{in} = E \left\{ \gamma_{i} \left[ \frac{\partial \alpha_{ni}}{\partial \tau_{i}} \tanh (\alpha_{n}) + \alpha_{ni} \frac{\partial \alpha_{ni}}{\partial \tau_{i}} \sech^{2} (\alpha_{n}) \right] \right\}. \]

\[ \text{Finally} \]

\[ E \left\{ \frac{\partial^{2} \ln \Lambda (r | \theta)}{\partial \tau^{2}} \right\} = \frac{2M \Gamma \gamma_{i} \gamma_{j}}{\tau_{i} \tau_{j}} - \frac{1}{\gamma_{i}} \sum_{n=0}^{M-1} \frac{P_{i}}{ \sum_{m=0}^{M-1} P_{m} } \]

\[ \alpha_{ni}, \alpha_{ni}, \alpha_{ni}, \alpha_{ni}, \alpha_{ni}, \frac{\partial \alpha_{ni}}{\partial \tau_{i}}, \frac{\partial \alpha_{ni}}{\partial \tau_{i}}, \frac{\partial \alpha_{ni}}{\partial \tau_{i}}, \frac{\partial^{2} \alpha_{ni}}{\partial \tau^{2}} \text{ and } \frac{\partial^{2} \alpha_{ni}}{\partial \tau^{2}} \text{ are linear functions of the channel noise and are therefore jointly dependent Gaussian random variables. Based on this, the expressions of (19) can be evaluated numerically. To avoid multiple order integrals over the joint PDF functions it is necessary to algebraically consider each term in (19) individually. In [12, Appendix B], general identities for the expectation operations are derived and omitted here because of space limitations. Unfortunately, the closed form insightful expressions of (19) relating the inflation of the CRLB of the multipath gain and delay parameters by the clock offset are not possible for the NDA case.} \]

\[ \text{V. SIMULATION RESULTS} \]

\[ \text{Results have been calculated based on the derived CRLB relations for the DA and NDA cases for various combinations of multipath amplitude, delay and clock offset. For the calculations, } M = 100 \text{ random symbols of } \pm 1 \text{ with PAM modulation as in (1) were used, } \Delta \text{ was assumed to have a Gaussian distribution with zero mean and the standard} \]
deviation of $\sigma_\gamma = 10$ ppm $= 10 \times 10^{-6}$, $N$, the length of the time-hopping code is taken equal to 20 and the code symbols are randomly picked up in the interval $0 \leq c \leq 19$ with the frame period of $T_f = 40$ nsec as in [6] and chip time of 1 nsec. The second order Gaussian monocycles, denoted as $g(t)$ is given as

$$g(t) = \left[ 1 - 16\pi \left( \frac{t - 0.5D_g}{D_g} \right)^2 \right] \exp \left[ - 8\pi \left( \frac{t - 0.5D_g}{D_g} \right)^2 \right]$$

(20)

which is used in the evaluation with the pulse width parameter, $D_g$ set to 1 nsec [4-6]. A simple multipath model consisting of $L_c$ paths equally spaced in delay such that $\tau_i = i \cdot \tau_0$, $1 \leq i \leq L_c$ is used. Also the gains are equal amplitude and normalized such that $\gamma_i = 1/\sqrt{L_c}, \quad 1 \leq i \leq L_c$. Two sets of plots corresponding to $L_c = 1$ and $L_c = 10$ are given in Figures 1 and 2, respectively to obtain an insight regarding impact of large number of multipath components on CRLB.

Comparing Figures 1 and 2, it is observed that increasing $L_c$ increases the square root of CRLB (SR-CRLB) for the $\tau_i$ estimation which is intuitive as there is less energy per multipath component. However, increasing $L_c$ doesn’t change the SR-CRLB for the clock offset estimation. The reason is that increasing $L_c$ doesn’t significantly change the support duration of the total received signal. The estimation of $\Delta$ depends primarily on the temporal duration of the received signal and not on the detailed structure of it. However, on one side increasing the length of packet for the synchronization gives us a good accuracy for clock offset estimation, but on the other side causes a long overall drift (even larger than chip time) of received signal which makes great performance degradation from BER perspective [9-11].

Another consequence of increasing $L_c$ is that the impact of clock offset on the SR-CRLB of $\tau_i$ decreases and becomes negligible for moderately large values of $L_c$. As the figures show the ratio between the SR-CRLB for $\tau_i$ in the DA case with known and unknown clock offset is always constant over the whole range of SNR values and approaches unity as $L_c$ increases. As Figures 1 and 2 show, the difference between the SR-CRLB of clock offset estimation for the DA and NDA cases deviates for low SNR. However, at very low SNR SR-CRLB of the $\Delta$ for the generalized one is saturated due to the a priori information regarding clock offset which is intuitive since very noisy signal cannot give us any useful information. Another observation is that this saturation happens at lower SNR for the DA case which is also intuitive.

At high SNR there is no difference between the SR-CRLB of clock or timing offset of known or unknown clock offset with or without having a priori information regarding clock offset. For low SNR the data no longer contributes to the
estimate of $\Delta$ as it is too noisy. When $I^{(D)} \ll I^{(P)}$ data is insignificant and when $I^{(D)} \gg I^{(P)}$ then priori knowledge becomes insignificant. Finally having or not having clock offset doesn’t influence the CRLB for the path gain estimation which is also intuitive.

**VI. CONCLUSIONS**

In this paper the CRLB of an UWB PAM signal with TH was determined for the DA and NDA cases where an unknown clock offset between the receiver and the transmitter was imposed. However, it was assumed that the receiver has priori information regarding clock offset. Computable expressions for the CRLB of the jointly unknown parameters pertaining to the channel and the clock offset were derived. Numerical results based on these relations indicate that the inflationary influence of the unknown clock offset on the joint CRLB becomes insignificant as the SNR increases and as the number of multipath components, $L_c$, increases. The later is of significance for UWB wireless communications as $L_c$ is typically large therefore indicating that cold start synchronization schemes can in principle be realized where the initial unknown clock offset has minimal impact. Also the derived CRLB expressions indicate that the unknown clock offset doesn’t have any inflationary influence on the variance of the path gain estimation. At low SNR, CRLB of clock offset for both DA and NDA in generalized case is saturated due to the a priori information of clock offset. Finally, the CRLB of the estimation of the clock offset itself is not significantly influenced by the multipath structure of the channel.

**REFERENCES**