Resource Allocation for Two-Way OFDM Relay Networks with Fairness Constraints

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Abstract—In this paper, a new resource allocation scheme is proposed for a two-way OFDM relay network. In the proposed scheme, two terminals exchange information with the help of multiple relays, and subcarriers and their power are successively allocated to the relays to maximize sum capacity. To achieve fairness for the relays, power constraint on each relay is imposed. Simulation results show that the proposed scheme improves sum capacity and fairness significantly.

I. INTRODUCTION
Orthogonal frequency division multiplexing (OFDM) provides reliable and high data rate communications in wireless networks [1]. In an OFDM system, bandwidth for data transmission is divided into a number of orthogonal subcarriers and each subcarrier experiences independent frequency-flat fading. In the multi-user OFDM system, frequency diversity and multi-user diversity are obtained by adaptive resource allocation [2], [3].

Cooperative relaying obtains spatial diversity without having a multiple input multiple output (MIMO) antenna array but additional channel resources are required for relay transmission [4]. By combining cooperative relaying with either superposition coding or network coding, two-way relaying provides improved spectral efficiency compared with one-way cooperative relaying [5].

In two-way relaying, two terminals exchange information with each other via a single or multiple relays. In a single relay network, various relaying protocols are proposed to reduce the required channel resources [5]. In [6], two-way relaying is combined with OFDM so that sum capacity is increased by power allocation and tone permutation. In a multiple relay network, it is known that diversity gain is increased as the number of relays increases, and the distributed space time coding scheme and the opportunistic relaying scheme are proposed for two-way relaying [7], [8]. However, combining two-way relaying with OFDM in the multiple relay network has not been investigated yet.

In this paper, we propose a new resource allocation scheme for a two-way OFDM relay network. In the proposed scheme, a number of relays in good channel condition are selected, and the subcarriers and their power are successively allocated to the selected relays based on the instantaneous channel condition. Compared with the opportunistic relaying scheme in which a single best relay is selected, multi-user diversity may be exploited as the number of selected relays increases.

To achieve fairness for the relays, maximum power constraint is imposed on each relay.

This paper is organized as follows. Section II describes a system model and problem formulation. A new resource allocation scheme is proposed in Section III. Section IV shows simulation results and Section V concludes this paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION
A. System Model
Consider a two-way OFDM relay network which consists of two terminals and \(M\) potential relays. Assume that the two terminals, \(A\) and \(B\), exchange information with each other and the \(M\) potential relays are uniformly distributed between the two terminals. Assume that there is no direct link between two terminals and suppose that \(K\) relays out of \(M\) potential relays are selected to support communication between the two terminals. Assume that an amplify-and-forward protocol is used for relaying, and all terminals and relays are symbol synchronous. Also assume that each of terminals and relays has a single antenna and do not transmit and receive simultaneously.

Assume that an OFDM symbol consists of \(N\) subcarriers and the channel of each subcarrier is frequency-flat. Let \(f_k^{(n)}\) and \(g_k^{(n)}\) denote the instantaneous channel coefficients of the \(n\)-th subcarrier from the terminal \(A\) to the relay \(k\) and from the terminal \(B\) to the relay \(k\), respectively. Assume that the instantaneous channel coefficient of each subcarrier is constant over an OFDM frame which consists of several OFDM symbols so that centralized resource allocation is feasible. Assume that resource allocation is performed by a centralized scheduler and perfect channel state information of all links is available at all terminals, relays, and the scheduler.

Assume that a symbol is transmitted in two phases: the multiple-access (MA) and broadcast (BC) phases. In a TDD mode, MA and BC phases occupy two time slots of equal duration and the channel reciprocity is assumed so that each of \(f_k^{(n)}\) and \(g_k^{(n)}\) is same for both phases. In the MA phase, both terminals \(A\) and \(B\) transmit symbols to relays simultaneously. Let \(x_A^{(n)}\) and \(x_B^{(n)}\) denote the transmit symbols, and \(p_A^{(n)}\) and \(p_B^{(n)}\) denote the transmit power of the two terminals \(A\) and \(B\) on the \(n\)-th subcarrier, respectively. The received symbol at the relay \(k\) on the \(n\)-th subcarrier is given by

\[
y_k^{(n)} = \sqrt{p_A^{(n)} f_k^{(n)}} x_A^{(n)} + \sqrt{p_B^{(n)} g_k^{(n)}} x_B^{(n)} + n_k^{(n)} \quad (1)
\]
where $n_k^{(n)}$ is the zero-mean complex Gaussian noise with variance $N_0$ on the $n$-th subcarrier at the relay $k$.

In the BC phase, each relay amplifies the received signal and forwards it to the two terminals $A$ and $B$. The amplification factor $\beta(n)$ of the relay $k$ on the $n$-th subcarrier is given by [5]

$$\beta(n) = \frac{p_k^{(n)}}{\sqrt{p_A^{(n)}|f_k^{(n)}|^2 + p_B^{(n)}|g_k^{(n)}|^2 + N_0}}$$

(2)

where $p_k^{(n)}$ is the transmit power of the relay $k$ on the $n$-th subcarrier. The received symbols at the two terminals $A$ and $B$ from the relay $k$ are given by

$$y_A^{(n)} = \beta(n) f_k^{(n)} y_k^{(n)} + n_A^{(n)}$$

(3)

and

$$y_B^{(n)} = \beta(n) g_k^{(n)} y_k^{(n)} + n_B^{(n)}$$

(4)

respectively, where $n_A^{(n)}$ and $n_B^{(n)}$ are zero-mean complex Gaussian noise with variance $N_0$ on the $n$-th subcarrier at the two terminals $A$ and $B$, respectively. Let $P_A$, $P_B$, and $P_k$ denote the maximum transmit power of the terminal $A$, terminal $B$, and the relay $k$, respectively. Then, the total maximum transmit power of the $K$ selected relays is given by $P_R = \sum_{k=1}^{K} P_k$.

Assuming perfect self-interference cancelation, the capacity of the two-way relay network on the $n$-th subcarrier is given by [5]

$$c_k^{(n)} = \frac{1}{2} \log_2 \left( 1 + \frac{\left| \beta(n) f_k^{(n)} g_k^{(n)} \right|^2 P_B^{(n)}}{\left| \beta(n) f_k^{(n)} g_k^{(n)} \right|^2 + 1 + N_0} \right)$$

(5)

$$+ \frac{1}{2} \log_2 \left( 1 + \frac{\left| \beta(n) f_k^{(n)} g_k^{(n)} \right|^2 P_A^{(n)}}{\left| \beta(n) g_k^{(n)} \right|^2 + 1 + N_0} \right)$$

Let $\rho_k^{(n)}$ denote the subcarrier assignment indicator variable for the relay $k$ on the $n$-th subcarrier. The $n$-th subcarrier is assigned to the relay $k$, $\rho_k^{(n)} = 1$; otherwise, $\rho_k^{(n)} = 0$. Assume that each subcarrier is allocated to only one relay so that there is no interference between relays. Then, the sum capacity over all assigned subcarriers is given by

$$C = \sum_{k=1}^{K} \sum_{n=1}^{N} p_k^{(n)} c_k^{(n)}$$

(6)

B. Problem Formulation

Assume that $K$ relays out of $M$ potential relays are selected prior to resource allocation. In order to select the relays which are in good channel condition, we utilize the relay selection metric of the opportunistic relaying scheme which is given by [9], [10]

$$W_m = \min \left\{ \sum_{n=1}^{N} |f_m^{(n)}|^2, \sum_{n=1}^{N} |g_m^{(n)}|^2 \right\}, m = 1, \ldots, M.$$  (7)

$K$ relays which have the $K$ largest metric values are selected to participate in relaying.

To enhance the fairness of relay utilization, power constraints are imposed on the selected relays. The optimization problem with fairness constraints is formulated as

$$C^* = \max \sum_{k=1}^{K} \sum_{n=1}^{N} \rho_k^{(n)} c_k^{(n)}$$

(8)

subject to: $\rho_k^{(n)} \in \{0, 1\}, \forall k, n$  

$$\sum_{k=1}^{K} \rho_k^{(n)} = 1, \forall n$$  

$$\sum_{n=1}^{N} p_k^{(n)} \leq P_A$$  

$$\sum_{n=1}^{N} p_k^{(n)} \leq P_B$$  

$$\sum_{n=1}^{N} \rho_k^{(n)} p_k^{(n)} \leq P_k, \forall k$$  

$$p_A^{(n)} + p_B^{(n)} + p_k^{(n)} \geq 0, \forall k, n.$$  

(9)

III. PROPOSED RESOURCE ALLOCATION SCHEME

The optimization problem in (8) jointly considers subcarrier allocation and power distribution. To reduce computational complexity of the joint optimization problem, we propose a suboptimal resource allocation scheme which separates the optimization problem into two steps, i.e., subcarrier allocation and power distribution.

A. Subcarrier Allocation Criterion and Proposed Subcarrier Allocation Algorithm

In this subsection, we obtain a subcarrier allocation criterion by using a convex optimization technique and propose a new heuristic subcarrier allocation algorithm.

1) Subcarrier Allocation Criterion: The problem in (8) is a combinatorial optimization problem having both discrete and continuous variables, which is too complex to obtain an optimal solution. To make the problem tractable, let the constraint on $\rho_k^{(n)}$ be relaxed so that it takes a real value in $[0, 1]$ [2]. Then, the Lagrangian for the optimization problem with the relaxed constraint is given by

$$L = -\sum_{k=1}^{K} \sum_{n=1}^{N} \rho_k^{(n)} f_k^{(n)} + \sum_{n=1}^{N} \lambda_n \left( \sum_{k=1}^{K} \rho_k^{(n)} - 1 \right)$$

$$+ \eta \left( \sum_{n=1}^{N} p_A^{(n)} - P_A \right) + \nu \left( \sum_{n=1}^{N} p_B^{(n)} - P_B \right)$$

$$+ \mu \left( \sum_{n=1}^{N} \rho_k^{(n)} p_k^{(n)} - P_k \right) - \sum_{n=1}^{N} \alpha_n p_A^{(n)}$$

$$- \sum_{n=1}^{N} \beta_n p_B^{(n)} - \sum_{k=1}^{K} \sum_{n=1}^{N} \gamma_k n p_k^{(n)} - \sum_{k=1}^{K} \sum_{n=1}^{N} \zeta_k n \rho_k^{(n)}$$

(10)

where $\lambda_n, \eta, \nu, \mu, \alpha_n, \beta_n, \gamma_k, n,$ and $\zeta_k, n$ are nonnegative Lagrange multipliers.
Any optimal solution to the relaxed problem has to satisfy the following Karush-Kuhn-Tucker (KKT) conditions [16]:

$$
\frac{\partial L}{\partial p_k^{(n)}} = -c_k^{(n)} + \lambda_n + \mu_k p_k^{(n)} - \zeta_{k,n} = 0, \forall k, n
$$

$$
\frac{\partial L}{\partial p_A^{(n)}} = \frac{\partial L}{\partial p_B^{(n)}} = \frac{\partial L}{\partial p_k^{(n)}} = 0, \forall k, n
$$

$$
\eta \left( \sum_{n=1}^{N} p_A^{(n)} - P_A \right) = \nu \left( \sum_{n=1}^{N} p_B^{(n)} - P_B \right) = 0
$$

$$
\mu_k \left( \sum_{n=1}^{N} p_k^{(n)} p_k^{(n)} - P_k \right) = 0, \forall k
$$

$$
\alpha_n p_A^{(n)} = \beta_n p_B^{(n)} = 0, \forall n
$$

$$
\gamma_{k,n} p_k^{(n)} = 0, \forall k, n
$$

$$
\zeta_{k,n} p_k^{(n)} = 0, \forall k, n.
$$

Because $\zeta_{k,n}$ is nonnegative, (11) becomes

$$
\lambda_n \geq c_k^{(n)} - \mu_k p_k^{(n)}.
$$

From (11) and (17), we obtain

$$
\rho_k^{(n)} \left( \lambda_n - c_k^{(n)} + \mu_k p_k^{(n)} \right) = 0.
$$

If the $n$-th subcarrier is allocated to the relay $k$, i.e., $\rho_k^{(n)}$ is positive, then $\lambda_n - c_k^{(n)} + \mu_k p_k^{(n)} = 0$ which implies that the equality holds in (18).

Then, to maximize the sum capacity and meet the power constraint of each relay, the $n$-th subcarrier is allocated to the relay $k^*$ such that

$$
k^* = \arg \max_k \left( c_k^{(n)} - \mu_k p_k^{(n)} \right).
$$

To obtain the optimal solution for subcarrier allocation by using (20), it is needed to know $\mu_k$. It is known that the exact value of the Lagrange multiplier is obtained by iterative searching which has excessive computational complexity [2]. A substitute value of $\mu_k$ is obtained by a KKT condition in (14). By the complementary slackness condition, if $\sum_{n=1}^{N} \rho_k^{(n)} p_k^{(n)} < P_k$, i.e., the relay $k$ meets the power constraints, $\mu_k$ is zero. Otherwise, $\mu_k$ becomes positive. Then, $\mu_k$ can be substituted as the difference between the assigned power for the relay $k$ and the maximum transmit power of the relay $k$, which is denoted by $\tau_k$ and given by

$$
\tau_k = \sum_{n=1}^{N} \rho_k^{(n)} p_k^{(n)} - P_k.
$$

### Algorithm 1: Proposed subcarrier allocation algorithm

#### Step 1: Initialization

Set $\mathcal{N} = \{1, 2, \ldots, N\}$, $\mathcal{K} = \{1, 2, \ldots, K\}$, and $\mu_k^{(n)} = 0, \forall k, n$.

#### Step 2: Subcarrier allocation under fairness constraints

while $\mathcal{K} \neq \emptyset$ do

$$(k^*, n^*) = \arg \max_{k,n} c_k^{(n)}, \ k \in \mathcal{K}, \ n \in \mathcal{N};$$

$$\mu_k^{(n^*)} = 1, \mathcal{N} = \mathcal{N} - \{n^*\};$$

update $|\tau_k^*|$;

if $|\tau_k^*| < \frac{P_k}{N}$ then

$$\mathcal{K} = \mathcal{K} - \{k^*\};$$

#### Step 3: Remaining subcarrier allocation

Set $\mathcal{K} = \{1, 2, \ldots, K\}$

while $\mathcal{N} \neq \emptyset$ do

$$(k^*, n^*) = \arg \max_{k,n} c_k^{(n)}, \ k \in \mathcal{K}, \ n \in \mathcal{N};$$

$$\mu_k^{(n^*)} = 1;$$

$$\mathcal{N} = \mathcal{N} - \{n^*\}, \mathcal{K} = \mathcal{K} - \{k^*\};$$

end

2) Proposed Subcarrier Allocation Algorithm: Based on the subcarrier allocation criterion, a heuristic algorithm is proposed. In the proposed algorithm, assume that the transmit power of each relay is equally allocated to the assigned subcarriers. Also assume that each relay has the same maximum transmit power, i.e., $P_k = P_R / K, \forall k$.

The proposed algorithm consists of three steps as shown in Algorithm 1. In the first step, all sets and variables are initialized. In the second step, each subcarrier is allocated to the relay which maximizes the capacity $c_k^{(n)}$ under the maximum transmit power constraint of each relay. If the difference between maximum transmit power and allocated power of a relay is smaller than the power of a single subcarrier, the relay is not allocated with additional subcarriers. In the third step, each of the remaining subcarriers is allocated to the relay which maximizes the capacity $c_k^{(n)}$, until all subcarriers are allocated.

### B. Power Allocation for a determined Subcarrier Allocation

In this subsection, we propose a power allocation scheme based on the dual decomposition technique [6], [13]. Let $p_A = [p_A^{(1)}, p_A^{(2)}, \ldots, p_A^{(N)}]^T$, $p_B = [p_B^{(1)}, p_B^{(2)}, \ldots, p_B^{(N)}]^T$, $p_k = [p_k^{(1)}, p_k^{(2)}, \ldots, p_k^{(N)}]^T, \forall k$, $\rho_k = [\rho_k^{(1)}, \rho_k^{(2)}, \ldots, \rho_k^{(1)}]^T, \forall k$, and $1 = [1, 1, \ldots, 1]^T$. After subcarrier allocation is determined (i.e., $\rho_k$ is determined for all $k$), the optimization problem for power allocation is formulated as

$$
C^* = \max \sum_{k=1}^{K} \sum_{n=1}^{N} \rho_k^{(n)} c_k^{(n)}
$$
subject to: $1^T p_A \leq P_A$ \hspace{1cm} (23a)
$1^T p_B \leq P_B$ \hspace{1cm} (23b)
$\rho_k^T p_k \leq P_k, \forall k$ \hspace{1cm} (23c)
$p_A, p_B, p_k \geq 0, \forall k$. \hspace{1cm} (23d)

The Lagrangian for the problem in (22) is given by

$$L_p(p_A, p_B, p_1, \cdots, p_K; \eta, \nu, \mu) = \sum_{k=1}^{K} \sum_{n=1}^{N} \rho_k^{(n)} c_k^{(n)} - \eta p_A^{(n)} - \nu p_B^{(n)} - \sum_{k=1}^{K} \mu_k (\rho_k^T p_k - P_k)$$

where $\eta, \nu, \mu$ are nonnegative Lagrange multipliers, and $\mu = [\mu_1, \mu_2, \cdots, \mu_K]^T$.

Then, the Lagrange dual function is given by

$$f(\eta, \nu, \mu) = \max L_p(p_A, p_B, p_1, \cdots, p_K; \eta, \nu, \mu)$$

subject to: $p_A, p_B, p_1, \cdots, p_K \geq 0$. \hspace{1cm} (25)

From a slight modification of $L_p$ in (24), the Lagrange dual function is decomposed into the set of $N$ independent subproblems which is given by

$$f^{(n)}(\eta, \nu, \mu) = \max \left\{ \sum_{k=1}^{K} \rho_k^{(n)} c_k^{(n)} - \eta p_A^{(n)} - \nu p_B^{(n)} - \sum_{k=1}^{K} \mu_k (\rho_k^T p_k - P_k) \right\}, \forall n. \hspace{1cm} (26)$$

Then, (25) becomes

$$f(\eta, \nu, \mu) = \sum_{n=1}^{N} f^{(n)}(\eta, \nu, \mu) + \eta P_A + \nu P_B + \sum_{k=1}^{K} \mu_k P_k. \hspace{1cm} (27)$$

The Lagrange dual problem is given by

$$f^* = \min f(\eta, \nu, \mu)$$

subject to: $\eta, \nu, \mu \geq 0$. \hspace{1cm} (28)

The difference between two optimal values, $C^*$ in (22) and $f^*$ in (28), is called duality gap. It is known that if the number of subcarriers $N$ is large, the duality gap is asymptotically zero [12]. Hence, the solution of the optimization problem in (22) can be obtained by solving the dual problem in (28).

To solve the dual problem in (28), we use a subgradient method [17] which is based on iterative searching. Given the initial values of $\eta, \nu, \mu$, we solve the $N$ subproblems independently to obtain a solution for power allocation. Based on the obtained power allocation, the values of $\eta, \nu, \mu$ are updated as

$$\eta^{l+1} = [\eta^l + \epsilon^l (1^T p_A - P_A)]^+, \hspace{1cm} \nu^{l+1} = [\nu^l + \epsilon^l (1^T p_B - P_B)]^+, \hspace{1cm} \mu_k^{l+1} = [\mu_k^l + \epsilon^l (\rho_k^T p_k - P_k)]^+, \forall k. \hspace{1cm} (29)$$

where $l$ is the iteration number and $\epsilon^l$ is the step size.

It is known that if $\epsilon^l$ is sufficiently small, each of $\eta, \nu, \mu$ is guaranteed to converge to its optimal value after a number of iterations [17]. Based on the optimal values of $\eta, \nu, \mu$, we obtain the optimal power allocation which maximizes $L_p$ in (25). Then, we normalize $p_A, p_B,$ and $p_k$ so that the power constraints (23a), (23b), and (23c) are satisfied.

IV. SIMULATION RESULTS

Suppose that two terminals are located at the boundary of a circular cell and the distance between them is 1 km. Suppose that the number of potential relays is 100 and they are uniformly distributed in the cell. We adopt the ITU pedestrian B model for the frequency-selective channel [15]. Suppose that the number of subcarriers is 64, the path loss exponent is 4, and $P_A = P_B = P_R = KP_k$. Similar to the worst possible average signal to noise ratio (WSNR), the received SNR is defined as the average received SNR at the relay which is located on the middle of two terminals [3].

The performance of the proposed scheme is compared with those of the static allocation, opportunistic relaying, and greedy allocation schemes [2], [10], [11]. In the opportunistic relaying scheme, a single best relay is selected and allocated with all subcarriers. In the greedy allocation scheme, a subcarrier is allocated to the relay which maximizes the capacity, while fairness is not considered. In the static allocation scheme, each relay is allocated with predetermined subcarriers regardless of the instantaneous channel condition.

Fig. 1 shows the average capacity per subcarrier versus SNR for $K = 6$. It is shown that the proposed scheme achieves much larger average capacity per subcarrier than both the static allocation scheme and the opportunistic relaying scheme. It is shown that the average capacity per subcarrier of the proposed scheme is slightly smaller than that of the greedy allocation scheme. It is also shown that the subcarrier allocation (SA) gives much larger improvement of the performance than the power allocation (PA).

Fig. 2 shows the fairness index versus the number of selected relays at SNR = 10 dB. The fairness index is defined
as $F = \frac{1}{K \sum_{k=1}^{K} (\sum_{n=1}^{N} \rho_k^n (p_k^n))^2}$ [14]. It is shown that the fairness index of the proposed scheme is 1 for all $K$, which implies that the maximum fairness is achieved. It is also shown that the fairness index of the greedy allocation scheme decreases as the number of selected relays increases.

Fig. 3 shows the average capacity per subcarrier versus the number of selected relays at SNR = 10 dB. It is shown that the average capacity per subcarrier of the proposed scheme increases as the number of selected relays increases. It is also shown that the capacity gain of the proposed scheme decreases as the number of selected relays increases. The performance of the greedy allocation scheme is also shown for comparison.

V. CONCLUSIONS

In this paper, we propose a new resource allocation scheme for a two-way OFDM relay network. In the proposed scheme, the optimization problem to maximize the sum capacity is formulated under the fairness constraints. Due to the prohibitive computational complexity of the optimization problem, a suboptimal resource allocation scheme is proposed by separating the subcarrier allocation and power distribution into two steps. Simulation results show that the proposed scheme achieves higher average capacity per subcarrier than the opportunistic relaying scheme, and also achieves higher fairness index than the greedy allocation scheme.

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