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SPHERES Control of Attitude Control System using Reaction Wheels

Abstract:

The Synchronized Position Hold, Engage, Reorient Experimental Satellites (SPHERES) laboratory, which works towards the development and testing of algorithms which control autonomous microsatellites, and the Space Systems Engineering class, an Astronautical-optional class needed to complete the undergraduate Aero/Astro baccalaureate degree at MIT, have been working together in the Space System Laboratory to develop an attitude control mechanism by using reaction wheels. The technology aims to control unexpected changes in satellite orientation caused by uncontrollable spatial circumstances without the use of developed propulsion systems, and aims to be applied to control algorithms for innovative satellite technology to be used in lunar orbit. The system, however, was incapable of imitating attitude control tests due to severe miscalculations in the chase's center of mass. The paper is a detailed description of the conception, design, and implementation of a CM protection mount that adjusts the center of mass of the ACS to a desired accuracy, and suggestions for future work in attitude control mount assembly. The paper also discusses in depth the background, design, and implementation of a controller of the ACS using the SPHERES satellites that functions in 1 degree of freedom (DOF). Techniques for creating a communication algorithm between the SPHERES satellites and the reaction wheels will be discussed, as well as future applications and tests for 1- and 3-DOF attitude control.

Introduction

The Space Systems Laboratory of the MIT Department of Aeronautics and Astronautics has had two active laboratories working in conjunction to develop technologies for further attitude control development - the SPHERES Laboratory, and the 16.83 class, known as Space Systems Engineering. The Synchronized Position Hold, Engage, Reorient Experimental Satellites (SPHERES) Laboratory, started in the Spring 1999 semester, work with full functioning autonomous satellite systems and algorithms based in formation flight. Each SPHERE, with a mass of approximately 4 kg and a diameter of .25 meters (SPHERES: A platform), has all the features that would be found on functioning man made satellites orbiting the Earth: active propulsion through twelve CO₂ thrusters, communication to users through a satellite-computer interface, communication between individual satellites, a power source via rechargeable Duracell batteries, and positional and attitude control using global and inertial sensors (Miller). The SPHERES run tests in two frames of motion; on earth, and on the International Space Station (ISS). The SPHERE functions in reference frames called Degrees of Freedom (DOF), which are defined as axes the SPHERE can move along without affecting movement on another axis (SPHERES: A platform). For example, on the ISS, the SPHERE can move left and right without affecting forward-back or up-down motion, or the sphere can rotate along the x-axis without affecting y-axis or z-axis rotation. On the earth, the SPHERE runs tests on a 'flat floor' and specially manufactured glass table which

effectively eliminates friction for SPHERE movement (SPHERES: A platform). On the ISS, the SPHERE satellite has 6 DOF and conducts full control tests, while the SPHERE has 3 DOF on the Earth and is used for troubleshooting and testing code. Tests are coded in C Programming language onto the SPHERE computer system, and with effective setup, the SPHERE makes maneuvers based on this code, obtaining information from a defined global frame and from inertial information obtained from accelerometers and rate gyroscopes (Nolet).

The Space Systems Engineering Laboratory, also known simply as 16.83, is a three semester class that works towards creation of full space systems from scratch. The SPHERES Lab, in fact, started as a 16.83 Project that grew to become a full laboratory (Open CourseWare). The 16.83 laboratory created the Attitude Control System (ACS) to test with SPHERES, as shown in Fig. 1. The system is a chaise connected to four reaction wheels in a tetrahedron formation, with an air bearing near the geometric center of the chaise. The air bearing is used to create an effectively frictionless surface that can rotate 360° about the Z axis as defined by Fig. 2, and up to 30° about the X and Y axes as defined as Fig. 3. The reaction wheels are used to actively control the ACS by constantly changing acceleration. Using the SPHERE's computer as the 'brain', a control mechanism can effectively be used to control the orientation of the ACS in three angular DOFs (16.83 Design Document).



Fig. 1: Attitude Control System.

The research in this paper concerns the necessary steps taken to create a SPHERES controller for the ACS that works in 1 DOF. Steps include the assembly of a protection mount used to actively test and change the center of mass of the system, the creation of the controller, and the mechanical equations that dictate the movement of the ACS.



Figs. 2 & 3: Reference axes of the ACS.

Mechanics behind Attitude Control System Movement

Before any further explanation of control of the ACS is explained, it is important to mention significant equations to describe mechanical motions of the ACS through classical mechanics. An essential equation to the dynamics of the ACS is Newton's Third Law of Angular motion, which declares that a force on an object creates a force of equal magnitude in the opposite direction. The reaction wheels, when accelerated, cause the entire ACS to torque in the opposite angular direction, with minimal torque lost by friction between the air bearing

and its mount. Another necessary equation for the centering of the air bearing is the idealized equation for the center of mass (Serway):

$$\mathbf{r}_{CM} = \frac{1}{M} \int \mathbf{r} \, dm$$

If the center of rotation of the air bearing is thought of as the origin of the system, the position vector \mathbf{r}_{CM} of the center of mass is given a simple point of reference. The forces exercised on a system by gravity from the center of mass can be described through the equation:

$$\sum \mathbf{F}_{ext} = M\mathbf{a}_{CM}$$

This relationship, when describing the ACS, is better described as a relationship of torque;

$$\sum \tau_{ext} = I\alpha_{CM}$$

where I is the moment of inertia of the system and α_{CM} is the acceleration of the system due to the center of mass. The torque of the system due to its center of mass sufficiently describes why a balanced ACS is necessary for the accurate control testing of the reaction wheels; if the center of mass is a distance from the center of rotation, the system feels an external torque due to gravity. If the ACS is unbalanced, the reaction wheels would have to exercise more torque to balance the ACS. This deters from the reaction wheel's ability to control the angular mechanics of the system and increases the wheels' chances for saturating, or reaching its top speed and being unable to exercise more torque.

Building the Center of Mass/Protection Mount

Unfortunately, the Control System suffered severe center of mass miscalculations when the chaise was fully assembled and a SPHERE was mounted on top for initial testing, which means the full chaise was nowhere near balanced on the air bearing. This event revealed another large problem; it seemed that if the ACS' system had a chance to reach some extreme angular velocity, the whole chaise had a chance to jump and cause the air bearing to jump out of its mount. This incident created a novel engineering predicament; a design needed to be manufactured that both protected the ACS's air bearing from jumping out of its mount, and could be used to actively change the center of mass of the system. The center of mass of the ACS would be located at the center of rotation of the air bearing, and thus there would be no exercised torques on the system caused by gravity.

After many changes in design, a model was created to both protect the ACS and actively change its center of mass. As shown in Fig. 4, offsets are mounted from the sides of the system, which hold a



Fig. 4: ACS with installed Protection/CM Mount.

circular ring that circles the post holding the chase. The design is made so the ring can hit the post of the ACS before any other feature closer to the air bearing can hit, and thus inhibits any air bearing 'jumps.' The design is also created with threaded rods and battery mounts which are outfitted with the same batteries used to power electronics on the ACS. The threaded rod mount system is used to dynamically change the center of mass of the ACS in the Z-direction by moving the mounts up and down the threaded rods. Battery mounts are used instead of standardized weights because their mass availability. The combined masses of the batteries are near to the mass of the SPHERE that would be attached for control mechanisms, and so they can be switched with an uncharged battery to power the ACS electronics.

To first run tests to balance the ACS, a dummy figure needed to be designed that had the same mass and center of mass as the SPHERE, so if the ACS were to topple, a SPHERE Satellite would not be lost in the wreckage. A dummy made from 6061 aluminum, chosen for its density and availability, was machined to have a difference in mass from an actual SPHERE by 56 g, or by 1.2%. The dummy was also engineered to sit on the top of the ACS mount the same way a SPHERE would, with approximately the same center of gravity of a SPHERE with an installed CO₂ tank on top of the ACS. Effectively, any CM measurement made with the SPHERES dummy would be balanced with a SPHERE as well.

To balance the ACS, a laser was shined horizontally at measured heights on the trusses holding the reaction wheels, which can be assumed to be equal in height and distance from the ground. The center of mass was raised using the battery mounts to a position close to, but below, the center of mass, so the tilt of the ACS caused by gravity torque could be observed. Standardized weights were added onto the triangular plate which the air bearing was attached at precise places until the laser measured an equal height on each of the sides of the three reaction wheel trusses. It is also important to note that an interesting engineering question with the design and balancing of the ACS deals with the relationship between the center of mass of the ACS and the moment of inertia of the equation. To relate to the previous equation, an idealized equation describing the moment of inertia can be described as:

$$I = \int r^2 dm$$

where r is the distance between the center of mass and points of mass of any object (Serway). It is important to note the equation of rotational inertia is proportional to distance from the origin to the second power, while the center of mass which defines the center of mass as proportional to the distance to the first power. This information highlights an interesting engineering design problem; though it might seem simple to add smaller weights further distances away from the rotational center of the ACS, those further distances will affect the rotational inertia greatly, and thus will hinder effectiveness of the reaction wheels in torquing the system.

Implementing Oscillation to Pinpoint the Center of Mass

It is also important to note an interesting characteristic of the ACS mount that is related to the position of the center of mass of the object. The simplest way to balance the ACS, also known as creating a stable system, is to balance the X and Y axes as to make the ACS parallel with the ground, and to locate the center of mass in the negative Z direction. Because the center of mass always works to be directly below the center of rotation due to the force of gravity, when agitated, the ACS acts as a physical pendulum and begins to oscillate. Contrary to popular thought, as the center of mass approaches the center of rotation of the ACS, the frequency of the oscillations slow down. The dynamics of oscillation of the ACS as a physical pendulum can be derived through the former derived equation for Newton's third law for angular motion.

Consider fig. 5, which displays an object being rotated about a pivot point by the force of gravity, with its center of mass being some distance d from the center of rotation and at some angle with the vertical axis. This displays another form of the equation for torque, which is equal to the force mg multiplied by the moment arm $d \sin \theta$ of the system (Serway):

$$\sum \tau_{\text{ext}} = I \alpha_{\text{CM}} = -mgd \sin \theta$$

Because α_{CM} is the second derivative of θ with respect to time t , the equation can be further derived as:

$$\frac{d^2 \theta}{dt^2} = -\frac{mgd \sin \theta}{I}$$

By assuming the angle θ is small, and thus the relationship $\sin \theta = \theta$ is valid, the relationship can be simplified to:

$$\frac{d^2 \theta}{dt^2} = -\left(\frac{mgd}{I}\right) \theta = -\omega^2 \theta$$

The variable ω , which designates the angular frequency of an oscillation, can be used because this equation represents the relationship for simple harmonic motion, where the acceleration is directly proportional and opposite in direction to the distance from some equilibrium point, which is the center of rotation for the ACS. Thus, the solution for the differential equation can be solved through the equation $\theta = \theta_{\text{max}} \cos(\omega t + \varphi)$, where φ is a constant, θ_{max} is the largest θ value, and:

$$\omega = \sqrt{\frac{mgd}{I}}$$

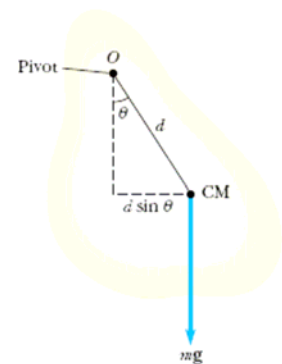


Fig. 5: A physical pendulum, with center of mass CM and pivot point O. Credit: Serway and Jewett

Thus, the period of oscillation T of the ACS can be described as;

$$T = \frac{2\pi}{\omega} = \sqrt{\frac{I}{mgd}}$$

We see as the distance between the center of rotation and the center of mass decreases, the period of the oscillation becomes proportionally larger. Also, by calculating the moment of inertia of the ACS and by measuring the time of oscillation of the ACS, one can calculate how close the center of mass is to the center of rotation. If the ACS is balanced in the X and Y axes as well, this information can be used to minimize the torques caused by gravity when full 3-DOF attitude tests are implemented.

Control System of 1 Reaction Wheel in 1 DOF

When the ACS is effectively balanced, the control mechanism that can be used to control the program can be effectively developed and tested. It is important to first define basic control theory and how it is used in our system. The SPHERE satellite expresses information about its angular states in two ways relevant to attitude control. The orientation of the state is expressed in the form of a quaternion, which is a four dimensional vector that dictates a unit vector through which an object rotates around, and to what degree the rotation is executed. The SPHERE also collects angular rates in terms of ω_x , ω_y , and ω_z . The user inputs a wanted state, which is designated for simplification as θ_0 . The SPHERE uses global and inertial sensors to estimate its current state, designated as $\hat{\theta}$, which is assumed to be close enough, but not equal to its actual orientation θ . The control system then uses the correction $\theta_e = \theta_0 - \hat{\theta}$, where θ_e is the state error the SPHERE satellite aims to correct.

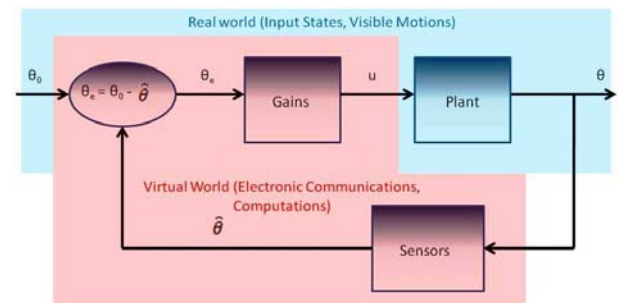


Fig. 6: Closed loop control system.

To correct this value, the SPHERE then can use a proportional-differential (PD) controller, which is used to calculate the necessary movement towards the wanted state. A PD controller follows the equation:

$$u = K_P \theta_e + K_D \dot{\theta}_e$$

where u is the input to the plant, or the needed values needed to change the current state into the wanted state. In terms of the SPHERE, u is expressed in torque values. θ_e and $\dot{\theta}_e$ are the angular errors and angular rate errors, respectively. K_P and K_D are known as gains, or proportionality constants that dictate how powerful the orientation and angular rates affect u , and thus the correction torques. The control algorithm is constantly updated

by a loop running at a user-defined frequency, then the SPHERE on the ACS should be able to rotate towards a state of $\tilde{\theta}$ that is equal to the wanted state of θ_0 (Basics of Control Theory).

The first control objective of the SPHERE was to develop an algorithm that controlled the SPHERE in 1 DOF, with the reaction wheel at the top of the assembly. Designated wheel 0, this specific reaction wheel was chosen because the wheel has 360° freedom in its DOF. The way the ACS has been assembled, this mechanism is the simplest way to fashion a simple control algorithm. As explained earlier in the paper, the ACS has been deliberately balanced so the center of mass is in the negative Z direction, or directly below its center of rotation of the air bearing, which causes oscillations if agitated. Another test orientation of the ACS is to make it critically stable, or to adjust the center of mass so it would be centered on the X, Y, and Z axes. While critically stable, the center of mass is not affected by gravity because there no moment arm between the center of mass and center of rotation, and would rotate as a satellite in space. Setting up and maintaining a critically stable system, however, is extremely time consuming. For the purposes of this study, a developed controller for 1-DOF by using a stable system and one reaction wheel is more useful than losing time on a complicated system with all four reaction wheels.

A particular problem with the motor controllers was found when we began to assemble the electronics on the ACS. We found the controllers are sent commands through packets of bytes that set a specific velocity, but cannot set accelerations. For short tests, the wheels can equate an average acceleration to a designated wheel speed, and cause the needed torque on the system. After long tests, however, this has the potential to turn into a serious error because some of the torque of the wheels will be lost through friction in the bearings of the wheels. To compensate, the wheels will turn faster with less torque output, until the wheels reach a saturation speed. For this experiment, however, it is assumed the experiment will run for relatively small amounts of time, so necessary wheels speeds can implement a proportional torque and hopefully arrive at a desired control state. Though there are many other problems dealing with specific of the motor controllers, much of those problems will deal with more complicated control systems and are beyond the scope of this document.

One process integral to the creation of the control system was the algorithm that turned the correction torques into byte commands that could be sent to the motor controllers, and thus translate into needed wheel speeds. The algorithm runs as follows:

```
{
```

```
Const Max_Speed;
```

```
Float New_Speed, Old_Speed;
```

```
Int Controller_Amt, Controller_Dir;
```

```
\\Old speed replaced with new speed with  
\\dynamic change.
```

New_Speed = Old Speed + Torque * Constant;
formerly made

\\The torque is found through

\\PD controllers.

If (New_Speed > Max_Speed)

Controller_Amt = Max_Cmd;

\\The speeds are translated into commands
the \\wheels understand.

Else

Controller_Amt = (Max_Cmd/MaxSpeed * New_Speed);

If (New_Speed > 0)

\\The direction of the wheels is dictated.

Controller_Dir = Forward,

Else

Controller_Dir = Backward

Send_Controller_Dir();

Send_Controller_Amt();

}

Angular Momentum of the ACS

Once it was tested that the reaction wheel could be controlled successfully, an important task was to find the maximum angular momentum the reaction wheel can reach, as this value represents the largest angular momentum the ACS can reach and still be controllable by the reaction wheels. First, it is important to explain why angular momentum

is the valued unit to measure the ACS control. Angular momentum is defined as angular velocity multiplied by the moment of inertia, or the first integral of the torque of the system over a specified time (Serway):

$$L = I\omega = \int \tau dt$$

Any control algorithm dealing with the ACS, in fact, can be thought of as a complex model of the law of conservation of angular momentum, where $I_1\omega_1 = I_2\omega_2$ of a closed system. The same relationship is also true for the changes in angular momentum, which can be expressed through the equation:

$$I_{REAC}\Delta\omega_{REAC} = I_{ACS}\Delta\omega_{ACS}$$

where the subscript $_{REAC}$ designates the reaction wheel, and $_{ACS}$ designates the attitude control system. The aim of the attitude control algorithm in any test is for the estimated state to equal the wanted state, which is expressed through angular velocities and orientations. If orientations are controlled singularly, they can be thought of as a state when the angular velocity is zero. Thus, the important values of the control system are changes in angular momentum caused by the reaction wheels.

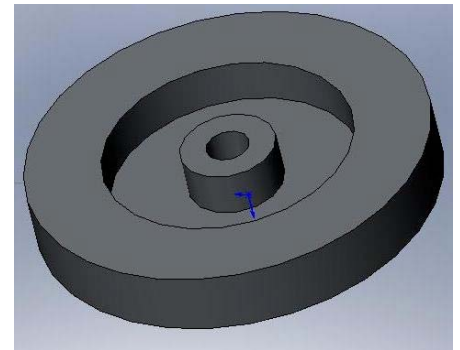


Fig. 6: CAD Drawing of Reaction Wheel. Credit: 16.83 Class Spring

Fortunately, the only changing value of the ACS during testing is the angular rate, as the moment of inertia is a constant value. First, the moment of inertia of the reaction wheel can be found by using the equation for the moment of inertia of a disc (Serway):

$$I = \frac{1}{2}\pi\rho LR^4$$

where ρ is the density of the object, L is the depth of the disc, and R is the radius of the disc. Due to the particular shape of the reaction wheel as shown in Fig. 6, the equation of the moment of inertia was found to be as follows:

$$I_{REAC} = \frac{1}{2}\pi\rho(L_1R_1^4 - L_2R_2^4 + L_2R_3^4 - L_1R_4^4)$$

Using the exact dimensions of the reaction wheel as found from the CAD model of the reaction wheels, I_{REAC} was found to be $7.426 \times 10^{-6} \text{ kg}\cdot\text{m}^2$.

The maximum change in angular velocity the reaction wheel ever reaches is when the wheel brakes from turning at maximum speed in one direction to maximum speed in the other. The maximum speed of the reaction wheel was calculated to be approximately 3529 rpm, or 369.55 rad/s. Thus, the angular momentum of the reaction wheel, and thus the

maximum angular momentum magnitude controllable on the ACS in the Z-axis, is approximately .00274 Newton-meter-seconds.

Conclusion

By developing the ACS protection mount, many lessons were learned about what needs to be done to completely balance the system. If a SPHERE is taken out and put back in the top of the ACS, it is highly likely the center of mass of the SPHERE has moved compared to where it was before. This means it is likely the center of mass of the ACS needs to be slightly adjusted to account for the slight change in the center of mass. As a result, an important task of the ACS should be another threaded rod design mounted parallel with the sides of the triangle mount, with movable weights mounted to the system as to actively change the center of mass in the X and Y axes. Another much more complex task is making the system critically stable, or setting the center of mass in the Z-axis to zero. The battery mounts would need to be adjusted extremely accurately and a more precise center of mass measuring design would need to be added. Desaturation tests of the reaction wheels by using the thrusters on the SPHERE create an interesting center of mass problem; as the thrusters use the CO₂, the center of mass of the SPHERE, and thus the ACS, moves downwards during testing. Whether or not the change in center of mass would greatly affect control capability is left for future testing.

Learning to implement and control the reaction wheels with the SPHERES satellite revealed many new important lessons in electronic and computer engineering. The first lessons revealed the importance of congruence between communicating electronic systems. We realized late into the project though the SPHERE outputs control information through torques, the motor controller can only read commands that communicate wheel velocities. Because there was no simple algorithm to relate the differing units, the problem created needless complexity that might have been avoided if the capabilities of the two had been better researched. By learning to construct the attitude controller, the necessity to construct robust control algorithms was understood. There were many assumptions placed on the algorithm that controlled the ACS, and it is highly unlikely the algorithm could be used to control the ACS for more complicated tests without taking the assumptions into consideration. Future work in attitude control would be to set up controllers for extended periods of time, for controlling orientations, or in full angular measurements from 0 to 360°. More interesting research would be the necessity and efficiency in creating a controller that controls in smaller intervals in time, so the motors can control its state more smoothly. Analytic research needs to be completed to find the three-dimensional moment of inertia in all three dimensions of the ACS, so angular momentum can be calculated accurately and more work can be developed in creating an ACS controller in all three DOF.

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