A Modular Code for Real Time Dynamic Simulation of Gas Turbines in Simulink

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1 Introduction

Real-time simulation can be used as a powerful tool in developing, testing and tuning control devices of gas turbines. A high-fidelity computer simulation can be used to substitute a real engine for many applications such as:

- Rapid-prototyping of control or diagnostic systems;
- Verifying the functionality of mechanic and electronic devices;
- Reducing costs and time for control equipment setup.

Moreover, it can be possible to simulate critical transients that should be avoided on the actual plant due to risk of damage. The block diagram in Fig. 1 shows how real-time software can be linked to control devices by means of suitable interfaces. The main features of the real-time simulation are:

- The simulation is carried out in part via software and in part through hardware devices; for such a reason, it is called “hardware-in-the-loop” simulation;
- The computer program outputs have to be generated at least as fast as the predicted physical phenomena impose [1].

Other applications of real-time gas turbine models include the use as onboard observer in aircraft engines or as engine simulator in flight simulators. For real-time applications, in order to reduce the computational time, techniques based on the linearization of the model have been often adopted [2,3]. However, the computer programs based on linearized models should be used only in the neighborhood of the working point about which the linearization has been carried out. Recently, real-time engine models based on nonlinear aero-thermal relations have been proposed [4,5]. In these works, the set of nonlinear algebraic equations has been solved by means of the Newton-Raphson method, using the Broyden method for updating the Jacobian matrix. This solving procedure, already adopted by the authors in previous works [6], has shown some limits when applied to real-time simulation. The main problem is that the time required by the iterative procedure to complete all the calculations for a given time step cannot be a priori determined, as it is depending on the convergence rate that is variable with the working point.

The present work shows a novel approach for developing a real time simulation code characterized by the following key features:

- The mathematical model and the numerical scheme are specially developed in order to obtain, at the same time, high-fidelity and computational efficiency;
- The code is modular and can be applied to any kind of gas turbine layout.

The program has been developed using an object-oriented approach already adopted by the authors for gas turbine simulation [7]. The code has been developed in Matlab-Simulink® [8] that provides a graphical user interface for developing the program. The proposed mathematical model is also compatible with Real-Time-Workshop®, a software package available in Simulink®. Through this tool, it is possible to automatically generate a source code in C language from the Simulink model of the gas turbine. Such source code can be compiled in order to generate an executable program much faster than the original Simulink program. Moreover it is possible to produce programs capable of driving input/output interfaces for data acquisition and control. It is believed that the time needed for developing new applications or to modify existing ones by means of this procedure will be much shorter than the time required by traditional programming technique.

In the following sections the mathematical model and the solving procedure are described. Then, two gas turbines for power generation are simulated in order to verify the dynamic response and the numerical performance.

2 Gas Turbine Mathematical Model

The mathematical model adopted in this work can be defined as an “aero-thermal model” with a lumped representation of the gas turbine components [2]. This approach has been adopted by many
Compressors and turbines are modeled as volume-less elements; a capacity (plenum) is introduced between these elements in order to take into account the unsteady mass balance. The burner is modeled as a pure energy accumulator. Other components, such as the cooled turbine module, the bleed air extraction module, the electric load module and so on, have been developed to complete the engine model.

For all the components, the algebraic equations are arranged in such a way that the output values can be obtained from the input variables without iterative calculations. In most cases, the input variables are referred to the entering flow and, similarly, the output variables are referred to the exiting flow. However, in order to determine the matching between compressor and first stage turbine nozzle or the enthalpy drop subdivision among the subsonic turbine stages, it is necessary to take into account the pressure in the downstream blocks. For this reason, in some cases, a variable referred to the inlet flow is evaluated as a function of the properties of the exiting flow.

2.1 Thermodynamic Fluid Properties. Due to the high turbine inlet temperature and fuel-to-air ratio that characterize modern gas turbines for power generation, the variation of the specific heats with temperature and composition should be included in the mathematical model, not only for design and steady-state off-design calculations, but also for the accurate simulations of the transients.

In the model adopted in the present work, the working gas is assumed to behave as a mixture of semi-perfect gases having the following properties:

The specific heats of the individual component gases are only functions of temperature;

The pressure of the mixture is the sum of the partial pressures of the individual gas components.

Under such hypothesis the thermodynamic properties of the working fluids are functions of temperature and mixture composition. As a consequence of such an approach, the mass fraction of each chemical component is necessary. Unfortunately, in modern gas turbines with blade air cooling, the composition of the expanding gas is not constant due to the mixing of the hot gas with the cooling air. Consequently, it is necessary to evaluate the mixture composition in every turbine stage.

The number of variables required to determine the composition can be reduced recognizing that the combustion in gas turbine engines is always operated with a large excess of air and that the combustion process can be considered completed at the exit of the combustion chamber. Consequently, the mixtures can be considered in thermodynamic equilibrium and the unburned products are neglected. Therefore, for gas turbines without water or steam injection, it is possible to assume that the expanding gas is composed of only two components that are mixtures of constant composition themselves: stoichiometric combustion products and dilution air [6,7]. The composition of the stoichiometric combustion products can be evaluated before running the simulation, if the fuel composition is known. Therefore, the actual composition of the combustion gas can be identified once either the fraction of stoichiometric products or the fraction of dilution air is known.

In this work, the following variables have been chosen to identify the thermodynamic properties of the working fluids: the stagnation temperature, \( T \), the stagnation pressure, \( p \), the mass flow rate, \( g \), and the fraction of stoichiometric products, \( x \). The stoichiometric combustion product fraction \( x \) has been chosen instead of the fuel-to-air ratio \( f \) used in Ref. [7], since it seems to give a more understandable representation of the mixing process in the cooled turbine.

For gas turbine power plants with water or steam injection, the water fraction that is not a combustion product can be adopted as third component of the mixture.

The thermodynamic properties of the individual chemical species are evaluated against the temperature using a polynomial law. The gas constant, \( R \), the specific heat, \( c_p \), and the enthalpy, \( h \), of the mixture can be obtained considering the actual gas composition.

2.2 The Compressor. During the transient, the compressor module, considered as a volume-less component, is assumed to behave as a quasi-steady component, so that a steady state compressor map is used. The map provides the mass air flow rate (\( g \)) and the isentropic efficiency (\( \eta_{is} \)) as a function of pressure ratio (\( \beta_i = p_{out}/p_{in} \)), corrected rotational speed \( (n/\sqrt{\theta}) \), and inlet guide vane position (IGV)
\[
\frac{g \sqrt{\vartheta}}{\delta} = f_1(\beta_c, n_l/\vartheta, \text{IGV}), \quad \eta_{n_c} = f_2(\beta_c, n_l/\vartheta, \text{IGV})
\] (1)

Such functions are numerically evaluated through interpolation on digitized maps. Section 3 will describe the adopted interpolation procedure. The air temperature at the compressor exit is given by

\[
T_{\text{out}} = T_{\text{in}} \left[ 1 + \frac{1}{\eta_{n_c}} \left( \frac{p_{\text{out}}^{k-1/k}}{p_{\text{in}}} - 1 \right) \right]
\] (2)

where the specific heat ratio \( k \) is evaluated at the temperature \( T_{\text{in}} \) that is the arithmetic average between the inlet \( T_{\text{in}} \) and the outlet \( T_{\text{out}} \) temperature. In order to avoid an iterative procedure for calculating \( T_{\text{out}} \), the specific heat ratio \( k \) is evaluated using the value of the temperature \( T_{\text{out}} \) determined at the previous time step. This assumption does not introduce an appreciable error because the temperature \( T_{\text{out}} \) has low influence on the specific heat ratio; moreover, the error by the suggested procedure can be considered negligible because of the very small time step used in real time simulation. The compressor mechanical power is given by

\[
P_c = g(h_{\text{out}} - h_{\text{in}})
\] (3)

where the enthalpy values \( h_{\text{out}} \) and \( h_{\text{in}} \) are evaluated at the temperatures \( T_{\text{out}} \) and \( T_{\text{in}} \), respectively.

### 2.3 Plenum

Since turbomachinery, compressor and turbine units are considered as volume-less elements, the unsteady mass balance is modeled through an adiabatic capacity (plenum). A plenum is placed at the compressor outlet in order to take into account the unsteady mass balance within compressor ducts, compressor discharge and combustion chamber. Other plena are placed between the turbine stages.

The flow speed is assumed to be negligible inside the plenum, while temperature and pressure are supposed to vary with a polytropic law with exponent \( m \). Applying the mass conservation law, results in

\[
\frac{V_p}{mRT_{\text{out}}} \cdot \frac{dp_{\text{out}}}{dt} = g_{\text{in}} - g_{\text{out}}
\] (4)

where \( V_p \) is the volume of the plenum. The polytropic coefficient \( m \) can be approximated by the specific heat ratio \( k \). The energy accumulation inside the plenum is neglected, therefore

\[
T_{\text{out}} = T_{\text{in}}
\] (5)

Moreover, since neither pressure losses nor momentum effects are considered in the plenum,

\[
p_{\text{in}} = p_{\text{out}}
\] (6)

This value is needed to provide the pressure datum to the component placed upstream of the compressor plenum. The volume \( V_p \) influences the pressure \( p_{\text{out}} \) variation due to the amount of the mass accumulated in the plenum. Using the gas state equation, Eq. (4) can be rearranged in the form

\[
\frac{dp_{\text{out}}}{p_{\text{out}} dt} \cdot M_p = \frac{g_{\text{in}} - g_{\text{out}}}{s_{\text{out}}}
\] (7)

where the characteristic time \( \tau_p \) is given by

\[
\tau_p = \frac{M_p}{mS_{\text{out}}}
\] (8)

and \( M_p \) represents the mass inside the plenum.

### 2.4 Splitter

The compressed air extracted for turbine cooling is evaluated in a module called splitter. According to experimental data available about gas turbine cooling systems [12], the air mass flow, is evaluated from

\[
\frac{g_{\text{cool}} V_{\text{in}}}{p_{\text{in}}} = K \sqrt{1 - \frac{p'_{\text{out}}}{p_{\text{in}}}}
\] (9)

where \( K \) represents the discharge coefficient, \( p_{\text{in}} \) and \( T_{\text{in}} \) are the pressure and the temperature at the bleed point, respectively, and \( p'_{\text{out}} \) is the static pressure at the exit of the cooling circuit. The \( K \) coefficient is estimated on the basis of the cooling air mass flow rate, needed for blade cooling at the gas turbine design point. For simplicity, in this work, the value of \( p'_{\text{out}} \) is approximated by the value of the pressure at the stage exit.

The main flow mass flow rate exiting from the splitter is evaluated by subtracting the cooling flow rate from the entering mass flow.

### 2.5 Burner

The burner has been represented as a pure energy accumulator, neglecting the mass balance that is instead attributed to the upstream plenum block. The inside temperature and pressure have been assumed homogeneous and equal to the combustor respective outlet values.

The equation that describes the burner dynamics is obtained from the unsteady energy conservation

\[
\frac{d(M_c u_c)}{dt} = g_{\text{in}} h_{\text{in}} + g_b (h_b + \eta LHV) - g_{\text{out}} h_{\text{out}}
\] (10)

where \( M_c \) and \( u_c \) are the mass inside the burner and the internal specific energy, respectively; \( LHV \) is the lower heating value of the fuel. Neglecting the variations of the mass and of the specific heat functions, Eq. (10) can be rearranged thus obtaining

\[
\tau_{cc} \frac{dT_{\text{out}}}{dt} = \left( g_{\text{in}} h_{\text{in}} + g_b (h_b + \eta LHV) - g_{\text{out}} h_{\text{out}} \right) \frac{s_{\text{out}}}{p_{\text{out}}}
\] (11)

where the time constant \( \tau_{cc} \) can be evaluated from

\[
\tau_{cc} = \frac{M_c}{Ks_{\text{out}}}
\] (12)

A time delay \( \epsilon_{cc} \) has been introduced to take into account of the flame delay that exists from the fuel injection to the heat release.

Under the hypothesis of negligible variation of the mass present in the combustor, the continuity equation gives

\[
g_{\text{out}} = g_{\text{in}} + g_b
\] (13)

This equation is rearranged in order to obtain the feedback signal going from the turbine first stage to the compressor plenum

\[
g_{\text{in}} = g_{\text{out}} - g_b
\] (14)

The pressure drop across the burner has been evaluated from

\[
p_{\text{out}} = \gamma_{cc} p_{\text{in}}
\] (15)

where the loss coefficient, \( \gamma_{cc} \), depending on the square mass flow rate is evaluated by means of the value of the mass flow at the previous time step.

### 2.6 Multistage Turbine With Blade Cooling

In modern gas turbines, either “heavy-duty” or “aero-derivative,” the gas expansion is distributed on more stages, characterized by blade cooling systems. Under the hypothesis that the turbine module behavior can be considered quasi-steady, the turbine characteristic curves can be used in order to estimate the mass flow parameter \( \Gamma \) as a function of the expansion ratio \( \beta_c = p_{\text{out}}/p_{\text{in}} \) and the turbine corrected speed. Knowing the value of mass flow parameter \( \Gamma \), it is possible to evaluate the mass flow rate across turbine as a function of temperature and pressure at the turbine inlet

\[
g_{\text{in}} = \frac{\Gamma p_{\text{in}}}{\sqrt{T_{\text{in}}}}
\] (16)

In order to subdivide the enthalpy drop among more stages, the mass flow parameter has been evaluated for each stage using the
method described in Ref. [6] and in Ref. [13]. A plenum is placed between two consecutive stages in order to model the unsteady mass balance.

For each stage the cooled turbine expansion has been modeled according to the scheme proposed by El Mastri [14] and already adopted both for steady-state off-design simulation [13,15] and for dynamic simulation [6]. A graphical representation of the cooled expansion is provided in Fig. 2. The model can be represented using three distinguishable phases as follows:

- the mixing between main flow and cooling air of the stator blades (from point A to point C in Fig. 2);
- the expansion of such mixed gas flow (from point C to point D);
- the mixing between the new main flow and rotor cooling air (from D to F).

These three phases have been represented by three components; in particular there are two mixing unit (one for the stator cooling air and the other for the rotor cooling air), and a component representing the adiabatic gas expansion. These two components are called “mixer” and “adiabatic turbine,” respectively. Figure 3 shows the Simulink representation of these blocks as they have been implemented in the cooled stage block. The mathematical models are described below.

2.6.1 Mixer. The mixer block is used to evaluate the enthalpy and the pressure drop due to the mixing of the main gas flow with the cooling air. The mass flow at the mixer outlet is given by the continuity equation

\[ g_{\text{out}} = g_{\text{in}} + g_{\text{cool}} \]  

(17)

The composition of the gas exiting the mixer, considering that the stoichiometric product fraction in the air is nil, is obtained from the relation

\[ x_{\text{out}} = \frac{g_{\text{in}} x_{\text{in}}}{g_{\text{out}}} \]  

(18)

The outlet gas enthalpy \( h_{\text{out}} \) after the mixing process is evaluated from the enthalpy balance

\[ h_{\text{out}} = \frac{g_{\text{in}} h_{\text{in}} + g_{\text{cool}} h_{\text{cool}}}{g_{\text{out}}} \]  

(19)

The stagnation temperature \( T_{\text{out}} \) is obtained as a function of the outlet enthalpy \( h_{\text{out}} \) and of the dilution air \( x_{\text{out}} \).

The pressure drop in the mixer is evaluated from the momentum loss equation

\[ p_{\text{out}} = p_{\text{in}} \left[ 1 - \frac{g_{\text{cool}}}{g_{\text{in}}} \cdot (1 - Y) \cdot K \cdot M_a^2 \right] \]  

(20)

where \( Y \) is the momentum loss coefficient depending on blade geometry, and \( M_a \) is the Mach number of the gas. For the stator row, the Mach number is evaluated from absolute velocity in the throat section while in the rotor row, the Mach number is evaluated from the relative velocity in the rotor throat section. For simplicity, it has been assumed that the Mach numbers remain constant during the simulations.

2.6.2 Adiabatic Expansion. In order to evaluate the final temperature of the adiabatic expansion, a suitable expression for the isentropic efficiency \( \eta_t \) as a function of the expansion ratio \( \beta_t \) and of the corrected rotational speed is needed. The adopted expression provides a law for isentropic efficiency variation as a function of the efficiency of both stator and rotor rows. Such row efficiencies are evaluated in relation to the main flow angle of attack.

The temperature at the end of expansion is given by the relationship

\[ T_{\text{out}} = T_{\text{in}} \left[ 1 - \eta_t (1 - \beta_t^{1-1/b_t}) \right] \]  

(21)

where the specific heat ratio \( k \) is evaluated, for the actual gas composition, at the mean temperature \( T_m \) between \( T_{\text{in}} \) and \( T_{\text{out}} \).
using, for this last temperature, the value obtained at the previous time step.

The power $P_t$ produced by the expanding gas is given by

$$P_t = \dot{m} \left( h_{in} - h_{out} \right)$$

(22)

where the enthalpy values $h_{in}$ and $h_{out}$ are evaluated from the temperatures $T_{in}$ and $T_{out}$, respectively.

### 2.7 Shaft Dynamic

The angular acceleration of the shaft joining compressor, turbine and, eventually, applied load is given by the angular momentum equilibrium, depending on moment of inertia $J$ which includes the inertia effects of the shaft and of the other connected devices. Thus

$$\frac{d\omega}{dt} = \frac{1}{I} \left( P_t - P_c - P_j - P_E \right)$$

(23)

where $P_t$ represents the internal mechanical power of the turbine, $P_c$ is the internal mechanical power required by the compressor, $P_j$ is the mechanical power required by the electrical load and, finally, $P_E$ represents a sum of power losses due to mechanical friction, friction on the rotor disks, and power to drive auxiliary equipments.

For gas turbines with two or more shafts, Eq. (23) is to be applied to each shaft. Arranging Eq. (23) in nondimensional form, the characteristic time constant $\tau$ is obtained

$$\frac{d\eta}{dt} = \frac{\eta^2}{\tau} \left( P_t - P_c - P_j - P_E \right)$$

(24)

where $\eta$ is given by

$$\eta = \frac{J \cdot \omega_{des}}{P_{a,des}}$$

(25)

and $\omega_{des}$ and $P_{a,des}$ are the shaft angular speed and the turbine output power at the design point, respectively. The power losses considered in the term $P_j$ are estimated by a loss-factor that is a function of the angular speed. This formulation has been preferred to the solving of the algebraic equations, the constraint about iterative methods make it impossible to use a solution scheme like the Newton-Raphson one. As has been shown in the previous section, the algebraic equations are arranged in order to explicitly generate the output values. It will be shown that if the algebraic equations are properly ordered, it is possible to obtain the solution by forward substitution. Supposing that the system consists of $N$ ordinary differential equations and $M$ algebraic equations, the equations are arranged and sorted as follows

$$\frac{dy_i}{dt} = \frac{\eta^2}{\tau} \left( P_t - P_c - P_j - P_E \right)$$

(26)

At each time step, the differential equations are integrated by means of an explicit scheme, thus obtaining the new values of the first $N$ variables. Then, through the scheme in Eq. (26), the remaining $M$ variables are sequentially evaluated by forward substitution. In other words, any variable $y_i$ is calculated as an explicit function of the variables $y_1, \ldots, y_{N-i}$ that are already known.

Most of the algebraic equations of the proposed mathematical model result to be properly ordered without numerical manipulation, if they are ordered in accordance with the stream direction in the components that compose the gas turbine model. The Simulink environment offers the possibility of easily ordering the equations by means of a graphical user interface [7].

To illustrate how the solution proceeds, a simple single-shaft gas turbine is considered. The model, shown in Fig. 5, consists of the following components: compressor, plenum, burner, adiabatic turbine, and a simple proportional-integral (PI) speed governor. The aim is to show how it is possible to deal with the matching between the compressor and the turbine without using an iterative solver. The arrows in the Simulink scheme represent the transfer of the data values from one block to the other. Bold arrows represent the transfer of multiple data referred to the variables that identify the fluid properties: the stagnation temperature, $T$, the stagnation pressure, $p$, the mass flow rate, $g$, and the fraction of dilution air, $x$. Often, the transfer of the data has the same direction as the gas stream but, in some cases, since the downstream block interacts with the upstream one, one datum may be transferred in the opposite direction to the stream one. Examples of
these feedbacks are the plenum discharge pressure $p_2$, the mass flow rate entering the turbine $g_3$, the mass flow rate entering the combustion chamber $g_2$.

The flow chart given in Fig. 6 shows how the sequential solution proceeds. At each time step, the ordinary differential equations are integrated. For sake of simplicity, in this example, the Euler scheme is considered. Specifically, the pressure $p_2$ at the compressor discharge, which is equal to the pressure $p_2$ at the plenum exit, is obtained from the unsteady mass balance of the plenum. The temperature $T_3$ at the combustor outlet is obtained from the unsteady energy balance of the combustor. The rotational speed, $n$, is obtained from the shaft dynamics. The fuel rate, $g_f$, is determined from the speed governor equations. Once these variables are known, it is possible to evaluate all the other unknown quantities by substitution in the algebraic equations, as shown by the flow chart. It can be observed that the working point of the compressor is determined by the values of the rotational speed, $n$, and the pressure ratio $\beta = p_2/p_1$. The mass flow rate entering the turbine, $g_3$, is determined, from the known mass flow parameter, the temperature $T_3$ and the pressure $p_3$.

In turbines with more stages, for all the stages following the first one, the inlet pressure is determined from Eq. (16), instead of the mass flow rate, since the pressure is necessary to evaluate the pressure ratio in the upstream stage.

The flow chart in Fig. 6 shows that the solution has been reached without the iterative procedure that is generally required for the solution of the nonlinear equation system. It should be observed that this result is made possible by means of the approximations that are applicable to real-time gas turbine dynamic models in consideration of the very short time-step. Specifically, as has been shown in the mathematical model, the specific heats are determined on the basis of temperature and composition evaluated at the previous time step. To this purpose the “Memory Block,” available in Simulink, has been used. The Simulink environment is able to check if iterative calculations are needed to determine the value of one or more variables: this procedure is useful also to verify if the assembly of the components or a new component is compatible with real-time operations.

The proposed formulation is also compatible with the Real-Time-Workshop® toolbox, that is a toolbox available in Matlab-Simulink environment able to automatically generate a source code in C language from the Simulink scheme. This feature can be useful for developing a code for hardware-in-the-loop applications.

Finally, it is useful to describe the method adopted for loading the data of the compressor map in the computer code. This method has been developed in order to make as short as possible the computation time needed for interpolation and to make this time independent from the number of points of the experimental map. This technique, similar to the so-called “beta-lines” method [1], is applied to a map in terms of nondimensional groups in order to take into account the effects of ambient air conditions.
This technique is based upon two distinct phases: the first phase consists of off-line calculation on the digitized map and it is done only once; the second phase is executed at every time step. The preliminary analysis strives for obtaining efficiency in the data extraction and consists of transforming the domain of the physical variables, in which the map is defined, into a Cartesian uniform domain. The transformation adopted in this work associates any point \( \beta, n \) of the map with a point \( \xi, \eta \) in the transformed domain by means of the relations

\[
\eta = \frac{n - n_{\min}}{n_{\max} - n_{\min}}, \quad \xi = \frac{\beta - \beta_{\min}(n)}{\beta_{\max}(n) - \beta_{\min}(n)}
\]

where:

- \( n_{\min} \): minimum rotational speed in the compressor map;
- \( n_{\max} \): maximum rotational speed;
- \( \beta_{\min}(n) \): pressure ratio at intersection between the choking line and the constant speed curve referred to the speed \( n \);
- \( \beta_{\max}(n) \): pressure ratio at intersection between the surge line and the constant speed curve referred to \( n \).

In this way, the compressor working range can be represented by a grid as shown in Fig. 7. The transformed domain \( (\xi, \eta) \) results in a uniform Cartesian grid in which is possible to interpolate, by means of a bilinear technique, with a small computational effort. Moreover, since the computational effort is not dependent on the number of point of the map, it is possible to make use of very refined map only charging the computer memory but not increasing the computation time. This technique has been generalized for the map of a compressor with variable inlet guide vanes.

### 4 Simulation Results

The aim of this section is to show that the code is able to accurately simulate the dynamic behavior of a gas turbine under very large load variations.

Two gas turbines for power generation have been chosen in order to test the code: the first one is a large heavy duty gas turbine and the second one is an aero-derivative double-shaft engine. Preliminarily steady-state off-design behavior of the large heavy duty gas turbine has been simulated in order to show that the nonlinear mathematical model characteristics of the code make it able to reproduce the behavior of the gas turbine under working conditions varying from idle to full-load.

#### 4.1 Single Shaft Gas Turbine

In order to overcome the lack of complete experimental data from transient tests in the open literature, the code has been undertaken to a validation against steady-state off-design data, by simulating a very slow transient. The experimental data given by Jansen et al. in [19] concerning the gas turbine V64.3 manufactured by Siemens have been chosen. The gas turbine has a single shaft gas arrangement for electric power generation (Fig. 8); it consists of 17 stages axial flow compressor, of which four stages are provided of variable inlet guide vanes (IGV) in order to vary the air mass flow. Seven of the eight turbine rows are cooled, although, for sake of simplicity, only the
first two stages are assumed to be cooled in the code. As given in Ref. [20], the design specifications of the engine are summarized in Table 1. The turbine expander has been represented by making use of three different blocks: the first two blocks represent two cooled stages while the third one simulates the adiabatic expansion. Subsequently the code has been developed under the Simulink environment, some parameters needed for the simulation have been determined on the basis of the off-design performance given in Ref. [19] by making use of an adaptive procedure, needed to obtain that the design parameters can produce a reasonable fitting to the experimental data, on the basis of the technological characteristics of the considered engine. The simulation results given by Kim et al., [20], as well as the results obtained by using a code previously developed in FORTRAN by two of the authors [6,13] have been provided as useful for comparison. The part-load simulation form full load to 110% of nominal load is presented in nondimensional form in Fig. 9. The IGV control system keep the exit temperature constant from 50% to 100% of nominal load: for lower power output, the IGV position is kept constant. The illustrated parameters concern compressor pressure ratio, exhaust gas mass flow and temperature, turbine inlet temperature and overall LHV efficiency. A good agreement between simulation and experiments is obtained for all the measured variables, as well as with the numerical results obtained with the other two above mentioned codes that are, instead, not suitable for real time applications. Moreover it clearly appears that, due to its intrinsic nonlinear features, the code is able to accurately simulate the whole transient from idle to full load, while the linearized real time codes are able to model only a limited region around the working point chosen for linearization.

A load rejection test from full load to idle is simulated in order to verify that the code is able to reliably reproduce very hard dynamics. Figure 10(a) shows the time-plot of the electrical power demand ($P_{\text{load}}$) in nondimensional form, which is obtained by dividing the power demand by its design point value. The time unit for the x axis is the second. The power demand variation is supposed to occur instantaneously following the imposed law. In Fig. 10(b) the controlled variables are represented: the rotational speed of gas generator and power turbine. The curve clearly shows the plant behavior in the two different stages of load drop and load rising. The curves representing the control variables are drawn in Fig. 10(c). The fuel mass flow rate ($g_f$) is normalized to its design point value, while the inlet guide vane position (IGV), varying from 0 to 1, is given by the rotation angle of inlet guide vanes and is normalized to its value at full open position. It appears that the fuel flow rate rapidly decreases as a consequence of the electric power drop since the fuel system has a short characteristic time; on the other side, the IGV variation is quite slow since the temperature transducer characteristic time is quite long as shown by observing the curve of $T_{\text{TXM}}$ in Fig. 10(b). Figure 10(d) finally shows temperature and pressure at the compressor plenum exit ($T_2$ and $P_2$) and at the turbine inlet ($T_3$ and $P_3$), respectively. The important information that can be predicted by using a complete thermo-fluid dynamic code, like the proposed one, which can take into account the dynamic matching among the turbomachinery components of the engine, may be very useful for the setup of the control system, such as the anti-surge control system.

In order to evaluate the numerical accuracy of the code, the simulation has been repeated using a Heun second order scheme: the results concerning the most important variables have been compared showing that the maximum relative difference between the solutions obtained adopting the Euler scheme and that obtained adopting the Heun one is smaller than $5 \times 10^{-4}$.  

### 4.2 Double Shaft Gas Turbine

A double shaft aeroderivative gas turbine for power generation is considered. The main engine characteristics adopted for developing the simulation model are referred to the gas turbine LM2500 by General Electric [21]: specifically, suitable values of efficiencies, compressor mass flow rate and pressure ratio, turbine inlet temperature, have been chosen in order to allow the simulation model to meet the engine performance given by the manufacturer. Moreover, such values can be adapted to take into account the degradation of the engine performance due, for example, to the compressor fouling.

The model developed for simulating such an engine is shown in Fig. 9. For the sake of clarity, the gas turbine model is displayed as a network of subsystems composed of many blocks. The compressor block is a subsystem composed of two compressor units with an intermediate air extraction for turbine blade cooling; it includes also two plena and two splitter blocks for the cooling air flows. The high-pressure turbine block is a subsystem composed of two air-cooled stages. The power turbine, that is considered not to be cooled, is modeled by a single adiabatic expander. A plenum, placed between the exit of the HP turbine and the inlet of the power turbine, takes into account the volume of both the turbines and the connecting ducts. The map of the compressor, including variable inlet guide vanes varies with the rotational speed, has been adapted from Ref. [22]. The control variable is the fuel flow rate, $g_f$, and a PI controller (not shown in the figure) has been designed to control the power turbine rotational speed. Similar to the previously shown models, numbers in the scheme are given to identify the different engine sections.

As far as concerns the specific features of the real-time model, Fig. 11 shows that the value of the mass flow, $g_3$, at the HP turbine inlet, is addressed backwards from the HP turbine block to the burner. Then, from this block, the air mass flow, $g_2$, is addressed backwards to the compressor subsystem, where it is needed for the mass balance of the plenum. It can be observed also that the pressure at the power turbine inlet is addressed upstream to the HP turbine block, in order to determine the enthalpy drop on the HP turbine.

A varying electric load, Fig. 12(a), has been applied in order to describe the dynamic behavior of the gas turbine. First, starting from the design point condition, a load rejection of 50% of full-load at $t=5$ s is simulated. Then, 25 s later, a load rising ramp is applied bringing back the power to full load. Figure 12(b) shows the time plots of the rotational speed of gas generator and power turbine. Figure 12(c) shows the control signal VCE sent by the controller to the fuel system. In the same figure, the fuel flow rate $g_f$ appears to be slightly delayed in comparison to the fuel signal due to the dynamics of the fuel system. The simultaneous variations of the compressor speed and the pressure ratio on the compressor working point are described in Fig. 12(d) where part of the compressor map is plotted. The three curves are referred to the load rejection (inverted triangles), to the load rising ramp (circles), and to a quasi-steady load variation (crosses). Finally, time plots of temperature and pressure referred to the main points of the gas turbine are plotted in Figs. 12(e) and 12(f). These figures show that the program provides a detailed representation of the gas turbine dynamics, including the effects on the intermediate temperature and pressure. The plotted results have been obtained using the Euler explicit first order scheme for integrating the ordinary differential equations. Also for this gas turbine, the simulation has been repeated using the Heun second order scheme; the

### Table 1 Design specifications assumed for the gas turbine Siemens V64.3

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft speed, rpm</td>
<td>5400</td>
</tr>
<tr>
<td>Electrical power, MW</td>
<td>61.5</td>
</tr>
<tr>
<td>Cycle efficiency, %</td>
<td>35.8</td>
</tr>
<tr>
<td>Inlet air flow, kg/s</td>
<td>185</td>
</tr>
<tr>
<td>Pressure ratio</td>
<td>15.6</td>
</tr>
<tr>
<td>Turbine inlet temp, °C</td>
<td>1250</td>
</tr>
<tr>
<td>Turbine exit temp, °C</td>
<td>534</td>
</tr>
</tbody>
</table>
results obtained confirm that the small time step guarantee accurate solutions even adopting a first order scheme. Further tests carried out using a previously developed simulation code making use of a fully implicit solving procedure have given again results undistinguishable from the above described ones.

4.3 Computer Time Performance. The Simulink scheme has been used to produce a source code in C-language by means of the RealTimeWorkshop toolbox. This code has been compiled to obtain an executable program that has been tested to verify the real-time capability.

The bar chart in Fig. 13 outlines the average single step calculation time measured on a personal computer based on Pentium III® 800 MHz with a 128 Mbyte memory. The results are referred to the simple single shaft gas turbine in Fig. 5, to the single shaft gas turbine in Fig. 8, and to the aero-derivative engine in Fig. 11. For these last two simulations the comparison between the results obtained by means of the Euler and the Heun schemes is given in order to evaluate the computational effort resulting by using a higher order scheme.

Taking into account that a time step of 1 ms was assumed and

Fig. 9 Simulation results for a slow load increase for the Siemens V64.3 gas turbine comparison of the real-time Simulink model with experimental data by Jansen et al. [19], results by Kim et al. [20], and, finally, with the result obtained by using a proven FORTRAN code [6]
Fig. 10 Dimensionless results for the single-shaft gas turbine; (a) applied load; (b) controlled variables; (c) control variables; (d) temperature and pressure at the compressor discharge ($T_2, p_2$) and the combustor exit ($T_3, p_3$).

Fig. 11 Double shaft gas turbine scheme
that the maximum calculation time resulted about 0.17 ms, it appears that the computer simulation runs 6 to 20 times faster than the simulated system. This result allows us to adopt the described technique in "hardware in the loop" applications, since enough time seems to remain available for data exchange through an input-output interface, within the assumed time step.

5 Conclusions

A novel technique for developing a high-fidelity real time code for gas turbine simulation has been presented. It is shown that the set of nonlinear algebraic equations and ordinary differential equations that compose the mathematical model of the gas turbine can be solved by means of a forward substitution procedure. The mathematical model and the solving procedure are described in the paper. The code is based on the object-oriented approach and is realized under the Matlab-Simulink graphical environment: a library of blocks that simulate the gas turbine components has been set up and a description of main blocks has been provided. These blocks can be easily assembled by means of the graphical interface.

The developed code is compatible with the software package called Real-Time-Workshop toolbox. By means of this tool, a source code in C language is automatically generated from the Simulink model and can be compiled. This feature represents an opportunity to modify an existing code or to realize a new one in a relatively short time from models already tested in the Simulink environment.

Two different gas turbines for power generation have been simulated with a relatively high level of details. Some tests have been carried out on the programs obtained by compiling the C codes generated by means of the described procedure. Such tests showed that the codes are able to provide the results in real time, even using a simple personal computer.

The source code in C language can be also modified in order to realize "hardware-in-the-loop" applications for testing control equipments or for control embedded applications or diagnostic purposes.
Acknowledgment

This work has been supported by the Italian Government, Ministry for University and Scientific and Technological Research, under the research project “Dynamic Modeling of Energy Systems.”

Nomenclature

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_p$ =</td>
<td>specific heat at constant pressure</td>
</tr>
<tr>
<td>$c_v$ =</td>
<td>specific heat at constant volume</td>
</tr>
<tr>
<td>$g$ =</td>
<td>mass flow rate</td>
</tr>
<tr>
<td>$h$ =</td>
<td>specific enthalpy</td>
</tr>
<tr>
<td>IGV =</td>
<td>normalized inlet guide vane position</td>
</tr>
<tr>
<td>$K$ =</td>
<td>specific heat ratio $=c_{p}/c_{v}$</td>
</tr>
<tr>
<td>LHV =</td>
<td>lower heating value</td>
</tr>
<tr>
<td>$M$ =</td>
<td>mass</td>
</tr>
<tr>
<td>$Ma$ =</td>
<td>Mach number</td>
</tr>
<tr>
<td>$m$ =</td>
<td>polytropic coefficient</td>
</tr>
<tr>
<td>$n$ =</td>
<td>shaft rotational speed</td>
</tr>
<tr>
<td>$P$ =</td>
<td>power</td>
</tr>
<tr>
<td>$p$ =</td>
<td>pressure</td>
</tr>
<tr>
<td>$R$ =</td>
<td>gas constant</td>
</tr>
<tr>
<td>$t$ =</td>
<td>time</td>
</tr>
<tr>
<td>$T$ =</td>
<td>temperature</td>
</tr>
<tr>
<td>$u$ =</td>
<td>internal energy</td>
</tr>
<tr>
<td>$V$ =</td>
<td>volume</td>
</tr>
<tr>
<td>VCE =</td>
<td>fuel valve input signal</td>
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<tr>
<td>$x$ =</td>
<td>stoichiometric products mass fraction</td>
</tr>
<tr>
<td>$y$ =</td>
<td>pressure loss coefficient</td>
</tr>
<tr>
<td>$Y$ =</td>
<td>momentum loss coefficient</td>
</tr>
<tr>
<td>$\beta$ =</td>
<td>pressure ratio</td>
</tr>
<tr>
<td>$\delta$ =</td>
<td>$P_{amb}/P_0$</td>
</tr>
<tr>
<td>$\theta$ =</td>
<td>$T_{amb}/T_0$</td>
</tr>
<tr>
<td>$\eta$ =</td>
<td>efficiency</td>
</tr>
<tr>
<td>$\tau$ =</td>
<td>time constant</td>
</tr>
<tr>
<td>$\Gamma$ =</td>
<td>mass flow parameter</td>
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</table>

Subscripts

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
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<tbody>
<tr>
<td>$a$ =</td>
<td>air</td>
</tr>
<tr>
<td>$amb$ =</td>
<td>ambient conditions</td>
</tr>
<tr>
<td>$b$ =</td>
<td>fuel</td>
</tr>
<tr>
<td>$c$ =</td>
<td>compressor</td>
</tr>
<tr>
<td>$cc$ =</td>
<td>combustion chamber</td>
</tr>
<tr>
<td>cool =</td>
<td>cooling air</td>
</tr>
<tr>
<td>des =</td>
<td>design-point</td>
</tr>
<tr>
<td>$in$ =</td>
<td>inlet</td>
</tr>
<tr>
<td>is =</td>
<td>isentropic</td>
</tr>
<tr>
<td>mis =</td>
<td>measured</td>
</tr>
<tr>
<td>$out$ =</td>
<td>outlet</td>
</tr>
<tr>
<td>$p$ =</td>
<td>plenum</td>
</tr>
</tbody>
</table>

$t$ = turbine

0 = standard conditions

References