An algorithm for multiresolution pyramid decomposition is described. At each stage, the smoothed (“lowpass”) image is obtained by combining morphological grayscale opening and closing. Using this technique, we avoid the systematic bias of traditional approaches, as illustrated by an example. As our application, we perform image enhancement by modifying the reconstruction scheme using a morphological edge detector. The processing scheme offers a method for edge-preserving noise (speckle) suppression, in which only a minor number of multiplications is required.

1. INTRODUCTION

Multiresolution techniques are widely used in algorithms for processing, analysis and coding of images and other types of signals. There are essentially two (partly overlapping) motives for adopting such techniques. First, they can improve the efficiency in terms of decreased computational load and storage requirements. Second, in a multiresolution decomposition, the information contained in a signal is arranged into a conceptually meaningful hierarchy, which can be used, e.g., as a basis for coarse-to-fine processing or analysis.

In a pyramid scheme, the input to each stage is decomposed into one lowpass and one detail signal, where the former of these is downsampled and passed on to the next decomposition stage [1]. Since the detail signal is defined by the difference between the original signal and the reconstructed lowpass signal, perfect reconstruction can be achieved without imposing any particular filter demands. An approach where each decomposition stage for a two-dimensional signal involves decimation by a factor of $\sqrt{2}$ in each dimension has been presented by Feauveau [2].

In morphological image and signal processing, geometric properties such as size and shape are emphasized rather than, e.g., the frequency properties of signals [3]. Most algorithms are built up by simple operations such as addition, sign-shift and comparison, which make them suitable for fast implementations in hardware or software. Many properties of morphological filters have analogies in the traditional theory of linear, shift-invariant filters [4]. Moreover, statements similar to the sampling theorem have been formulated for morphological reconstruction of a signal from its samples [5].

In this paper, a morphological approach is adopted in a multiresolution decomposition scheme for images. The amount of information preserved at different resolution levels is estimated and compared to that of other approaches. In our application example, edge-preserving speckle suppression is achieved by means of modified reconstruction. This is a further development of the method by Sattar et al., where a decomposition scheme based on traditional lowpass filtering was used [6]. A morphological edge detector is thereby applied to the lowpass image of each reconstruction stage, and the detail image is masked by the detected edges.

2. MULTIRESOLUTION PYRAMID SCHEME

In our pyramid scheme, we have adopted Feauveau’s idea where the image resolution at each stage is reduced by a factor of $\sqrt{2}$ in each dimension [2]. Every second pixel is retained and, in order to make the lower resolution image fit to the Cartesian grid, subsampling includes rotation of the image by 45°.

In Fig. 1, decomposition stage number $m$ in the multiresolution pyramid scheme is displayed. The input image is denoted by $u_{m-1}$, while the output images are denoted by $u_m$ (lower resolution image) and $d_m$ (detail image), respectively. As smoothing filter before decimation, we use a combination of morphological closing and opening. For both operations, a cross-shaped symmetric five-pixel structuring set,

$$B = \{(n_1, n_2) \in \mathbb{Z}^2 : |n_1| + |n_2| \leq 1\} = B^*$$

is used ($B^*$ denotes the symmetric set with respect to the origin). From the classical theory for morphological filters, we know that opening is an antiextensive operation, whereas closing is an extensive operation. Thus, if $u_B$ and $u_B^*$ denote opening and closing of the image $u$ by $B$, it is true that

$$u_B \leq u \leq u_B^*$$

for each pixel in the image. Furthermore, a theory giving conditions for when an opened or closed image can be reconstructed from its samples has been developed (see [5]). For the combination $(u_B + u_B^*)/2$, no similar theory exists. However, its general behavior is “lowpass,” which makes it suitable as presampling filter in our pyramid decomposition scheme. Moreover, in contrast to either opening or closing alone, it involves essentially bias-free smoothing.
The detail image \( d_m \) in Fig. 1 is obtained as the difference between the input image \( u_m \) and a filtered, upsampled version of \( u_{m-1} \). As interpolation filter, a combination of closing and dilation is used. Like the previously described presampling filter, it yields an essentially bias-free approximation of \( u_m \).

![Figure 1. One morphological decomposition stage.](image)

By cascading a number of decomposition stages, we obtain a multiresolution pyramid representation consisting of one detail image from each stage plus the lowest resolution image from the last stage. Perfect reconstruction of the original image can be obtained by cascading reconstruction stages as in Fig. 2, however without modification of the detail images. Contrary to this, in the following, algorithms involving modified reconstruction are described.

![Figure 2. One morphological reconstruction stage.](image)

### 3. MODIFIED RECONSTRUCTION

Two examples involving modified reconstruction were carried out. In the first one, all detail images were discarded and only the lowest resolution image was used in the reconstruction. Comparison with the original image was performed and the similarity was interpreted as a measure of how well the filters worked as bias-free lowpass filters.

In our second example — the application — we performed modified reconstruction by masking the detail image by the output from morphological edge detection as described by Lee et al. [7]. By doing this through an appropriate number of stages, noise could be suppressed while meaningful edges were preserved. At each stage \( m \), an edge strength image was computed as

\[
E_m = \min(\tilde{a}_m \ominus B^* - \tilde{a}_m * g, \tilde{a}_m * g - \tilde{a}_m \ominus B^*) \quad (3)
\]

where \( \ominus \) and \( \oplus \) denote morphological dilation and erosion while \( * \) denotes linear convolution and \( g \) is an average filter over the domain \( B \). The edges were then defined as the pixels for which \( E_m > T_m \), where the threshold \( T_m \) was selected automatically from the histogram.

### 4. RESULTS

#### 4.1. Reconstruction from Decimated Image

The 512×512 pixel grayscale image “Lenna” was decomposed into an 8-level multiresolution pyramid, in which the lowest resolution image from the last stage had size 32×32 pixels. In Fig. 3, we see the original image (upper left) and, as an example, the decimated image from the fifth decomposition stage (right). The latter has a size of approximately 91×91 pixels and after five-stage reconstruction with all detail images discarded, the lower left image in Fig. 3 was obtained.

![Figure 3. Original image “Lenna” (upper left), image after five-stage decimation (right), and image after five-stage reconstruction with detail images suppressed (lower left).](image)
for various reconstructed images.

In Fig. 4, the signal-to-distortion ratios are plotted for the present scheme, as well as other filtering approaches. Only linear lowpass filtering produced more accurate reconstruction from decimated representations. Since linear convolution involves a large number of multiplications, the computationally less complex morphological approach can be a reasonable alternative in a pyramid decomposition scheme. The two curves marked by '◦' and '◦*' correspond to the maximal and minimal morphological reconstructions in [5] and their poor performances in terms of SDR can be explained by the inherent bias.

![Graph of signal-to-distortion ratios](image)

**Figure 4.** Signal-to-distortion ratios of images reconstructed from decimated versions of various sizes. Linear 15×15 pixel lowpass filter: 'x.' The present morphological approach: '◦.' Zero-order hold: '+' Morphological opening-dilation: 'o.' Morphological closing-closing: 'w.'

### 4.2. Edge-Preserving Speckle Suppression

Our application of the multiresolution pyramid was speckle suppression by modified reconstruction as described in Section 3. We thereby used the artificial image in Fig. 5(a), which was obtained after multiplying a reference image containing areas of constant brightness levels by a Rayleigh distributed random field with unit mean. The signal-to-noise ratio of such images has been defined as $\mu/\sigma$, where $\mu$ is mean and $\sigma$ is standard deviation, and it is equal to $(4/\pi - 1)^{-1/2} = 1.91$ (5.63 dB) (see [8]).

Modified multiscale reconstruction was carried out for different multiresolution depths and the signal-to-noise ratios of the output images were evaluated in a rectangular region containing the objects to the left in the image. From these figures, which are given in Table 1, one could perhaps draw the conclusion that increased multiresolution depth implies improved performance. However, the signal-to-noise ratio reflects essentially the amount of noise suppression, while edge preservation is not taken into consideration. Therefore, the highest overall performance was achieved using three stages. The corresponding output image is displayed in Fig. 5(b), where it can be noticed that the speckle has been considerably suppressed while the edges have essentially been preserved.

![Image of speckle suppression](image)

**Figure 5.** (a) Speckle image. (b) Enhanced image after modified three-stage reconstruction.
Table 1. SNR (in dB) for enhanced images after modified reconstruction using different multiresolution depths.

<table>
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<tr>
<th>Depth</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR</td>
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<td>7.3</td>
<td>12.6</td>
<td>12.9</td>
<td>13.1</td>
<td>13.0</td>
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</table>

5. SUMMARY AND DISCUSSION

A multiresolution pyramid scheme based on morphological operations has been described. As presampling filter, we used a combination of closing and opening while as reconstruction filter, we used a combination of closing and dilation. At each stage of the pyramid, the input image was decomposed into one lowpass and one detail component. The accordance in information between the original and each lower resolution image was investigated in an example. In this respect, the present decomposition scheme was found to yield essentially bias-free approximations at various resolution levels. Furthermore, these were found only marginally poorer than those obtained using computationally demanding linear filtering.

As our application, the multiresolution pyramid was used for edge-preserving speckle suppression, which was achieved by modified reconstruction. The detail images were thereby masked by the outputs from morphological edge detection on the reconstructed lowpass images. The effect of this modification procedure was that useful detail information, representing, e.g., object boundaries, could be preserved while noise was suppressed.

Our main motive for adopting morphological operations in the present algorithm was their property of requiring no multiplications, which make them simple to implement in hardware or software. By using the average of opening and closing as presampling filter, we could exploit this advantage but also avoid systematic bias. Due to the fact that “pure” morphological filters were not used, we could not apply the corresponding results concerning reconstruction from samples. However, for the described pyramid decomposition scheme, this was not a crucial matter.

REFERENCES