A Game Approach for Multi-Channel Allocation in Multi-Hop Wireless Networks

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ABSTRACT
Channel allocation was extensively investigated in the framework of cellular networks, but it was rarely studied in the wireless ad-hoc networks, especially in the multi-hop ad-hoc networks. In this paper, we study the competitive multi-radio channel allocation problem in multi-hop wireless networks in detail. We model the channel allocation problem as a static cooperative game, in which some players collaborate to achieve high date rate. We propose the min-max coalition-proof Nash equilibrium (MMCPNE) channel allocation scheme in the game, which aims to max the achieved date rates of communication links. We analyze the existence of MMCPNE and prove the necessary conditions for MMCPNE. Furthermore, we propose several algorithms that enable the selfish players to converge to MMCPNE. Simulation results show that MMCPNE outperforms CPNE and NE schemes in terms of achieved data rates of the multi-hop links due to cooperation gain.

Categories and Subject Descriptors
H.4 [Information Systems Applications]: Communications Applications

General Terms
Theory

Keywords
Multi-Radio, Channel Allocation, Game Theory, Nash Equilibrium

1. INTRODUCTION
Wireless communication system is often assigned a certain range of communication medium (e.g., frequency band). Usually this medium is shared by different users through multiple access techniques. Frequency Division Multiple Access (FDMA), which enables more than one users to share a given frequency band, is one of the extensively used techniques in wireless networks [1], [2]. In FDMA, the total available bandwidth is divided permanently into a number of distinct sub-bands named channels. Commonly, we refer to the assignment of radio transceivers to these channels as the channel allocation problem. An efficient channel allocation is essential for the design of wireless networks.

In this paper, we present a game-theoretic analysis of fixed channel allocation strategies of devices that use multiple radios in the multi-hop wireless networks. Static non-cooperative game is a novel approach to solve the channel allocation problem in single-hop networks and Nash equilibrium (NE) provides an efficient criterion to evaluate a given channel allocation (e.g., in [3]). In multi-hop networks, however, non-cooperative game results in low achieved date rate of multi-hop links for the reasons mentioned in Section 4. Hence, we introduce static cooperative game with perfect information into our system. We mainly focus on the performance improvement of the multi-hop links, which is induced by cooperation gain, without sacrificing the performance of single-hop links. We first define the min-max coalition-proof Nash equilibrium (MMCPNE) in this game, which is aiming to achieve the maximal date rate of all links (single-hop links and multi-hop links). We also define three other equilibria schemes that approximate to MMCPNE, named as MCPNE, ACPNE and ICPNE respectively. Then, we study the existence of MMCPNE in the static cooperative game and our main result, Theorem 2, shows the necessary conditions for the existence of MMCPNE.

Furthermore, we propose the MMCP algorithm which enables the selfish players to converge to MMCPNE from an arbitrary initial configuration and the DCP-x algorithms which enable the players converge to approximate MMCPNE states (e.g., MCPNE, ACPNE and ICPNE). Finally, we present the simulation results of the previous algorithms, which show that MMCPNE outperforms CPNE and NE channel allocation schemes in terms of achieved data rates of multi-hop links due to cooperative gain.

The paper is organized as follows. In Section 2, we present related work on channel allocation and channel access in wireless networks. In Section 3, we introduce the system model which contains multi-hop links. In Section 4, we introduce the game-theoretic description of competitive channel allocation problem in multi-hop wireless networks. In Section 5, we provide a comprehensive analysis of the Nash equilibrium and min-max coalition-proof Nash equilibrium in the channel allocation game. Additionally, we propose several algorithms to reach the exact and approximate MM-
2. RELATED WORK

There has been a considerable amount of research on channel allocation in wireless networks, especially in cellular networks. Three major categories of channel allocation schemes are always used in cellular networks: fixed channel allocation (FCA, e.g., as present in [4]), dynamic channel allocation (DCA, e.g., as present in [5]) and hybrid channel allocation (HCA, e.g., as present in [6]) which is a combination of both FCA and DCA techniques.

Recently, channel allocation problem is becoming a focus of research again due to the appearance of new communication technologies, e.g., wireless local area networks (WLANs), wireless mesh networks (WMNs, e.g., as present in [7] and [8]) and wireless sensor networks (WSNs, e.g., as present in [9] and [10]). Using weighted graph coloring method, Mishra et al. propose a channel allocation method for WLANs in [11]. In wireless mesh networks, many researchers have considered devices using multiple radios. Equipping multiple radios in the devices in WMNs, especially the devices acting as wireless routers, can improve the capacity by transmitting over multiple radios simultaneously using orthogonal channels. In the multi-radio communication context, channel allocation and access are also considered as the vital topics. By joint considering the channel assignment and routing problem, Alcherry et al. propose an algorithm to optimize the overall throughput of WMNs in [12].

In the above cited work, the authors make the assumption that the devices cooperate with the purpose of the achievement of high system performance. However, this assumption might not hold for the following two reasons. In one hand, players are usually selfish who would like to maximize their own performance without considering the other players’ objective. In the other hand, the full cooperation of arbitrary devices is difficult to achieve due to the transmission distance limitation and transmission interference of neighboring devices.

Game theory provides a straightforward tool to study channel allocation problems in competitive wireless networks. As far as we know, game theory has been applied to the CSMA /CA protocol [13], [14], to the Aloha protocol [15] and to the peer-to-peer system [16]. Furthermore, on the basis of graph coloring, Halldorsson et al. use game theory to solve a fixed channel allocation problem in [17]. Unfortunately, their model does not apply to multi-radio devices. In wireless ad-hoc networks (WANETs), Felegyhazi et al. present a game-theoretic analysis of fixed channel allocation strategies of devices that use multiple radios in [3]. However, their results can be only applied to single-hop wireless networks without considering multi-hop networks.

3. SYSTEM MODEL

We assume that the available frequency band is divided into $N$ orthogonal channels of the same bandwidth using the FDMA method (e.g., 8 orthogonal channels in case of the IEEE 802.11a protocol). We denote the set of available orthogonal channels by $C = \{c_1, c_2, \ldots, c_N\}$.

We assume that there exist $L$ communication sessions$^1$ in our model and we denote the set of communication sessions by $L = \{l_1, \ldots, l_L\}$. We further assume each user participates in only one session. Hence we can divide all users into $L$ disjoint groups, denoted by $g_i$, according to different sessions. We denote the set of groups by $G = \{g_1, \ldots, g_L\}$ and $g_i = \{l_i\}$, where “=” denotes a one-one mapping. Additionally, we denote the set of senders in all groups by $S$ and the set of relaying users by $R$. It is easy to see that $|S| = |R| = |G| = L$.

Figure 1 presents an example with three communication sessions, where $L = 3$, $L = \{l_1, l_2, l_3\}$, $G = \{\{s_1, d_1\}, \{s_2, r_2, d_2\}, \{s_3, d_3\}\}$, $S = \{s_1, s_2, s_3\}$ and $R = \{r_2\}$.

![Figure 1: An example of 3 communication sessions.](image)

We assume each user owns a device equipped with two independent sets of radio transceivers, denoted by $T_s$ and $R_s$, which used to originate and receive the data packets respectively. Each transceivers set contains $k < |C|$ radio transceivers, all having the same communication capabilities. The communication between two devices is bidirectional and they always have some packets to exchange. Due to the bidirectional links, the originator radios in the sender and the receptor radios in the receiver are able to coordinate and thus to select the same channels to communicate. Hence, we omit the behaviors of $R_s$ since they well correspond with the $T_s$ in pre-hop users, and accordingly omit the behavior of receivers. Thus we define the players set, denoted by $U$, as the summation of senders set and relay users set, i.e., $U = S \cup R$. We assume that there is a finite number of players. We further assume that each device can hear the transmissions of any other device if they are using the same channel. This means that the players reside in a single collision domain. Note however that one device cannot communicate directly with other devices except its neighboring devices, e.g., the device equipped by pre-hop or post-hop player. For a comprehensive understanding of this phenomenon, please refer to the definitions of sensing range and transmission range in [17].

We assume that there is a mechanism that enables the multiple radios in any $T_s$ (or $R_s$) to communicate simultaneously by using orthogonal channels (as it is implemented in [18] for example). We denote the number of radios of player $u_i$ using channel $c$ by $k_{u_i,c}$ for every $c \in C$. For the sake of suppressing co-radios interference in device, we assume that different radios in any $T_s$ (or $R_s$) cannot use the same channel, i.e., $k_{u_i,c} \leq 1$ for arbitrary players and channels.

We formulate the channel allocation problem with a single stage game, which corresponds to a fixed channel allocation

$^1$Note that in our paper, the meaning of communication session is equivalent to the active communication link.
among the players. Each player’s strategy consists in defining the number of radios on each of the channels. Hence, we define the strategy of player $u_i$ as its channel allocation vector:

$$x_{u_i} = (k_{u_i,1}, k_{u_i,2}, \ldots, k_{u_i,|C|})$$

(1)

The strategy matrix, denoted by $X$, is defined by all players’ strategy vectors:

$$X = \left( x_{u_1}^T, \ldots, x_{u_1}^T, x_{u_2}^T, \ldots, x_{u_2}^T, \ldots, x_{u_{|U|}}^T, \ldots \right)^T$$

(2)

Furthermore, we denote the strategy matrix except for the strategy of player $u_i$ (or $c_i$) by $X_{-u_i}$ (or $X_{-c_i}$) and the strategy matrix of players set $U_u \subset U$ by $X_{U_u}$.

Figure 2 presents an example of channel allocation strategy in the system of Figure 1 with four available channels ($|C| = 4$) and four players ($|U| = |S| + |R| = 4$). Each player’s device equipped by two radios sets which contain three radios transceivers ($k = 3$) respectively. The tubers at the left of node denote the radios of $R_u$, and the remainder tubers denote the radios of $T_u$. The number on each radio link denotes the channel used by this radio transceivers pair. We can easily write the strategy of players $s_1$ and $s_2$ as $x_{s_1} = (1 \ 1 \ 0 \ 1)$ and $x_{s_2} = (1 \ 1 \ 0 \ 0)$ respectively.

![Figure 2: An example of channel allocation strategy, where $|C| = 4$, $|S| = 3$, $|R| = 1$ and $k = 3$.](image)

The total number of channels used by player $u_i$ can be written as $k_u = \sum k_{u_i,c}$ and $k_u \leq k$ obviously. Similarly, the total number of radios using a particular channel $c$ can be written as $k_c = \sum u_i k_{u_i,c}$. In Figure 2, $k_{s_1} = 3$, $k_{s_2} = k_{c_1} = k_{c_2} = 2$, $k_{c_3} = 3$, $k_{c_4} = 2$ and $k_{c_5} = 3$.

We denote the total available bandwidth on channel $c$ (i.e., the sum of the achieved rate of all players on channel $c$) by $R_c(k_c)$. In fact $R_c(k_c)$ is independent of $k_c$ for a TDMA protocol and for the CSMA/CA protocol using optimal backoff window values [19]. In practice, however, the backoff window values (e.g., in the 802.11 standard) are not optimal, and due to packet collisions $R_c(k_c)$ becomes a decreasing function of $k_c$ for $k_c > 1$. In our model, we assume that $R_c(k_c)$ is independent of $k_c$ and thus we can write $R_c(k_c)$ as $R_c$ by omitting the parameter $k_c$. Note however, that our simulation shows similar results when $R_c(k_c)$ is a slowly decreasing function of $k_c$ for $k_c > 1$.

We assume that the total available bandwidth on channel $c$ is shared equally among the radios deployed on this channel. We denote $R_{u_i,c}$ as the available bandwidth occupied by player $u_i$ on channel $c$ and we can write $R_{u_i,c}$ as following:

$$R_{u_i,c} = \frac{k_{u_i,c}}{k_c} \cdot R_c, \quad \forall u_i \in U, \ c \in C$$

(3)

>From Equation (3), we can easily find that the higher the number of radios on a given channel is, the lower the bandwidth per radio is. In Figure 2, we have $R_{s_1,c_1} = R_{s_1,c_2} < R_{s_2,c_1} < R_{s_2,c_2}$. We define the utility of player $u_i$ denote by $R_{u_i}^c$, as the total available bandwidth occupied by $u_i$ and we can write $R_{u_i}^c$ as follows.

$$R_{u_i}^c = \sum_{c \in C} u_i \cdot R_{u_i,c} \quad \forall u_i \in U$$

(4)

In fact any player’s utility is equivalent to its one-hop rate. In single-hop networks, the utility of any player exactly reflects its actual date rate. In multi-hop networks, however, the utility of any player may not reflect its achieved data rate. We define the end-to-end rate of a communication link, denoted by $R_{l_i}^e$, as the minimal utility of players in the link and we can write $R_{l_i}^e$ as follows.

$$R_{l_i}^e = \min_{u_i \in u_i} \forall l_i \in L$$

(5)

where $u_i$ is arbitrary player in group $g_i$, i.e., in communication link $l_i$. For single-hop link, $R_{l_i}^e = R_{u_i}^c$ since there is only one player $u_i$ in the link.

Recall the example in Figure 2, we can easily obtain the normalized one-hop rates: $R_{l_1}^e = \frac{4}{17} + \frac{1}{17} + \frac{1}{17} = 1.17$, $R_{l_2}^e = \frac{1}{7} + \frac{3}{7} = 0.83$, $R_{l_3}^e = \frac{1}{7} + \frac{3}{7} = 0.67$ and $R_{l_3}^e = 1 + \frac{1}{7} = 1.33$. Accordingly, we can obtain the normalized end-to-end rate of communication links: $R_{l_1}^e = R_{l_2}^e = 1.17$, $R_{l_2}^e = \min\{R_{l_2}^e, R_{l_1}^e\} = 0.83$ and $R_{l_3}^e = R_{l_3}^e = 0.67$.

4. NASH EQUILIBRIA

We refer to each player as a rational and self-interested player, who will always choose action that maximize its payoff. Thus we can formulate the multi-radio channel allocation problem as a static game, which corresponds to a fixed channel allocation among the players.

In single-hop networks, the multi-radio channel allocation problem can be formulated as a static non-cooperative game (e.g., in [3]). We define the payoff of player $u_i$, denoted by $P_{u_i}(X)$, as the utility of $u_i$ in the strategy matrix $X$, i.e., $P_{u_i}(X) = R_{u_i}^c$. In order to study the strategic interaction of the players in static non-cooperative game, we first introduce the concepts of Nash equilibrium [20].

**Definition 1:** (Nash Equilibrium - NE): The strategy matrix $X^* = \{x_{u_1}^*, \ldots, x_{u_{|U|}}^*\}$ defines a Nash Equilibrium (NE), if for every player $u_i$, we have:

$$P_{u_i}(x_{u_i}^*, X_{-u_i}^*) \geq P_{u_i}(x_{u_i}, X_{-u_i})$$

(6)

for every strategy $x_{u_i}$.

The definition of NE expresses the resistance to the deviation of a single player in non-cooperative game. In other words, in a NE none of the players can unilaterally change its strategy to increase its utility.

However, non-cooperative game is not suitable for the multi-hop networks for the following two reasons. In one hand, the definition of payoff function is not suitable for multi-hop networks. Specifically, the achieved date rate of any player in multi-hop link is not only determined by the utility itself, but also by the utilities of other players in the same link. In the other hand, it is possible that the players in the same multi-hop link cooperatively choose their

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3Specifically, the players belonging to the multi-hop links.

4Strictly speaking, $u_i$ is arbitrary player in group $g_i = \{d_i\}$ since we do not consider the behaviors of $d_i$. 
strategies for the purpose of high achieved date rate. Thus we formulate the problem as a static cooperative game in multi-hop networks.

In cooperative game, it might be possible that some players collude to increase their payoff at the expense of other players. Such a collusion is called a coalition, denoted by $c_0$. We denote the set of coalitions by $Q = \{c_0, c_1, c_2, \ldots\}$. We can generalize the notion of classical coalition-proof Nash equilibrium as defined in [21].

**Definition 2:** (Coalition-Proof Nash Equilibrium - CPNE): The strategy matrix $X^{cp}$ defines a coalition-proof Nash Equilibrium, if for every coalition $c_0 \in Q$ we have:

$$P_{u_i}(X_{c_0}^{cp}, X_{-c_0}^{cp}) \geq P_{u_i}(X'_{c_0}, X_{-c_0}^{cp}), \quad \forall u_i \in c_0$$

(7)

for every strategy set $X_{c_0}^{cp}$.

This means that no coalition can deviate from $X^{cp}$ such that the payoff of at least one of its members increases and the payoffs of other members do not change. Note that our definition in (7) corresponds to the principle of weak deviation.

In our model, we define coalition as the players belonging to the same communication link, e.g., $s_2$ and $r_2$, in Figure 1, since they can easily build up the cooperative process. Thus each communication link $l$ corresponds to a coalition $c_0$. In fact a coalition can be seen as a subset of $U$, i.e., $\bigcup_{c_0} c_0 = U$ and $c_0 \cap c_0 = \emptyset, \forall i \neq j$. In Figure 1, $c_01 = \{s_1\}, c_02 = \{s_2, r_2\}$ and $c_03 = \{s_3\}$.

Unfortunately, we find that classical CPNE in definition 2 is not strictly suitable for multi-hop networks as we define the coalition as above. In detail, according to CPNE, it is not permission for any coalition to improve a member's (e.g., $u_i$) payoff with worsening any other member's (e.g., $v_i$) payoff, even the payoff of $u_i$ is much higher than $v_i$. Thus the payoffs of players in the same coalition might be imbalance in classical CPNE, which will lead to poor performance in terms of achieved data rate.

Figure 3 presents an example of CPNE where $|C| = 6, k = 3$, player $u_1$, $u_2$ formulate a coalition and $u_3$, $u_4$, $u_5$ are non-coalition players. In other word, the coalition (of $u_1$ and $u_2$) can not improve a member's payoff without worsening the other member's payoff by unilaterally changing the strategies of $u_1$ and $u_2$. It is obvious that $P_{u_1} = 1.0, P_{u_2} = 1.5$ and $P_{u_1} \ll P_{u_2}$. As we mentioned above, player $u_1$ and $u_2$ belong to the same communication link, and thus the actual data rates of $u_2$ is the minimal utility of $u_1$ and $u_2$ (i.e., 1.0), which is much lower than the payoff of itself.

**Figure 3:** An example of CPNE channel allocation, where $u_1$ and $u_2$ formulate a coalition.

To overcome the shortcoming of payoff imbalance in CPNE, we define a novel coalition-proof Nash equilibrium in cooperative game, named as min-max coalition-proof Nash equilibrium (MMCPNE), in which players make their decisions so as to improve the minimal payoff of players in the coalition. We generalize the notion of MMCPNE as following:

**Definition 3:** (Min-Max Cooperation Game - MMCPNE): The strategy matrix $X^{mm}$ defines a novel coalition-proof Nash Equilibrium, if for every coalition $c_0$, we have:

$$\min_{u_i \in c_0} R^i_{u_i}(X_{c_0}^{mm}, X_{-c_0}^{mm}) \geq \min_{u_i \in c_0} R^i_{u_i}(X'_{c_0}, X_{-c_0}^{mm})$$

for every strategy set $X_{c_0}^{mm}$.

It is notable that MMCPNE points are not always CPNEs and vice versa. In fact, MMCPNE can be seen as a special coalition-proof Nash equilibrium with a judiciously designed payoff function, i.e., end-to-end rate $R^e_{u_i}$. Recall the example in Figure 3, if we do $(u_1, c_1) \Rightarrow (u_2, c_4)^6$, we obtain the MMCPNE channel allocation and we find the actual data rates of $u_1$ and $u_2$ increase to 1.71.

However, it is very difficult to find such a MMCPNE (or CPNE) strategy since we must jointly search the strategy in the strategies set of $|c_0|$ players. The computation of achieving MMCPNE (or CPNE) increases exponentially with the size of coalition, typically $O(2^{|c_0|})$ where $\omega$ is the expectation of $\omega$ and $\omega$ is the number of moves for a single player finding its best response strategy. In the worst case, a player must try all possible strategies to find its best response strategy, i.e., $\omega = \binom{|C|}{k}$.

To reduce the large computation in finding MMCPNE, we introduce three approximate solutions, denoted by minimal coalition-proof Nash equilibrium (MCPNE), average coalition-proof Nash equilibrium (ACPNE) and I coalition-proof Nash equilibrium (ICPNE). The definitions of MCPNE, ACPNE and ICPNE are shown as follows.

**Definition 4:** (Minimal Coalition-Proof Nash Equilibrium - MCPNE): The strategy matrix $X^m$ defines a special coalition-proof Nash Equilibrium, if for every player $u_i$, we have:

$$\min_{u_i \in c_0} R^i_{u_i}(X_{c_0}^{m}, X_{-c_0}^{m}) \geq \min_{u_i \in c_0} R^i_{u_i}(X'_{u_i}, X_{-c_0}^{m})$$

for every strategy $X_{u_i}^m$.

**Definition 5:** (Average Coalition-Proof Nash Equilibrium - ACPNE): The strategy matrix $X^a$ defines a special coalition-proof Nash Equilibrium, if for every player $u_i$, we have:

$$\min_{u_i \in c_0} R^i_{u_i}(X_{c_0}^{a}, X_{-c_0}^{a}) > \min_{u_i \in c_0} R^i_{u_i}(X'_{u_i}, X_{-c_0}^{a})$$

or

$$\sum_{u_i \in c_0} R^i_{u_i}(X_{c_0}^{a}, X_{-c_0}^{a}) \geq \sum_{u_i \in c_0} R^i_{u_i}(X'_{u_i}, X_{-c_0}^{a})$$

for every strategy $X_{u_i}^{a}$.

**Definition 6:** (I Coalition-Proof Nash Equilibrium - ICPNE): The strategy matrix $X^i$ defines a special coalition-proof Nash Equilibrium, if for every player $u_i$, we have:

$$\min_{u_i \in c_0} R^i_{u_i}(X_{c_0}^{i}, X_{-c_0}^{i}) > \min_{u_i \in c_0} R^i_{u_i}(X'_{u_i}, X_{-c_0}^{i})$$

Note that $(u_i, c_m) \Rightarrow (u_j, c_n)$ means exchanging the radio of $u_i$ in channel $c_m$ and the radio of $u_j$ in channel $c_n$. 

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6It is notable that the coalition may contain one player only.

Strictly speaking, $c_0 = g_i - \{d_i\}$ where $d_i$ is the destination user in group $g_i$. 

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or
\[
\begin{align*}
\min_{u_i \in \mathcal{C}_{\omega}} R_{u_i}^k \left( x_{u_i}^*, x_{-u_i}^* \right) &= \min_{u_i \in \mathcal{C}_{\omega}} R_{u_i}^k \left( x_{u_i}^*, x_{-u_i}^* \right) \\
R_{u_i}^k \left( x_{u_i}^*, x_{-u_i}^* \right) &\geq R_{u_i}^k \left( x_{u_i}^*, x_{-u_i}^* \right)
\end{align*}
\]

for every strategy \( x_{u_i}^* \).

It is notable that MCPNE, ACPNE and ICPNE are totally different although they seem similar. In MCPNE, players in a coalition select their strategies to maximize the minimal utilities of players in the coalition. In ACPNE, players in a coalition select their strategies to maximize the average utility while do not decrease the minimal utility of players in the same coalition. In ICPNE, however, players in a coalition select their strategies to maximize their own utilities while do not decrease the minimal utility of players in the same coalition.

Obviously, players can select their strategies independently to achieve the above three approximate MMCPNE situations and thus the computations increase linearly to the size of coalition, i.e., \( O(\omega \cdot |\mathcal{C}|) \). Strictly speaking, MCPNE (or ACPNE, ICPNE) is Nash equilibrium of non-cooperative game with a well-connected payoff function, rather than coalition-proof Nash equilibrium of cooperative game. In Section 6, we will show whether it is feasible to consider MCPNE (or ACPNE, ICPNE) as an approximation of MMCPNE through the simulation results.

5. EXISTENCE OF MMCPNE

In this section, we study the existence of Nash equilibria and min-max coalition-proof Nash equilibria in the static cooperative game.

In our model, we assume that \( |U| \cdot k > |C| \), hence the devices have a conflict during the channel allocation process. We first retrospect the work done by Mark Felegyhazi in [3]. The authors study in detail the problem of competitive multi-radio multi-channel allocation in single-hop wireless networks, i.e., \( \mathcal{R} = \emptyset \) or \( U = \emptyset \), and propose the conditions for Nash equilibria as the following theorem.

**Theorem 1:** Assume that \( |S| \cdot k > |C| \). Then a channel allocation \( \mathcal{X}^* \) is a NE iff the following conditions hold:

- \( k_{u_i,c} \leq 1 \) and \( k_{u_i,c} = k \) for any \( u_i \in S, c \in C \)
- \( \delta_{b,c} \leq 1 \) for any \( b, c \in C \)

where \( \delta_{b,c} = k_b - k_c \) denotes the difference of radios number between channel \( b \) and \( c \).

As mentioned in Section 4, the NE of non-cooperative game is not suitable for the multi-hop networks. In the following, we study the existence of MMCPNE for coalition set \( \mathcal{Q} \) in multi-hop networks. For simplicity, we assume that any communication session contains at most 2 hops, i.e., any coalition \( \mathcal{Q} \) contains at most 2 players. Note however, that it can be easily extended to the system in which any session contains more than 2 players.

Similar as Nash equilibria, there exist multiple MMCPNE states in the system. We divide the MMCPNE states into two sets according to theorem 1. We denote the MMCPNE states which satisfy the theorem 1 by MMCPNE-1 and denote the remainder MMCPNE states by MMCPNE-2. We find that the multi-hop links in MMCPNE-1 states always occupy more bandwidth compared with those in MMCPNE-2 states. We show this property as the following proposition.

**Proposition 1:** Assume that there exists a MMCPNE channel allocation \( \mathcal{X} \) with high coalition utility\(^7\) (for the multi-user coalitions), then \( \mathcal{X} \) is a Nash equilibrium, i.e., the conditions of theorem 1 hold.

**Proof.** It is straightforward to see that the first condition in theorem 1 always holds in MMCPNE due to the co-radios interference in device and the selfish nature of players. We validate the second condition in theorem 1 by contradiction. Assume there exist two channels \( b \) and \( c \) such that \( \delta_{b,c} \geq 2 \) in a high coalition utility) MMCPNE strategy \( \mathcal{X} \). We denote the set of individual players in channel \( b \) by \( U_b = \{ u_1, u_2, \ldots \} \), i.e., \( k_{u_i,b} = 1 \), \( \forall u_i \in U_b \). It is obvious that \( k_{u_i,c} = 1 \) otherwise \( u_i \) can improve its payoff by move its radio from channel \( b \) to \( c \). We denote the set of remainder players in channel \( b \) by \( U_{co} = \{ v_1, v_2, \ldots \} \). Similarly, we denote \( U_x \) as the remainder players in channel \( c \) excluding the players in \( U_b \). It is easy to see that \( |U_{co}| - |U_c| = k_b - k_c \geq 2 \), i.e., there exist at least two players \( v_1 \) and \( v_2 \) such that \( v_1 \in U_{co}, v_2 \in U_{co}, v_1 \not\in U_x, v_2 \not\in U_x \). We show this situation as Figure 4. Now suppose that player \( v_1 \) (or \( v_2 \)) moves its radio from channel \( b \) to \( c \), the utility \( U_b \) and \( U_x \) occupied decreases. Thus the utility of \( U_{co} \) increases since the total available bandwidth is constant.

![Figure 4: An example of MMCPNE channel allocation corresponding to Proposition 1.](image)

It is obvious that the coalition with high utility is likely to achieve high date rate. The value of Proposition 1 is that it provides a method to choose the MMCPNE with the high coalition utility\(^8\), i.e., MMCPNE-1. Thus we will focus on the MMCPNE-1 strategies for the remainder of the paper.

We divide the channels in NE channel allocation \( \mathcal{X}^* \) into two sets. We define the set of channels \( C^+ \) with the maximum number of radios, i.e., where any \( b \in C^+ \) has \( k_b = \max_{c \in C} k_c \). We denote the set of the remainder channels by \( C^- \). We denote the number of radios of any channel in \( C^+ \) and \( C^- \) by \( \delta^+ \) and \( \delta^- \) respectively. It is obvious that \( C = C^+ \cup C^- \) and \( \delta^+ = \delta^- = 1 \) according to theorem 1\(^9\).

Although none of the players can unilaterally change its strategy to increase its payoff in NE, it is possible that a player change its strategy to improve the payoff of another player he is in a coalition with, e.g., \( u_1 \) and \( u_2 \) in Figure 3.

\(^7\)Coalition utility is defined as the summation of all members’ utilities in the coalition. High coalition utility is defined as the fact that the coalition can not improve its utility by unilaterally changing its members’ strategies.

\(^9\)Note that the second equation holds when \( |C^+| > 0 \), otherwise \( \delta^- \) is meaningless.
Players in a coalition can help each other in two ways. The first possibility is that a player relocates its radios to improve the payoff of other when two players share any channels.

**Lemma 2:** Assume that there exists a coalition $c_0 = \{u_1, u_2\}$ and $R_{u_1}^i \neq R_{u_2}^i$ in a NE channel allocation $X$. If there exist two channels $c_1 \in C^+$ and $c_2 \in C^-$ such that $k_{u_1,c_1} = 1, \forall i$ and $k_{u_2,c_2} = 0, \forall i$, then $X$ is not MMCPNE.

*Proof.* Without loss of generality, we assume $R_{u_1}^i > R_{u_2}^i$ and thus the actual date rate $R_{u_1}^i(X) = \min\{R_{u_1}^i, R_{u_2}^i\} = R_{u_2}^i$. Suppose that $u_1$ moves its radio from channel $c_1$ to $c_2$, the rate of $u_1$ does not change whereas the rate of $u_2$ change to $R_{u_2}^i + 1/\delta^- - 1/\delta^+$. We can write the new rate as $R_{u_1}^i(X') = \min\{R_{u_1}^i, R_{u_2}^i + 1/\delta^- - 1/\delta^+\} > R_{u_2}^i$ since $\delta^- = \delta^+ + 1$. So we declare that $X$ is not MMCPNE. □

An example of any NE channel allocation corresponding to Lemma 2 is shown in Figure 5, where $|C| = 6, k = 4$ and $u_1, u_2$ formulate a coalition. According to Lemma 2, it cannot be a MMCPNE, since we can increase the actual date rates of $u_1$ and $u_2$ by moving $u_1$ from channel $c_4$ to $c_6$.

**Figure 5:** An example of a NE channel allocation corresponding to Lemma 2.

In some cases the assumption of unequal payoffs of two players, i.e., $R_{u_1}^i \neq R_{u_2}^i$, might not hold. In such cases, the Lemma 2 may no longer hold. Thus we show another necessary condition as follows.

**Lemma 3:** If there exists a coalition $c_0 = \{u_1, u_2\}$ and multiple channels $\{x_1, x_2, \ldots\} \in C^+$ and $\{y_1, y_2, \ldots\} \in C^-$ such that $k_{u_1,x_1} = 1, \forall i, j$ whereas $k_{u_2,y_1} = 0, \forall i, j$ in a NE channel allocation $X$, then $X$ is not MMCPNE.

*Proof.* Suppose that $u_1$ moves its radio in channel $x_1$ to $y_1$ and $u_2$ moves its radio in channel $x_2$ to $y_2$, the payoffs of player $u_1$ and $u_2$ both increase, and thus the minimal payoff (i.e., actual date rate) increases. □

The second possibility for coalition members helping each other is that they mutually exchange some radios with each other. We show this necessary condition as the following lemma.

**Lemma 4:** Assume that there exists a coalition $c_0 = \{u_1, u_2\}$ and $R_{u_1}^i - R_{u_2}^i > (1/\delta^- - 1/\delta^+)$ in a NE channel allocation $X$. If there exist two channels $c_1 \in C^+$ and $c_2 \in C^-$ such that $k_{u_1,c_1} = 1$ and $k_{u_2,c_1} = 0$ whereas $k_{u_1,c_2} = 0$ and $k_{u_2,c_2} = 1$, then $X$ is not MMCPNE.

We can prove the lemma by exchanging their radios in channel $c_1$ and $c_2$. Due to space limitation, we do not present the detail proof. We show an example of any NE channel allocation corresponding to Lemma 4 in Figure 6, where $|C| = 6, k = 3$ and $u_1, u_2$ formulate a coalition. According to Lemma 4, it cannot be a MMCPNE, since we can increase the actual date rates of $u_1$ and $u_2$ by exchanging their radios in channel $c_3$ and $c_5$.

**Figure 6:** An example of a NE channel allocation corresponding to Lemma 4.

We divide the radios of any player $u_i$ in a NE channel allocation $X^*$ into two sets. We denote the number of radios deployed in $C^+$ by $k_{u_i}^+$. Similarly, we denote the number of radios deployed in $C^-$ by $k_{u_i}^-$. In Figure 6, $k_{u_1}^+ = 3, k_{u_1}^- = 0, k_{u_2}^+ = 2$ and $k_{u_2}^- = 2$. Now we can extend Lemma 4 to more general situation.

**Lemma 5:** Assume that there exists a coalition $c_0 = \{u_1, u_2\}$ and $|k_{u_1}^+ - k_{u_2}^+| > 1$ in a NE channel allocation $X$, then $X$ is not MMCPNE.

*Proof.* Without loss of generality, we assume that $k_{u_1}^+ > k_{u_2}^+$. As mentioned above, any player cannot use multiple radios in the same channel, thus there exists at least one channel $c_1 \in C^+$ such that $k_{u_1,c_1} = 1$ and $k_{u_2,c_1} = 0$. Similarly, there exists at least one channel $c_2 \in C^-$ such that $k_{u_1,c_2} = 0$ and $k_{u_2,c_2} = 1$. Furthermore, we can write the utilities of two players as $R_{u_1}^i - R_{u_2}^i = k_{u_1}^+ - k_{u_1}^- + k_{u_2}^- - k_{u_2}^+$ and $R_{u_2}^i - R_{u_1}^i = k_{u_2}^+ - k_{u_2}^- + k_{u_1}^- - k_{u_1}^+$ respectively. Note that $k_{u_1}^+ + k_{u_1}^- = k$, thus we can write the utility difference of two players as:

$$R_{u_2}^i - R_{u_1}^i = \frac{k_{u_1}^+ - k_{u_1}^-}{1/\delta^+} + \frac{k_{u_2}^- - k_{u_2}^+}{1/\delta^-}$$

Using the conditions of the lemma, we can find that $R_{u_2}^i - R_{u_1}^i > (1/\delta^- - 1/\delta^+)$. Hence, the two conditions of Lemma 4 hold, and we achieve the proof directly from Lemma 4. □

>From equation (14), we can easily find that $|k_{u_1}^+ - k_{u_2}^+| > 1$ if $|R_{u_2}^i - R_{u_1}^i| > (1/\delta^- - 1/\delta^+).$ Thus we can immediately rule some restrictions in Lemma 4. We express this property as the following corollary.

**Corollary 6:** If there exists a coalition $c_0 = \{u_1, u_2\}$ and $R_{u_2}^i - R_{u_1}^i > (1/\delta^- - 1/\delta^+)$ in a NE channel allocation $X$, then $X$ is not MMCPNE.

It is notable that lemma 2 and lemma 3 are also available for CPNE state whereas the other lemmas are exclusively used in MMCPNE. Based on the previous lemmas, we prove the necessary conditions that enables a given NE allocation to be MMCPNE and we present it as the following theorem.

**Theorem 2:** Assume that there exists a coalition $c_0 = \{u_1, u_2\}$ and $R_{u_1}^i \geq R_{u_2}^i$ in a NE channel allocation $X$, if $X$ is MMCPNE, the following conditions hold:

- $R_{u_1}^i - R_{u_2}^i \leq (1/\delta^- - 1/\delta^+)$ and
- case 1: if $R_{u_1}^i \neq R_{u_2}^i$ then there does not exist two channels $b \in C^+$ and $c \in C^-$ such that $k_{u_1,b} = k_{u_2,b} = 1$ whereas $k_{u_1,c} = k_{u_2,c} = 0$,
- case 2: if $R_{u_1}^i = R_{u_2}^i$ then there does not exist four channels $\{b_1, b_2\} \in C^+$ and $\{c_1, c_2\} \in C^-$ such that
to converge to an approximate MMCPNE situation.

We could not prove that the conditions in Theorem 2 are sufficient to enable a NE channel allocation to be MMCPNE, neither could we find a counterexample, where the conditions hold and the NE channel allocation is not MMCPNE. Hence, we formulate the following conjecture.

**Conjecture 1:** Assume that there exists a coalition \( c_0 = \{u_1, u_2\} \) and \( R_{u_1} \geq R_{u_2} \) in a NE channel allocation \( X \). If the condition in theorem 2 hold, then \( X \) is MMCPNE. Hence the above conditions are necessary and sufficient conditions.

### 6. CONVERGENCE TO MMCPNE

We have demonstrated the necessary conditions to enable a NE channel allocation to be MMCPNE in Section 5. In this section, we propose a distributed MMCP Algorithm to enable the selfish players to converge to MMCPNE from an arbitrary initial configuration. We divide the algorithm into two stages. In the first stage, the coalitions move their radios to achieve high utility. Thus we call this stage as **inter-link competition** stage. In the second stage, players in the same link mutually adjust their radios to achieve higher date rate. We call this stage as **intra-link improvement** stage.

As mentioned previously, finding MMCPNE strategy may cost much time since we must jointly search the strategies of \( |c_0| \) players in the coalition. To overcome this limitation, we propose an distributed DCP Algorithm to enable the players to converge to an approximate MMCPNE situation.

#### 6.1 MMCP Algorithm

We present the pseudo-code of MMCP algorithm in Table A and B. Part I is the algorithm used in the **inter-link competition** stage. Note that for multi-hop link, the cooperative players move their radios to occupy more bandwidth under the restriction of without worsening the individual bandwidth any player occupied. Part II is the algorithm used in the **intra-link improvement** stage. For single-hop link, the player does nothing in this stage. For multi-hop link, the players in the link adjust their radios according to lemma 2 to 5 to improve their actual date rates. It is notable that the two stages are time overlapping.

To avoid the unstable channel allocations caused by simultaneously moving of different players, we use the technique of backoff mechanism well known in the IEEE 802.11 medium access technology similarly as [3]. We denote the backoff window by \( W \) and each coalition chooses a random initial value for his backoff counter with uniform probability from the set \( \{1, ..., W\} \).

#### 6.2 DCP Algorithm

In order to reduce the large computation of MMCP algorithm due to the mutual operation of \( |c_0| \) players, we propose a distributed low complexity algorithm, denoted by **DCP Algorithm**, to enable the selfish players to converge to an approximate MMCPNE situation. By transforming the mutual operation of \( |c_0| \) players into multiple independent operations of the players, DCP algorithm efficiently reduces the computational complexity, specifically, from exponentially increasing with \( |c_0| \) to linear increasing with \( |c_0| \).

We denote the DCP algorithm derived from definition 4 by **DCP-M Algorithm**. In other word, DCP-M algorithm enables the players to converge to MCPNE from an arbitrary initial configuration. We present the pseudo-code of DCP-M Algorithm in Table C. Similarly, we denote the DCP algorithm derived from definition 5 and 6 by **DCP-A Algorithm** and **DCP-I Algorithm** respectively. The processes of DCP-A and DCP-I algorithm are same as DCP-M algorithm except the rules of reorganizing the radios, i.e., line 7 to 15 in DCP-M algorithm. We do not present the detail pseudo-code of DCP-A and DCP-I algorithm due to space limitations.

### 7. SIMULATION RESULTS

We implemented the previous algorithms in MATLAB. In each simulation, we assume there exist 8 orthogonal channels, i.e., \(|C|=8\). We further assume that the system contains a two-hop link and multiple single-hop links, as shown in Figure 1, and we denote the coalition corresponding to the two-hop link by \( c_0 \).

We will focus on the performance of the multi-hop link.
C. DCP-M Algorithm

1: random channel allocation
2: while not in a MCPNE do
3: get the current channel allocation
4: for \( i = 1 \) to \( |U| \) do
5: if backoff counter is 0 then
6: assume that user \( u_i \) belongs to coalition \( co \)
7: \( \Omega \) = \{channel allocation\s\} according to Def.4:
8: for \( j = 1 \) to \( k \) do
9: assume that radio \( j \) uses channel \( b \)
10: \( \Omega := \{c|c \in C, k_{u_i,c} = 0\} \)
11: for \( m = 1 \) to \( |\Omega| \) do
12: assume the \( m_{\Omega} \) element in \( \Omega \) is \( c \)
13: suppose that move radio \( j \) from \( b \) to \( c \) and
14: record \( \mu_c = \min_{u_i \in co_j}(R_{ui}^c) \)
15: end for
16: move the radio \( j \) from \( b \) to channel \( a \) where \( a = \arg \max_{c \in \Omega} \mu_c \)
17: reset the backoff counter to a new value from the
18: set \( \{1, ..., W\} \)
19: else
20: decrease the backoff counter value by one
21: end if
22: end while

(i.e., the coalition \( co \)) in our simulation since there is no cooperative gain in the single-hop link. We first introduce three criterions, i.e., coalition utility, coalition efficiency and coalition usage factor, to evaluate the performance of links in the system and we present the concepts of them as follows.

- **Coalition Utility**: the coalition utility of any coalition \( co \) is defined as the ratio of the total bandwidth \( co \) occupied to the average bandwidth per user, denoted by \( \varphi_{co_i} = \frac{\sum_{e \in co_i}(R_{ei}^c)}{|C|/|U|} \).

- **Coalition Usage Factor**: the coalition usage factor of any coalition \( co \) is defined as the ratio of the achieved data rate to the total bandwidth \( co \) occupied, denoted by \( \tau_{co_i} = \frac{\min_{u_i \in co_j}(R_{ui}^c)}{\sum_{e \in co_i}(R_{ei}^c)} \).

- **Coalition Efficiency**: the coalition efficiency of any coalition \( co \) is defined as the product of coalition utility and usage factor, denoted by \( \phi_{co_i} = \varphi_{co_i} \times \tau_{co_i} \).

The criterion of coalition utility reflects the ability of any coalition to allocate bandwidth. The criterion of coalition usage factor is used to measure the usage ratio of total bandwidth \( co \) occupied. We can easily find that \( 1 \geq \tau_{co_i} \geq 1/|co| \) and \( 1/|co| \geq \tau_{co_i} \geq 1/|co| \). If \( \tau_{co_i} \neq 1/|co| \), the total bandwidth \( co \) occupied is not fully used, i.e., any bandwidth wasted. From Lemma 5 in Section 5, players in the same coalition tend to achieve the same utility in a MMCPNE channel allocation, and thus \( \tau_{co_i} \) is close to its up-bound, i.e., \( 1/|co| \). Furthermore, the criterion of coalition efficiency allows us to define the ability of a coalition to achieve a given data rate. It is easy to see that \( \phi_{co_i} = \frac{\min_{u_i \in co_j}(R_{ui}^c)}{|C|/|U|} \), i.e., the ratio of the end-to-end rate of any link to the average bandwidth per user.

We define **average coalition utility** as the average of coalition utility per round over a long period of time. Similarly, we define **average coalition usage factor** and **average coalition efficiency** as the average of coalition usage factor and efficiency respectively. We also introduce the notion of **Efficiency Ratio** defined in [3] to valid whether a channel allocation is Nash equilibrium. Due to space limitation, we do not present the definition in detail.

We assume that the duration of one round in the updating algorithm is 10ms. This duration of one round corresponds roughly to the time needed for all these devices to transmit one MAC layer packet, i.e., the time that the devices can learn about other devices in the channel. We run each simulation for 600 rounds, which corresponds to 6s according to the assumption above. Each average value is the result of 1000 simulation runs.

We firstly investigate the **efficiency ratio** of all algorithms. Figure 7 presents the simulation results in terms of efficiency ratio, where \( W = 15, |C| = 8, k = 4, |U| = 5 \) and players \( u_1 \) and \( u_2 \) formulate a coalition. From Figure 7, we find that MMCP, DCP-M, DCP-A and DCP-I algorithms all converge to the Nash equilibrium, i.e., their efficiency ratios converge to one.

Next, we present the simulation results of **coalition utility** of \( co \) in Figure 8. Note that the NE algorithm in the figure is the algorithm in [3], which enables the players converge to any Nash equilibrium from an arbitrary initial configuration. The CPNE algorithm is a algorithm which enables the players converge to conventional CPNE state. We do not present the pseudo-code of CPNE due to space limitation. From Figure 8, we find that MMCP, CPNE and DCP-A algorithms show higher coalition utility compared with other algorithms, specifically, the curves of MMCP and CPNE are almost overlapped. In other word, the coalition \( co_\alpha \) tends to occupy more bandwidth in the state of MMCPNE, CPNE and ACPNE. This phenomenon in MMCPNE (or CPNE) can be seen as the results of Lemma 2 and 3. In ACPNE, this phenomenon is caused by the fact that all players in the coalition are willing to improve the total bandwidth. The coalition utility of CPNE is a little higher than DCP-A due to the cooperation gain.

As mentioned above, the criterion of coalition utility cannot reflect the ability of a coalition to achieve a given data rate. We show this phenomenon by CPNE and DCP-A algorithms in Figure 9. We present the simulation results.
of coalition efficiency of $c_x$ in Figure 9, which exactly reflects the achieved data rate. We can observe that CPNE and DCP-A algorithms converge with low coalition efficiency (i.e., low data rate). However, MMCP algorithm still shows highest coalition efficiency. The NE and DCP-I algorithms show lowest performance in terms of both coalition utility and efficiency in the multi-hop networks.

Then we present the simulation results of coalition usage factor of $c_x$ in Figure 10. We find that MMCP algorithm converges to its up-bound, i.e., $\tau_{c_x} \approx 1/|c_x| = 0.5$. CPNE and DCP-A algorithms show low coalition usage factor because they occupy high total bandwidth while the available bandwidth is low. It is well coincident with former explanation. We find that NE algorithm also shows low coalition usage factor, specifically, the performance of NE is closely to DCP-A.

From Figure 8 to 10, we find that DCP-M algorithm and MMCP algorithm show tiny performance difference, e.g., less than 1% in terms of coalition usage factor and 5% in terms of coalition efficiency. The DCP-A algorithm and MMCP algorithm show large performance difference in terms of the coalition usage factor due to the large total bandwidth DCP-A occupied and low bandwidth it available used. The DCP-I algorithm and MMCP algorithm show large performance difference in terms of the coalition efficiency due to the low total bandwidth DCP-I occupied.

Finally, we present the effect of number of players on coalition usage factor in Figure 11. We can see that MMCP algorithm always keeps the system in a state of high coalition usage factor whereas CPNE, NE and DCP-A algorithms show low usage factor in most case. It is interesting that all algorithms converge to the same value when total players number $|U| = 6$ or 8. It is due to the fact that all channels are shared by the same number of radios, and thus the NE state is equivalent with MMCPNE (or ACPNE, MCPNE, ICPNE) state.

In summary, we can observe that, the proposed MMCP algorithm based on MMCPNE ensures high performance in terms of coalition efficiency and usage factor due to cooperation gain. NE algorithm proposed in [3] shows low performance in all terms in multi-hop system. Furthermore, we find that MCPNE can be seen as a feasible approximation of MMCPNE while ACPNE and ICPNE show poor per-
formance in terms of coalition efficiency and usage factor respectively compared with MMCPNE.

8. CONCLUSION

In this paper, we have studied the problem of competitive channel allocation among devices which use multiple radios in the multi-hop system. We first analyze that NE and CPNE channel allocation schemes cannot work in multi-hop networks due to the poor performance of achieved date rate of the multi-hop links. Then we propose a novel coalition-proof Nash equilibrium, denoted by MMCPNE, to ensure the multi-hop links to achieve high date rate without worsening the date rates of single-hop links. We investigate the existence of MMCPNE and propose the necessary conditions for the existence of MMCPNE. Finally, we provide several algorithms to achieve the exact and approximate MMCPNE states. We study their convergence properties theoretically. Simulation results show that MMCPNE outperforms CPNE and NE schemes in terms of achieved date rates of links due to cooperation gain.

9. ACKNOWLEDGEMENTS

We appreciate the valuable comments and constructive criticism from the anonymous reviewers. This work is supported by NSF China (No. 60702046); China Ministration of Education (No. 20070248095);

10. REFERENCES