Mathematical and Computational Applications, Vol. 10, No. 1, pp. 71-77, 2005.

MODIFIED LSM FOR CALIBRATION OF MEASURING APPARATUS

Chingiz Hajiyev
Faculty of Aeronautics and Astronautics, Istanbul Technical University
34469, Maslak, Istanbul, Turkey. cingiz@itu.edu.tr

Abstract - Modified least squares method (LSM) is developed that take into account errors in input standard signals for calibration of measuring devices. Calibration of the differential pressure gage by the standard control-point setting mechanisms is demonstrated in the paper as an example. The accuracy of the coefficient estimates changes when the errors of the control-point setting mechanisms are taken into account. Using the LSM in the classic form for the mentioned case gives wrong estimation errors in the final estimations of calibration curve coefficients. Therefore, for estimation of errors, characteristics of fractional errors are proposed use and as a variance of estimations errors guarantee the stability of the final estimation values.

Keywords - Least Squares Method, Calibration Characteristics, Differential Pressure Gage, Fractional Error

1 . INTRODUCTION

In practice, the least squares method (LSM) is most widely used for calibration of the measuring devices. However, there are several essential limitations of the LSM application [1-3] . During the formulation of the approximation $y = f(x)$, arguments can contain some errors in nature and due this fact LSM can produce additional errors and gives some bias in the estimations and , that more important, brings incorrect estimates of there errors.

Usually, the calibration of any measuring device is made by the help of etalon standard measuring instrument. However, each standard instrument reproduces signals with some definite errors (even though they are very small). Studies have shown that if these errors are not taken into account during calibration process, the final results will contain several additional errors.

In this study, some new algorithms are developed that take into account the errors of the standard instrument. For this purpose, the modified LSM is developed.

2. CALIBRATION OF MEASURING DEVICES WITH REGARDING INPUT SIGNALS ERRORS

2.1. Problem Statement

1) Calibration characteristics of a measuring device is described by m order polynomial as follows:

$$y(p) = a_0 + a_1 p + a_2 p^2 + ... + a_m p^m , \tag{1}$$

2) It is assumed, that the standard instrument used for calibration also have some small errors.
3) Measurements contain random Gaussian noise (δ_{yi}) with a zero mean

$$z_i = a_0 + a_1 p_i + a_2 p_i^2 + ... + a_m p^m + \delta_{yi} . \tag{2}$$

By taking into account all the above mentioned, it is required to develop special algorithms for the identification of a suitable calibration equation.

2.2. Algorithm of Solution

The coefficients in these polynomials are already evaluated in [4] by the least squares method . The expressions used to make the evaluation had the form:

$$\hat{\theta} = (X^T X)^{-1}(X^T z) \quad ; \tag{3}$$

$$D(\hat{\theta}) = (X^T X)^{-1} \sigma^2 , \tag{4}$$

where $\theta^T = [a_0 , a_1,..., a_m]$ is (mx1) dimensional vector of the estimate of the sought coefficients; $z^T = [z_1 , z_2 , ... , z_n]$ is (nx1) dimensional vector of the measurements; σ^2 is the variance of measurement error; and

$$X = \begin{bmatrix} 1 & p_1 & p_1^2 & . & . & . & p_1^m \\ 1 & p_2 & p_2^2 & . & . & . & p_2^m \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 1 & p_n & p_n^2 & . & . & . & p_n^m \end{bmatrix}$$

is [nx(n+1)] dimensional matrix of the known coordinates (here, $p_1 , p_2 ,..., p_n$ are values that are reproducible by the standard instruments).

In the use of the LSM to solve the above problem, it is assumed that the values of the arguments p_i are known exactly. However, since the arguments p_i that can be reproduced by the standard instruments are reproduced with an error, use of the LSM could produce inaccurate (biased) results. Specifically, the errors might be incorrectly evaluated.

To analyse the data in this case, it is preferable to use the modified LSM. With allowance for the error of the argument δ_{pi} ,we write Eqs. (1) and (2) in the implicit form as

$$y_i = f(p_i + \delta_{pi}), \tag{5}$$

and

$$z_i = f(p_i + \delta_{pi}) + \delta_{yi} . \tag{6}$$

respectively. Since the p_i values are nonrandom (i.e. , since we are examining a problem involving a factorial experiment), we can use the LSM to evaluate function (5). However, the characteristics of the fractional errors should be used to estimate the errors obtained from the evaluation.

After expanding nonlinear relation (1) into a Taylor series (and limit ourselves to terms of the first order in the expansion), we represent measurement equation (2) in the following form

$$z_i = a_0 + a_1 p_i + a_2 p_i^2 + ... + a_m p_i^m + \delta_{fi} \quad , \tag{7}$$

$$\delta_{fi} = \delta_{yi} + \delta_{pi}^* \tag{8}$$

$$D_{\delta fi} = D_{\delta yi} + D_{\delta yi}^* \tag{9}$$

$$D_{\delta pi}^* = a_1^2 D_{\delta pi} + (2a_2 p_i)^2 D_{\delta pi} + (3a\ 3p_i^2)^2 D_{\delta pi} + ... +(a_m m p_i^{m-1})^2 D_{\delta pi} \tag{10}$$

where δ_{fi} are the fractional errors due to the errors of measurements of the function and the argument; δ_{pi}^* is the error of the function y_i due to the error of reproduction of the argument p_i ; $D_{\delta fi}$, $D_{\delta yi}$ and $D_{\delta pi}^*$ are the variances of the errors δ_{fi} , δ_{yi} and δ_{pi}^*, respectively.

Inserting the values of the corresponding variances into (9), we obtain

$$D_{\delta fi} = \sigma_i^2 + a_1^2 D_{\delta pi} + 4a_2^2 p_i^2 D_{\delta pi} + ... + (a_m\, m\, p_i^{m-1})^2 D_{\delta pi} \quad (11)$$

Since variances are used as criteria of accuracy, we can take the following as a guaranteed estimate of the errors in y due to the errors of the signals reproduced by the standard instruments,

$$D_{\delta pi}^* = max_{\,pi}\{ a_1^2 D_{\delta pi} + 4a_2^2 p_i^2 D_{\delta pi} + ... + (a_m\, m\, p_i^{m-1})^2 D_{\delta pi} \} \quad 0\le p_i\le p_{max} \quad (12)$$

where p_{max} is the maximum value of the signals that can be reproduced by the standard instruments.

It is evident from (12) that the maximum value of $D_{\delta pi}^*$ is obtained at $p_i = p_{max}$. Thus, when the LSM is used in calibration problems of devices, the errors of the results should be evaluated by using fractional error characteristics. In this case, the guaranteed value of the variance will be determined by the following expression

$$D(\hat{\theta}) \le D_{gar} = (X^TX)^{-1}\{\sigma^2 + a_1^2 D_{\delta p} + 4a_2^2 p_{max}^2 D_{\delta p} + ... + (a_m m p_{max}^{m-1})^2 D_{\delta p}\} \quad (13)$$

2.3. Experimental Results

Shown below are results of a computer analysis of pressure measurements with and without allowance for the error of the argument of function (5). We used a differential pressure gage with a relative error of 0.5% (measurement range 1600 bar) as the pressure-measuring instrument. The control-point setting mechanisms used had a relative error of 0.05%. Standard pressures of 0, 200, 400, 600, 1000, 12000 and 1600 bar are reproduced using these mechanisms. Pressure gages errors are subjected to Gaussian distribution with zero mean [5]. The calibration characteristics of differential pressure gage is adequately described by a second order polynomial as follows:

$$y(p) = a_0 + a_1 p + a_2 p^2 . \quad (14)$$

The coefficients of polynomial (14) are estimated by the LSM.

The following estimates are obtained for the coefficients of polynomial (14) (the measurements are made at an ambient temperature of 22.6 °C):

$$\hat{a}_0 = 2,0051 ;\ \hat{a}_1 = 4,9118 ;\ \hat{a}_2 = -0,0052.$$

The variances of the estimates are as follows without and with allowance for the error of the argument of function (5):

a) without allowance for the standard pressures errors,

$$D(\hat{a}_0)=0,000006 ;\ D(\hat{a}_1) = 0,000104 ;\ D(\hat{a}_2) = 0,000028 ;$$

b) with allowance for the standard pressures errors,

$$D_{gar}(\hat{a}_0)=0,0000022 ;\ D_{gar}(\hat{a}_1) = 0,000129 ;\ D_{gar}(\hat{a}_2) = 0,000035 .$$

As it is seen from the results, the accuracy of the coefficient estimates changes when the errors of the control-point setting mechanisms are taken into account. Consideration must be given to the expediency of using modified LSM for calibration problems of the measuring devices.

3. RECURSIVE FORM OF THE LSM ACCOUNTING FOR ERRORS OF THE INPUT VARIABLES

As is known, recursive algorithms make it unnecessary to store the entire volume of measurement data in a problem. Instead, test parameter estimates are computed as new measurements are made. In some cases, this feature makes recursive algorithms more suited for arriving at quick results.

To obtain a recursive variant of the LSM, we represent (1) in vector form as,

$$y_i = x_i^T\theta \;,\; i = 1,2, \dots , n \tag{15}$$

where $x_i^T = [1, p_i, p_i^2, \dots , p_i^m]$ is the vector of the input variables [the argument of function (1)]; $\theta^T = [a_0, a_1, a_2, \dots , a_m]$ is the vector of the unknown parameters.

The output signal of the object y_i is recorded by a measuring instrument

$$z_i = y_i + \delta_{yi}\,, \tag{16}$$

where δ_{yi} has the same meaning as in the previous section.

The recursive form of the LSM that will be used to evaluate the vector of the system parameters (15) is as follows [6]:

$$\begin{aligned} \hat\theta_i &= \hat\theta_{i-1} + K_i\,(z_i - x_i^T\hat\theta_{i-1})\;, \\ K_i &= (p_{i-1}^{*}x_i)/(\sigma^2 + x_i^T p_{i-1}^{*}x_i) \\ p_i^{*} &= p_{i-1}^{*} - (p_{i-1}^{*}x_i x_i^T p_{i-1}^{*})/(\sigma^2 + x_i^T p_{i-1}^{*}x_i) \end{aligned} \tag{17}$$

where K_i is the gain of the filter being examined; p_i^{*} is the covariant matrix of the errors of the estimates.

Algorithm (17) does not account for the errors of the input variables. However, as shown above, the input variables are created by the standard instruments, which do have certain reproduction errors. Ignorance of these errors might lead to significant distortion of the parameter estimates, i.e. the estimates may turn out to be invalid due to bias.

The LSM can be used to estimate the parameters in Eq.(15), but fractional error characteristics should be used in any such attempt.

We find these characteristics by proceeding in a manner analogous to (8-10). When vectors are used, the corresponding expressions will have the following form:

$$\delta_{fi} = \delta_{yi} + \delta_{pi}^{*}\;; \qquad D_{\delta pi}^{*} = \theta^T D_{xi}\theta\;;$$

$$D_{xi} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & D_{\delta pi} & 0 & \dots & 0 \\ 0 & 0 & 4p_i^2 D_{\delta pi} & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & (mp_i^{m-1})^2 D_{\delta pi} \end{bmatrix} \tag{18}$$

$$D_{xi}(jj) = (\partial x_i(j)/\partial p_i)^2 D_{\delta pi} \quad \text{and} \quad D_{xi}(jk) = 0, \;\text{ if }\; j \neq k; \;\; D_{\delta fi} = \sigma_i^2 + \theta^T D_{xi}\theta$$

Insertion of the fractional error variances (18) into algorithm (17) instead of σ_i^2, leads to a new algorithm for evaluating the parameters of system (15) with allowance in the errors of the input variables:

$$\widehat{\theta}_i = \widehat{\theta}_{i-1} + K_i\,(y_i - x_i^T \widehat{\theta}_{i-1})\ ,$$
$$K_i = (p_{i-1}^{\ *} x_i)/(\sigma^2 + \widehat{\theta}_{i-1}^{\ T} D_{xi} \widehat{\theta}_{i-1} + x_i^T p_{i-1}^{\ *} x_i) \tag{19}$$
$$p_i^{\ *} = p_{i-1}^{\ *} - (p_{i-1}^{\ *} x_i x_i^T p_{i-1}^{\ *})/(\sigma^2 + \widehat{\theta}_{i-1}^{\ T} D_{xi} \widehat{\theta}_{i-1} + x_i^T p_{i-1}^{\ *} x_i)$$

3.1. An Illustrative Example

Consider the calibration problems, which have just been brought in the previous section for the differential pressure gage by standard pressure control-point setting mechanisms. Initial data and the values of the reproduced standard signals are taken as the same. To calibrate the differential pressure gage by the algorithm (19), the equation of calibration characteristics (14) cen be represented in the following vector form as

$$y_i = x_i^T \theta\ ,\quad i = 1,2,\ldots,n \tag{20}$$

where $x_i^T = [1, p_i, p_i^2]$ is the input signals vector; $\theta^T = [a_0, a_1, a_2]$ is the vector of the coefficient estimations.

Having in mind for the given case that

$$D_{xi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & D_{\delta pi} & 0 \\ 0 & 0 & 4p_i^2 D_{\delta pi} \end{bmatrix},$$

where D_{pi} is the variance of the reproduction error for standard pressure setters, estimation algorithm of (19) is applied.

During the calculations of the initial conditions, the following data are taken:

$$\sigma_{pi} = 0{,}00026\ ;\quad \sigma_i = 0{,}0026$$

The range of changing sample pressures is $0 \le p_i \le 1600$ *bar(0; 200; 400; 600; 1000; 1200; 1600 bar).*

The covariant matrix for input standard signal errors is

$$D_{xi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & (0{,}00026)^2 & 0 \\ 0 & 0 & 4p_i^2 (0{,}00026)^2 \end{bmatrix}.$$

As initial conditions, the following values $\theta^T = [1\quad 1\quad 1]$ are chosen.

$$p_0^* = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}.$$

The coefficients a_0, a_1 and a_2 found by estimation via algorithm (19), are given in Fig.1, and their errors variances in Fig.2 . The curves in Fig.1 are characterized behaviors of values $\hat{a}_0$, $\hat{a}_1$, $\hat{a}_2$ from iteration number. As it can be seen from the curves after some iterations, the deviations of the investigated values are very small. It proves the high convergence of the algorithm.

As it shown in Fig.2 the variance values decrease very fast and after the 2 nd iteration $D(\hat{a}_0)$, 3 ed iteration $D(\hat{a}_1)$ and 4 th iteration $D(\hat{a}_2)$ approaches to zero. It shows the high accuracy and fast convergence of the suggested algorithm.

The calculations are run on the IBM PC/AT computer. The time spent on every iteration is less than 0,02 second.

Thus, the results of the calculations by the personal computer have confirmed practical applicability of the suggested algorithm for calibration of measuring devices and have shown its high execution ability.

4. CONCLUSION

When calibrating measurement devices with a view to estimate coefficients of calibration characteristics, it is worthy to use modified LSM, taking into account argument errors of the measured function (in our case errors of standard input signals).

Using the LSM in the classic form for the mentioned case gives wrong estimation errors in the final estimations of calibration curve coefficients. Therefore, for estimation of errors, characteristics of fractional errors should be used and as a variance of estimations errors guarantee the stability of the final estimation values.

5. REFERENCES

1. R. W. Farebrother, *Linear Least Squares Computations*, Marcel Dekker, Inc., New York, 1988.
2. V. A. Granovskii and T. N. Siraya, *Methods of Processing Experimental Data in Measurements,* Energoatomizdat, Leningrad, 1990 (in Russian).
3. J. M. Mendel, *Lessons in Estimation Theory for Signal Processing, Communications, and Control,* Prentice-Hall, Inc., New Jersey, 1995.
4. A. A. Abdullaev and Ch. M. Gadzhiev (Hajiyev), Metrological support to data-acquisition systems for oil-product accounting, *Measurement Techniques* **36**, No. 9, 977-981, 1993.
5. Ch. M. Hajiyev and M. M. Isayev, *Measurement Processing Methods in Information-Measurement Systems of the Determination of Oil-Product Mass in Tanks*, Elm, Baku, 2000 (in Russian).
6. L. Ljung, *System Identification. Theory for the User,* Prentice-Hall, Inc., Sweden, 1987.

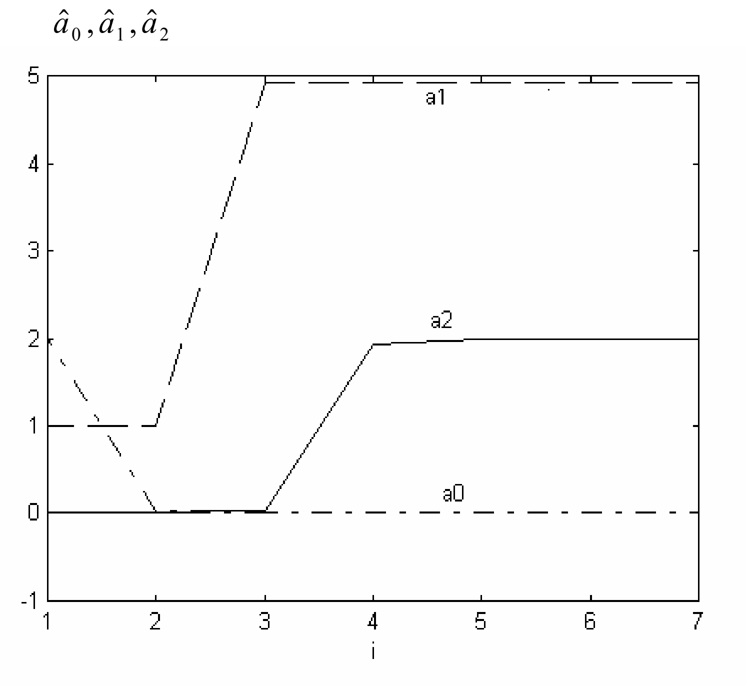

Fig. 1. $\hat{a}_0, \hat{a}_1, \hat{a}_2$ coefficients values behaviours

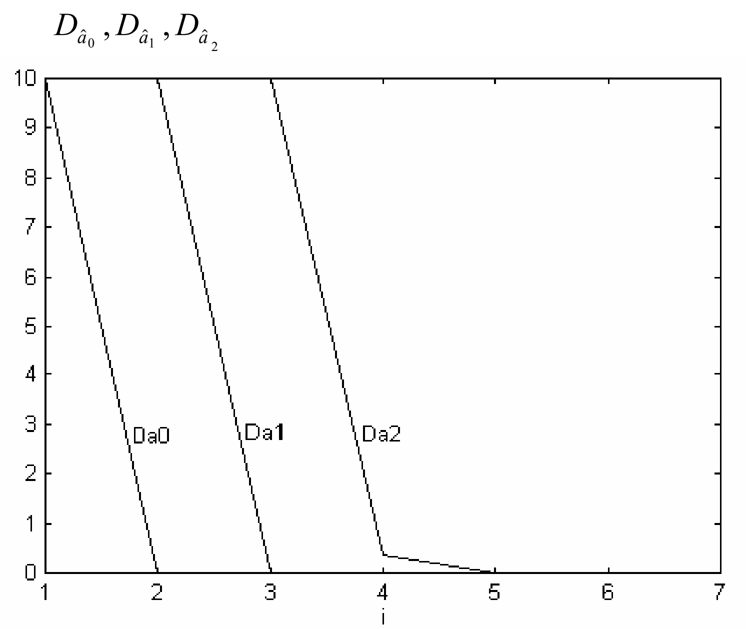

Fig. 2. Variances of the errors value $D_{\hat{a}_0}, D_{\hat{a}_1}, D_{\hat{a}_2}$