

Sturm-Liouville Problems Whose Leading Coefficient Function Changes Sign

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Abstract. For a given Sturm-Liouville equation whose leading coefficient function changes sign, we establish inequalities among the eigenvalues for any coupled self-adjoint boundary condition and those for two corresponding separated self-adjoint boundary conditions. By a recent result of Binding and Volkmer, the eigenvalues (unbounded from both below and above) for a separated self-adjoint boundary condition can be numbered in terms of the Prüfer angle; and our inequalities can then be used to index the eigenvalues for any coupled self-adjoint boundary condition. Under this indexing scheme, we determine the discontinuities of each eigenvalue as a function on the space of such Sturm-Liouville problems, and its range as a function on the space of self-adjoint boundary conditions. We also relate this indexing scheme to the number of zeros of eigenfunctions. In addition, we characterize the discontinuities of each eigenvalue under a different indexing scheme.

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