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Zero-Power Control of Parallel Magnetic Suspension Systems*

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Abstract

This paper discusses the feasibility of zero-power control in parallel magnetic suspension systems. In parallel magnetic suspension systems, multiple floators are suspend with a single power amplifier. The zero-power control has been used in magnetic suspension systems with permanent magnets providing bias flux. This control realizes the steady states where the attractive force produced by the permanent magnets balances the weight of the suspended object and the electromagnet coil current converges to zero. This paper shows that parallel magnetic suspension systems incorporating zero-power control can assign the poles arbitrary if the original parallel suspension systems are controllable. The feasibility of the zero-power control in a parallel magnetic suspension is demonstrated experimentally.

Key words: Magnetic Suspension, Magnetic Bearing, Parallel Suspension, Zero-Power Control, Controllability

1. Introduction

A basic magnetic suspension system has a single floator (object to be suspended) and electromagnet that is controlled with a single power amplifier [(1)]. In multiple magnetic suspension, multiple floators are controlled with a single power amplifier [(2)]. Multiple magnetis suspensions are classified into series and parallel. This paper focus on the latter. The controllability and observability of parallel magnetic suspension systems have been studied and the conditions under which the parallel magnetic suspension system are controllable and observable are clarified. The feasibility of parallel magnetic suspension has been also demonstrated experimentally [(3)].

This paper discusses on the applicability of zero-power control to the parallel magnetic suspension systems. The zero-power control has been used in magnetic suspension systems with permanent magnets providing bias flux [(3)-(7)]. This control realizes the steady states where the attractive force produced by the permanent magnets balances the weight of the floator and the electromagnet coil current converges to zero. Since there is no steady energy consumption for achieving stable levitation, it is used in the space environment [(4)-(6)], and magnetically levitated carrier systems in clean rooms [(7)]. A unique characteristic of the zero-power control system is that it behaves as if it has a negative stiffness; when an external force is applied to the floator in zero-power controlled systems, the floator moves to a new equilibrium position located in the direction opposite to the applied force. A novel vibration isolation system using this property is proposed which has high stiffness against direct disturbance [(8), (9)].

This paper shows that parallel magnetic suspension systems incorporating zero-power

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control can assign the poles arbitrary if the original parallel suspension systems are controllable. The feasibility of the zero-power control in a parallel magnetic suspension is demonstrated experimentally.

2. Basic magnetic suspension system

2.1 State equations

Figure 1 shows a single-degree-of-freedom-of-motion model for analysis. A hybrid magnet consisiting of permanet and electromagnets is used for generating and controlling suspension force. The floator is assumed to move only in the vertical direction translationally, and the effects of eddy-current flowing in the cores are neglected. When x presents the displacement of the mass from the equilibrium position where the attractive force generated by the permanent magnet balances the gravitational force, the equation of motion in the neighborhood of the equilibrium point is given by (10)

$$m\ddot{x}(t) = k_s x(t) + k_i i(t) + w(t)\,, \tag{1}$$

where m: mass of floator, k_s: gap-force coefficient of hybrid magnet, k_i: current-force coefficient of electromagnet, i: current flowing through the coil, w: disturbance acting on the floator.

The electrical circuit equation associated with the electromagnet becomes

$$L\frac{\mathrm{d}i}{\mathrm{d}t} + Ri + k_b\dot{x} = v(t)\,, \tag{2}$$

where L: inductance of coil, R: resistance of coil, k_b: velocity-voltage coefficient, v: voltage applied to the coil.

The magnetic suspension systems are classified into two types according to power amplifier. One uses a power amplifier with current output so that the coil current can be treated as control input and the control system is designed based solely on Eq.(1). This type is called as current-controlled magnetic suspension system. The state equation is given by

$$\dot{\boldsymbol{x}}_c(t) = \boldsymbol{A}_c\boldsymbol{x}_c(t) + \boldsymbol{b}_c i(t) + \boldsymbol{d}_c w(t)\,, \tag{3}$$

where

$$\boldsymbol{x}_c = \begin{bmatrix} x \\ \dot{x} \end{bmatrix},\quad \boldsymbol{A}_c = \begin{bmatrix} 0 & 1 \\ a_{21} & 0 \end{bmatrix},\quad \boldsymbol{b}_c = \begin{bmatrix} 0 \\ b \end{bmatrix},\quad \boldsymbol{d}_c = \begin{bmatrix} 0 \\ d \end{bmatrix},$$

$$a_{21} = \frac{k_s}{m},\quad b = \frac{k_i}{m},\quad d = \frac{1}{m}.$$

The other type uses a power amplifier with voltage output so that the voltage applied to the coil is treated as a control input while the coil current is treated as a state variable; the

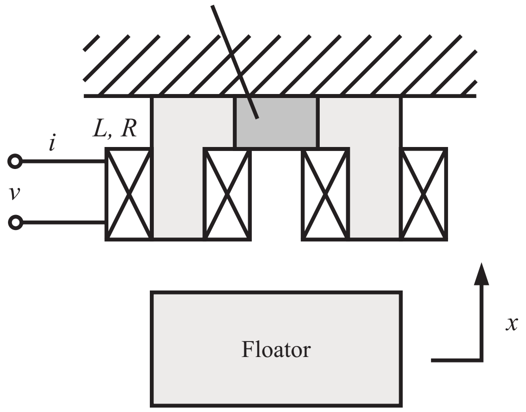

Fig.1 Basic model of magnetic suspension system

control system is designed based on Eqs.(1) and (2). This type is called as voltage-controlled magnetic suspension system and the state equation is given by

$$\dot{\boldsymbol{x}}_v(t) = \boldsymbol{A}_v \boldsymbol{x}_v(t) + \boldsymbol{b}_v v(t) + \boldsymbol{d}_v w(t)\,, \tag{4}$$

where

$$\boldsymbol{x}_v = \begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix},\ \boldsymbol{A}_v = \begin{bmatrix} 0 & 1 & 0 \\ a_{21} & 0 & a_{23} \\ 0 & -a_{32} & -a_{33} \end{bmatrix},\ \boldsymbol{b}_v = \begin{bmatrix} 0 \\ 0 \\ b_0 \end{bmatrix},\ \boldsymbol{d}_v = \begin{bmatrix} 0 \\ d_0 \\ 0 \end{bmatrix},$$

$$a_{23} = \frac{k_i}{m}(=b),\quad a_{32} = \frac{k_b}{L},\quad a_{33} = \frac{R}{L},\quad b_0 = \frac{1}{L},\quad d_0 = \frac{1}{m}.$$

2.2 Basic lemmas

The following conditions are critical for the controllability of each system.

$$b \neq 0 \tag{5}$$

$$a_{23} \neq 0,\quad b_0 \neq 0 \tag{6}$$

These conditions are fairly realistic in a practical point of view.

Lemma 1. *The current-controlled magnetic suspension system whose state equation is given by Eq.(3) is controllable when the condition (5) holds.*

Proof. It is apparent since the controllability matrix $\boldsymbol{C}_c$ is given by

$$\boldsymbol{C}_c = [\boldsymbol{b}_c \quad \boldsymbol{A}_c \boldsymbol{b}_c] = \begin{bmatrix} 0 & b \\ b & 0 \end{bmatrix}. \tag{7}$$

Q.E.D.

Lemma 2. *The voltage-controlled magnetic suspension system whose state equation is given by Eq.(4) is controllable when the condition (6) holds.*

Proof. It is apparent since the controllability matrix $\boldsymbol{C}_v$ is given by

$$\begin{aligned} \boldsymbol{C}_v &= [\boldsymbol{b}_v \quad \boldsymbol{A}_v \boldsymbol{b}_v \quad \boldsymbol{A}_v^2 \boldsymbol{b}_v] \\ &= \begin{bmatrix} 0 & 0 & a_{23} b_0 \\ 0 & a_{23} b_0 & -a_{23} a_{33} b_0 \\ b_0 & -a_{33} b_0 & (a_{33}^2 - a_{32} a_{23}) b_0 \end{bmatrix}. \end{aligned} \tag{8}$$

Q.E.D.

3. Parallel magnetic suspension system

The concept of *parallel magnetic suspension* is illustrated by Figs.2 and 3 in which the multiplicity is three [(2)]. It has three floators and electromagnets energized by a single power

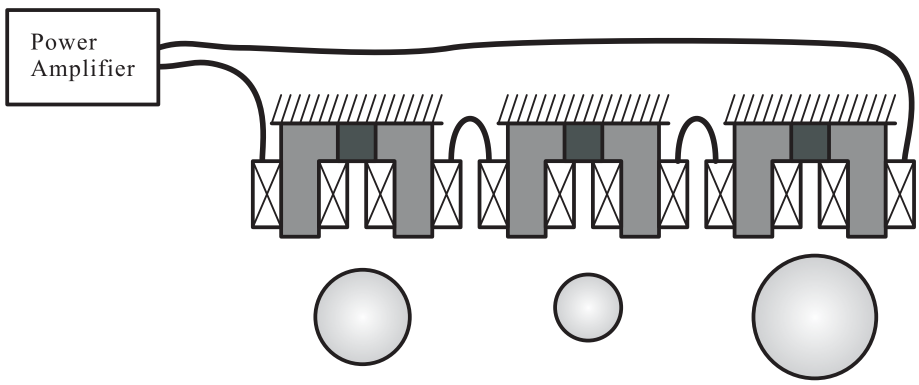

Fig.2 Parallel suspension system with series-connected coils

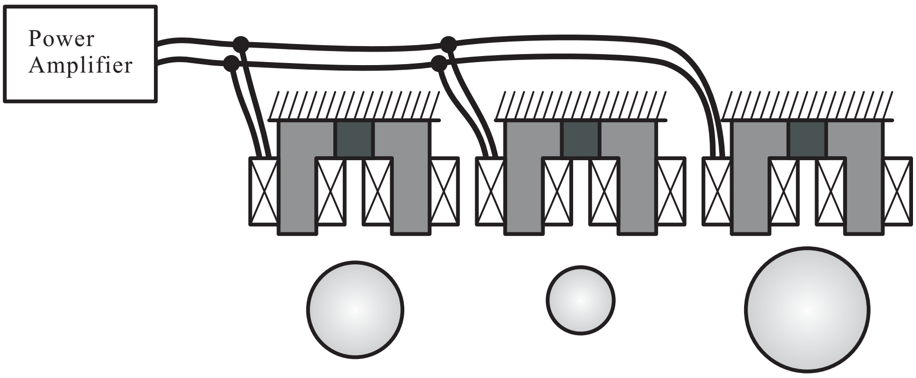

Fig.3 Parallel suspension system with parallel-connected coils

amplifier. In principle, the number of floators and electromagnets controlled by one amplifiler can be set *arbitrary*. The connection of coils is of two types: series (Fig.2) and parallel (Fig.3). In addition, there are two types of amplifier according to its output: current and voltage. In the following, we focus on **series-connected systems with two floators**.

In the following, matrix, vector, variable and parameter related to the kth subsystem consisting of a floator and the corresponding electromagnet is denoted by a parenthetic superscript k, for example, $\boldsymbol{A}^{(k)}$, $\boldsymbol{x}^{(k)}$, $x^{(k)}$ and $a^{(k)}$.

In discussing the controllability of parallel magnetic suspension systems, the following condition is critical, especially in series-connected systems [2].

$$a_{21}^{(i)} \neq a_{21}^{(j)} \quad (i \neq j) . \tag{9}$$

3.1 Current-controlled system with series connection

The state equation of current-controlled system is represented as follows.

$$\dot{\boldsymbol{x}}_{cpt2}(t) = \boldsymbol{A}_{cpt2}\boldsymbol{x}_{cpt2}(t) + \boldsymbol{b}_{cpt2}i(t) + \boldsymbol{D}_{cpt2}\boldsymbol{w}_{cpt2}(t) , \tag{10}$$

where

$$\boldsymbol{x}_{cpt2} = \begin{bmatrix} \boldsymbol{x}_c^{(1)} \\ \boldsymbol{x}_c^{(2)} \end{bmatrix}, \quad \boldsymbol{w}_{cpt2} = \begin{bmatrix} w^{(1)} \\ w^{(2)} \end{bmatrix}, \quad \boldsymbol{A}_{cpt2} = \begin{bmatrix} \boldsymbol{A}_c^{(1)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}_c^{(2)} \end{bmatrix}, \quad \boldsymbol{b}_{cpt2} = \begin{bmatrix} \boldsymbol{b}_c^{(1)} \\ \boldsymbol{b}_c^{(2)} \end{bmatrix},$$

$$\boldsymbol{D}_{cpt2} = \begin{bmatrix} \boldsymbol{d}_c^{(1)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{d}_c^{(2)} \end{bmatrix}.$$

The following fundamental theorem has already been proved [2].

Theorem 1. *The current-controlled series-connected suspension system whose state equation is given by (10) is controllable when Eqs.(5) and (9) hold.*

3.2 Voltage-controlled system with series connection

The state equation of voltage-controlled system is represented as follows.

$$\dot{\boldsymbol{x}}_{vpt2}(t) = \boldsymbol{A}_{vpt2}\boldsymbol{x}_{vpt2}(t) + \boldsymbol{b}_{vpt2}v(t) + \boldsymbol{D}_{vpt2}\boldsymbol{w}_{cpt2}(t) , \tag{11}$$

where

$$\boldsymbol{x}_{vpt2} = \begin{bmatrix} \boldsymbol{x}_c^{(1)} \\ \boldsymbol{x}_c^{(2)} \\ i \end{bmatrix}, \quad \boldsymbol{w}_{vpt2} = \begin{bmatrix} w^{(1)} \\ w^{(2)} \end{bmatrix}, \quad \boldsymbol{A}_{vpt2} = \begin{bmatrix} \boldsymbol{A}_c^{(1)} & \boldsymbol{0} & \boldsymbol{b}_c^{(1)} \\ \boldsymbol{0} & \boldsymbol{A}_c^{(2)} & \boldsymbol{b}_c^{(2)} \\ 0 \quad -\tilde{a}_{32}^{(1)} & 0 \quad -\tilde{a}_{32}^{(2)} & -\tilde{a}_{33} \end{bmatrix},$$

$$\boldsymbol{b}_{vpt2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \tilde{b}_0 \end{bmatrix}, \quad \boldsymbol{D}_{vpt2} = \begin{bmatrix} \boldsymbol{d}_v^{(1)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{d}_v^{(2)} \end{bmatrix},$$

$$\tilde{a}_{32}^{(k)} = k_b^{(k)} / \sum_{j=1}^{2} L^{(j)} \quad (k = 1, 2), \tag{12}$$

$$\tilde{a}_{33} = \sum_{j=1}^{2} R^{(j)} / \sum_{j=1}^{2} L^{(j)}, \tag{13}$$

$$\tilde{b}_0 = 1 / \sum_{j=1}^{2} L^{(j)}. \tag{14}$$

The following fundamental theorem has already been proved [2].

Theorem 2. *The voltage-controlled series-connected magnetic suspension system whose state equation is given by Eq.(11) is controllable when Eqs.(6) and (9) hold.*

4. Zero-power control

Zero-power control has been used mainly for energy saving to magnetic suspension with hybrid magnets [4)-(7]. In this section, the feasibility of such control in parallel magnetic suspension is discussed.

There are several methods of achieving the zero-power control [10].

1) Feeding back the velocity of the suspended object [4, 5, 10]
2) Introducing a minor feedback of the integral of current [6, 7, 10].
3) Building an observer-based controller with a function of estimating disturbance acting on the suspended object [11].
4) Using a low-pass filter [11].
5) Introducing a minor feedback of the integral of voltage [12]

In this paper, we apply the second method, which will be combined with state feedback.

4.1 Current-controlled system with series connection

The state space equation with a zero-power controller is given by

$$\dot{\boldsymbol{x}}_{cpt2}(t) = \overline{\boldsymbol{A}}_{cpt2}\overline{\boldsymbol{x}}_{cpt2}(t) + \overline{\boldsymbol{b}}_{cpt2}i(t) + \overline{\boldsymbol{D}}_{cpt2}\boldsymbol{w}(t), \tag{15}$$

where

$$\overline{\boldsymbol{x}}_{cpt2} = \begin{bmatrix} \boldsymbol{x}_{cpt2} \\ i_{\text{int}} \end{bmatrix}, \quad \overline{\boldsymbol{A}}_{cpt2} = \begin{bmatrix} \boldsymbol{A}_c^{(1)} & \boldsymbol{0} \\ \boldsymbol{0} & 0 \end{bmatrix}, \quad \overline{\boldsymbol{b}}_{cpt2} = \begin{bmatrix} \boldsymbol{b}_{cpt2} \\ 1 \end{bmatrix}, \quad \overline{\boldsymbol{D}}_{cpt2} = \begin{bmatrix} \boldsymbol{D}_{cpt2} \\ \boldsymbol{0} \end{bmatrix}.$$

The control input is represented by

$$\begin{aligned} i(t) &= -\boldsymbol{f}_{cpt2}\boldsymbol{x}_{cpt2} + k_c \int i(t)dt \\ &= -[\boldsymbol{f}_{cpt2} \quad -k_c]\overline{\boldsymbol{x}}_{cpt2}, \end{aligned} \tag{16}$$

where

$\boldsymbol{f}_{cpt2} = [p_d^{(1)} \quad p_v^{(1)} \quad p_d^{(2)} \quad p_v^{(2)}]$,

$p_d^{(k)}$, $p_v^{(k)}$: displacement and velocity feedback gains,

k_c : feedback gain of the integral of current.

Controllability

The following theorem is critical for realizing a stable zero-power control system.

Theorem 3. *The augmented system whose state equation is given by Eq.(15) is controllable* *when Eqs.(5) and (9) hold.*

Proof. The controllability matrix $\overline{C}_{cpt2}$ made of $(\overline{A}_{cpt2}, \overline{b}_{cpt2})$ becomes

$$\overline{C}_{cpt2} = \begin{bmatrix} 0 & b^{(1)} & 0 & a_{21}^{(1)}b^{(1)} & 0 \\ b^{(1)} & 0 & a_{21}^{(1)}b^{(1)} & 0 & (a_{21}^{(1)})^2 b^{(1)} \\ 0 & b^{(2)} & 0 & a_{21}^{(2)}b^{(2)} & 0 \\ b^{(2)} & 0 & a_{21}^{(2)}b^{(2)} & 0 & (a_{21}^{(2)})^2 b^{(2)} \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{17}$$

Calculating the determinant gives

$$\det \overline{C}_{cpt2} = -a_{21}^{(1)}a_{21}^{(2)}\left\{(a_{21}^{(1)} - a_{21}^{(2)})b^{(1)}b^{(2)}\right\}^2 \neq 0. \tag{18}$$

Therefore

$$\operatorname{rank} \overline{C}_{cpt2} = 5.$$

Q.E.D.

From Theorem 3, the closed-loop poles of the system described by Eq.(15) can be assigned arbitrary by the control law given by Eq.(16) that is a combination of a conventional state feedback and a minor feedback of the integral of current. In the following, it is assumed that the closed-loop system is stabilized by this controller.

Current regulation

To achieve zero-power performance, the coil current is made to converge to zero in the steady states. It will be shown that the controller given by Eq.(16) can achieve such performance. For the proof, transfer function representation [10] is applied. The equations describing the dynamics of the controlled object and the controller are given by

$$X^{(1)}(s) = \frac{b^{(1)}}{s^2 - a_{21}^{(1)}} I(s) + \frac{d^{(1)}}{s^2 - a_{21}^{(1)}} W^{(1)}(s), \tag{19}$$

$$X^{(2)}(s) = \frac{b^{(2)}}{s^2 - a_{21}^{(2)}} I(s) + \frac{d^{(2)}}{s^2 - a_{21}^{(2)}} W^{(2)}(s), \tag{20}$$

$$I(s) = -((p_d^{(1)} + p_v^{(1)}s)X^{(1)}(s) + (p_d^{(2)} + p_v^{(2)}s)X^{(2)}(s)) + \frac{k_c}{s} I(s), \tag{21}$$

where all the initial values are assumed to be zero for simplicity. From Eq.(21), we get

$$I(s) = -\frac{s}{s - k_c}((p_d^{(1)} + p_v^{(1)}s)X_1(s) + (p_d^{(2)} + p_v^{(2)}s)X_2(s)). \tag{22}$$

Substituting Eq.(22) into Eqs.(19) and (20) and rearranging them lead to

$$X^{(1)}(s) = \frac{s - k_c}{t_c(s)}(t_{22}(s)d^{(1)}W^{(1)}(s) - t_{12}(s)d^{(2)}W^{(2)}(s)), \tag{23}$$

$$X^{(2)}(s) = \frac{s - k_c}{t_c(s)}(t_{11}(s)d^{(2)}W^{(2)}(s) - t_{21}(s)d^{(1)}W^{(1)}(s)), \tag{24}$$

where

$$t_{11}(s) = (s^2 - a_{21}^{(1)})(s - k_c) + b^{(1)}(p_d^{(1)} + sp_v^{(1)})s, \tag{25}$$

$$t_{12}(s) = b^{(1)}(p_d^{(2)} + sp_v^{(2)})s, \tag{26}$$

$$t_{21}(s) = b^{(2)}(p_d^{(1)} + sp_v^{(1)})s, \tag{27}$$

$$t_{22}(s) = (s^2 - a_{21}^{(2)})(s - k_c) + b^{(2)}(p_d^{(2)} + sp_v^{(2)})s, \tag{28}$$

$$t_c(s) = t_{11}(s)t_{22}(s) - t_{12}(s)t_{21}(s). \tag{29}$$

Substituting Eqs.(23) and (24) into Eq.(22) gives

$$I(s) = -\frac{s}{t_c(s)}[\{(p_d^{(1)} + sp_v^{(1)})t_{22}(s) - (p_d^{(2)} + sp_v^{(2)})t_{21}(s)\}d^{(1)}W^{(1)}(s)$$

$$+\{(p_d^{(2)}+sp_v^{(2)})t_{11}(s)-(p_d^{(1)}+sp_v^{(1)})t_{12}(s)\}d^{(2)}W^{(2)}(s)]\,. \tag{30}$$

In discussing the zero-power control, the disturbances should be considered to be stepwise[8]. It can be modeled as

$$W^{(k)}(s)=\frac{w_0^{(k)}}{s}\quad (k=1,2)\,. \tag{31}$$

Since the closed-loop system is assumed to be stable, it is concluded from Eq.(30) the coil current converges to zero for arbitrary constant disturbances as

$$\lim_{t\to\infty} i(t)=\lim_{s\to 0} sI(s)=0 \qquad \text{for}\quad \forall w_0^{(k)}\quad (k=1,2)\,. \tag{32}$$

Therefore, the zero-power performance is achieved in this system. In addition, from Eqs. (23) and (24), we can show

$$\lim_{t\to\infty} x^{(k)}(t)=\lim_{s\to 0} sX^{(k)}(s)=-\frac{d^{(1)}}{a_{21}^{(1)}}w_0^{(k)}=-\frac{1}{k_s^{(k)}}w_0^{(k)}\quad (k=1,2)\,. \tag{33}$$

In the steady states, therefore, each floator is at the position where the increased force produced by the permanent magnet balances the applied disturbance force. The floator moves in the direction opposite to that of the applied force as if the suspension system has negative stiffness, which is a typical characteristic of the zero-power control system [8]-[10].

4.2 Voltage-controlled system with series connection

The discussion on the voltage-controlled system will be carried out in a way similar to the current-controlled system. The state space equation with a zero-power controller is given by

$$\dot{\bar{\boldsymbol{x}}}_{vpt2}(t)=\bar{\boldsymbol{A}}_{vpt2}\bar{\boldsymbol{x}}_{vpt2}(t)+\bar{\boldsymbol{b}}_{vpt2}i(t)+\bar{\boldsymbol{D}}_{vpt2}\bar{\boldsymbol{w}}(t)\,, \tag{34}$$

where

$$\bar{\boldsymbol{x}}_{vpt2}=\begin{bmatrix}\boldsymbol{x}_{vpt2}\\ i_{\text{int}}\end{bmatrix},\quad \bar{\boldsymbol{A}}_{vpt2}=\begin{bmatrix} & \boldsymbol{A}_{vpt2} & & & & \boldsymbol{0}\\ 0 & 0 & 0 & 0 & 1 & 0\end{bmatrix},\quad \bar{\boldsymbol{b}}_{vpt2}=\begin{bmatrix}\boldsymbol{b}_{vpt2}\\ 0\end{bmatrix},$$

$$\bar{\boldsymbol{D}}_{vpt2}=\begin{bmatrix}\boldsymbol{D}_{vpt2}\\ \boldsymbol{0}\end{bmatrix}.$$

The control input is represented by

$$\begin{aligned} i(t)&=-\boldsymbol{f}_{vpt2}\boldsymbol{x}_{vpt2}+k_c\int i(t)dt\\ &=-[\boldsymbol{f}_{vpt2}\quad -k_c]\bar{\boldsymbol{x}}_{vpt2}\,. \end{aligned} \tag{35}$$

Controllability

The following theorem is critical for stabilizing the control system with zero-power performance.

Theorem 4. *The augmented system whose state equation is given by Eq.(34) is controllable when Eqs.(6) and (9) hold.*

Proof. Calculating the determinant of the controllability matrix $\bar{\boldsymbol{C}}_{vpt2}$ made of $(\bar{\boldsymbol{A}}_{vpt2}, \bar{\boldsymbol{b}}_{vpt2})$ leads to

$$\det\bar{\boldsymbol{C}}_{vpt2}=\left|\bar{\boldsymbol{b}}_{vpt2}\quad \bar{\boldsymbol{A}}_{vpt2}\bar{\boldsymbol{b}}_{vpt2}\quad \bar{\boldsymbol{A}}_{vpt2}^2\bar{\boldsymbol{b}}_{vpt2}\quad \bar{\boldsymbol{A}}_{vpt2}^3\bar{\boldsymbol{b}}_{vpt2}\quad \bar{\boldsymbol{A}}_{vpt2}^4\bar{\boldsymbol{b}}_{vpt2}\quad \bar{\boldsymbol{A}}_{vpt2}^5\bar{\boldsymbol{b}}_{vpt2}\right|$$

$$=\begin{vmatrix} 0 & 0 & \tilde{b}_0 b^{(1)} & 0 & \tilde{b}_0 a_{21}^{(1)} b^{(1)} & 0\\ 0 & \tilde{b}_0 b^{(1)} & 0 & \tilde{b}_0 a_{21}^{(1)} b^{(1)} & 0 & \tilde{b}_0 (a_{21}^{(1)})^2 b^{(1)}\\ 0 & 0 & \tilde{b}_0 b^{(2)} & 0 & \tilde{b}_0 a_{21}^{(2)} b^{(2)} & 0\\ 0 & \tilde{b}_0 b^{(2)} & 0 & \tilde{b}_0 a_{21}^{(2)} b^{(2)} & 0 & \tilde{b}_0 (a_{21}^{(2)})^2 b^{(2)}\\ \tilde{b}_0 & 0 & 0 & 0 & 0 & 0\\ 0 & \tilde{b}_0 & 0 & 0 & 0 & 0 \end{vmatrix},$$

$$= -\tilde{b}_0^6 \det \overline{C}_{cpt2} . \quad (36)$$

Therefore, when Eqs.(6) and (9) hold,

$$\operatorname{rank} \overline{C}_{vpt2} = 6 .$$

Q.E.D.

Current regulation

To show the zero-power performance is achieved, transfer function representation is applied again. The equations describing the dynamics of the controlled object and the controller are is given by

$$X^{(1)}(s) = \frac{b^{(1)}}{s^2 - a_{21}^{(1)}} I(s) + \frac{d^{(1)}}{s^2 - a_{21}^{(1)}} W^{(1)}(s) , \quad (37)$$

$$X^{(2)}(s) = \frac{b^{(2)}}{s^2 - a_{21}^{(2)}} I(s) + \frac{d^{(2)}}{s^2 - a_{21}^{(2)}} W^{(2)}(s) , \quad (38)$$

$$sI(s) = -(\tilde{b}_0 p_d^{(1)} + (\tilde{a}_{32}^{(1)} + \tilde{b}_0 p_v^{(1)})s)X^{(1)}(s) - (\tilde{b}_0 p_d^{(2)} + (\tilde{a}_{32}^{(2)} + \tilde{b}_0 p_v^{(2)})s)X^{(2)}(s)$$
$$+ (\tilde{b}_0 \frac{k_c}{s} - \tilde{a}_{33}) I(s) . \quad (39)$$

From Eq.(39), we get

$$I(s) = -\frac{s}{s^2 + \tilde{a}_{33}s - k_c}((\tilde{b}_0 p_d^{(1)} + (\tilde{a}_{32}^{(1)} + \tilde{b}_0 p_v^{(1)})s)X^{(1)}(s)$$
$$- (\tilde{b}_0 p_d^{(2)} + (\tilde{a}_{32}^{(2)} + \tilde{b}_0 p_v^{(2)})s)X^{(2)}(s)) . \quad (40)$$

Substituting Eq.(40) into Eqs.(37) and (38) and rearranging them lead to

$$X^{(1)}(s) = \frac{s^2 + \tilde{a}_{33}s - k_c}{\hat{t}_c(s)} (\hat{t}_{22}(s) d^{(1)} W^{(1)}(s) - \hat{t}_{12}(s) d^{(2)} W^{(2)}(s)) , \quad (41)$$

$$X^{(2)}(s) = \frac{s^2 + \tilde{a}_{33}s - k_{cc}}{\hat{t}_c(s)} (\hat{t}_{22}(s) d^{(2)} W^{(2)}(s) - \hat{t}_{21}(s) d^{(1)} W^{(1)}(s)) , \quad (42)$$

where

$$\hat{t}_{11}(s) = (s^2 - a_{21}^{(1)})(s^2 + \tilde{a}_{33}s - k_c) + b^{(1)}(\tilde{b}_0 p_d^{(1)} + (\tilde{a}_{32}^{(1)} + \tilde{b}_0 p_v^{(1)})s)s , \quad (43)$$

$$\hat{t}_{12}(s) = b^{(1)}(\tilde{b}_0 p_d^{(2)} + (\tilde{a}_{32}^{(2)} + \tilde{b}_0 p_v^{(2)})s)s , \quad (44)$$

$$\hat{t}_{21}(s) = b^{(2)}(\tilde{b}_0 p_d^{(1)} + (\tilde{a}_{32}^{(1)} + \tilde{b}_0 p_v^{(1)})s)s , \quad (45)$$

$$\hat{t}_{22}(s) = (s^2 - a_{21}^{(2)})(s^2 + \tilde{a}_{33}s - k_c) + b^{(2)}(\tilde{b}_0 p_d^{(2)} + (\tilde{a}_{32}^{(2)} + \tilde{b}_0 p_v^{(2)})s)s , \quad (46)$$

$$\hat{t}_c(s) = \hat{t}_{11}(s)\hat{t}_{22}(s) - \hat{t}_{12}(s)\hat{t}_{21}(s) . \quad (47)$$

Substituting Eqs.(41) and (42) into Eq.(40) gives

$$I(s) = -\frac{s}{t_c(s)}[\{(\tilde{b}_0 p_d^{(1)} + (\tilde{a}_{32}^{(1)} + \tilde{b}_0 p_v^{(1)})s)\hat{t}_{11}(s)$$
$$- (\tilde{b}_0 p_d^{(2)} + (\tilde{a}_{32}^{(2)} + \tilde{b}_0 p_v^{(2)})s)\hat{t}_{21}(s)\} d^{(1)} W^{(1)}(s)$$
$$+ \{(\tilde{b}_0 p_d^{(2)} + (\tilde{a}_{32}^{(2)} + \tilde{b}_0 p_v^{(2)})s)\hat{t}_{22}(s)$$
$$- (\tilde{b}_0 p_d^{(1)} + (\tilde{a}_{32}^{(1)} + \tilde{b}_0 p_v^{(1)})s)\hat{t}_{12}(s)\} d^{(2)} W^{(2)}(s)] . \quad (48)$$

Therefore, it can be shown that the coil current converges to zero for arbitrary constant disturbances as

$$\lim_{t\to\infty} i(t) = \lim_{s\to 0} sI(s) = 0 \quad \text{for} \quad \forall w_0^{(k)} \quad (k = 1, 2) . \quad (49)$$

Therefore, the zero-power performance is achieved in this system. In addition, from Eqs. (41) and (42), we can show

$$\lim_{t\to\infty} x^{(k)}(t) = \lim_{s\to 0} sX^{(k)}(s) = -\frac{d^{(1)}}{a_{21}^{(1)}} w_0^{(k)} = -\frac{1}{k_s^{(k)}} w_0^{(k)} \quad (k = 1, 2) . \quad (50)$$

In the steady states, each floator is at the position where the increased force produced by the permanent magnet balances the applied disturbance force.

5. Experiment

5.1 Experimental system

Figures 4 and 5 shows a schematic diagram and a photo of the experimental system. It combines two magnetic suspension systems. One of them, which is denoted by apparatus 1, suspends a steel ball with a mass of 70g and a diameter of 26mm; a permanent magnet with a diameter of 10mm and a thickness of 0.5mm is attached to an electromagnet for providing bias flux. The other denoted by appratus 2 has an arm with a mass of 2.4kg and a length of 420mm as a floator and two electromagnets, one of which is for control and the other generates disturbance. A rectangular permanent magnet made of NdFeB material is attached to the electromagnet for control. In this appratus, the suspended object (arm) rotates around the fixed axis and does not float actually. The dynamics of this system is, however, equivalent to those of the typical magnetic suspension system such as the apparatus 1 when the angular displacement of the arm is small (10). Such appraratuses have been frequently used in basic study on magnetic suspension (13).

The displacement of each floator is detected by an eddy-current type gap sensor. The amplifier is of current-output type so that the following experiments treat current-controlled magnetic suspension.

Figure 6 shows a block diagram of the control system with zero-power performance.

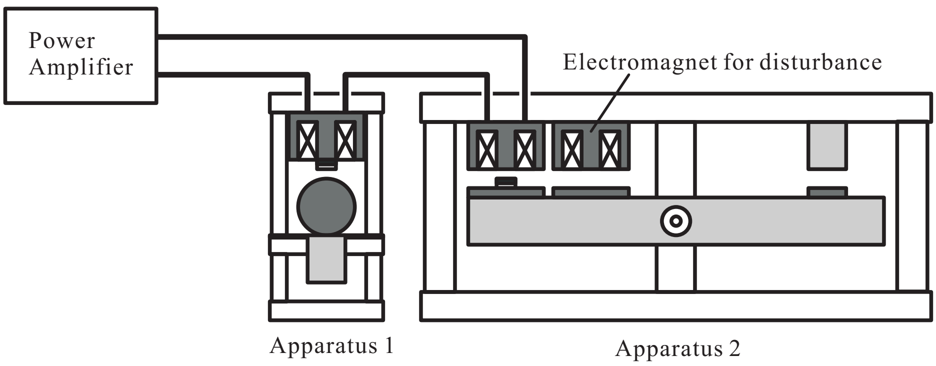

Fig.4 Experimental apparatus

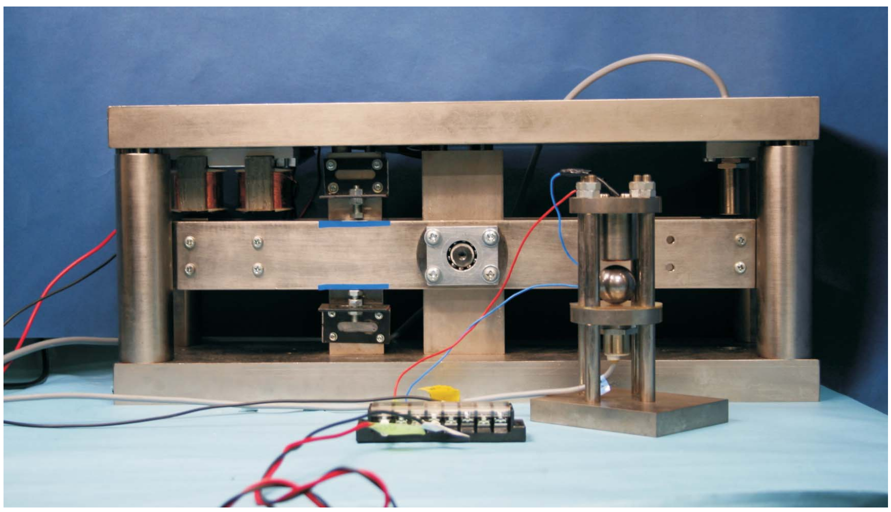

Fig.5 Photo of the apparatus

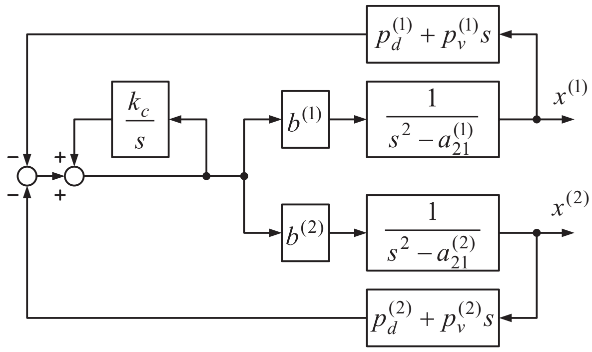

Fig.6 Block diagram of zero-power controller for parallel supension

The integral of control current is fed back, which composes a minor loop. The gains of the controller are tuned by trial and error. Designed control algorithms are implemented with a digital controller DS1103 manufactured by dSPACE^{TM}. The control period is 100μs.

5.2 Experimental result

First, the parameters of each apparatus were identified experimetally. The obtained values are given as follows.

[appratus 1] $a_{21}^{(1)} = 9.59 \times 10^3$ [N/m], $b^{(1)} = 10.9$ [N/A].

[appratus 2] $a_{21}^{(2)} = 0.783 \times 10^3$ [N/m], $b^{(2)} = 0.95$ [N/A].

These results show that the controllability condition (9) is satisfied in the developed experimental setup.

Second, the feedback gains were tuned by trial and error. The selected values are

$p_d^{(1)} = 3.0 \times 10^3$ [A/m], $p_v^{(1)} = 8.2$[As/m],

$p_d^{(2)} = -2.5 \times 10^3$ [A/m], $p_v^{(2)} = -8.1$[As/m],

$k_c = 1.0$ [s].

Noncontact zero-power suspension was achieved with these feedback gains.

Figure 7 shows responses to a stepwise disturbance that is generated by the electromagnet for generating disturbance acting on the floator 2 (arm). Fig.7(c) shows that the control current converges to zero in the steady states and the zero-power control is achieved. Even though the disturbance acts only on the arm, both the floators are displaced due to this disturbance in transient states. In contrast, only the arm is displaced to the opposite direction to applied force and the other floator return to the initial equilibrium position in the steady states. Such behaviour agree with the predicitons described in Section 4.1.

6. Conclusion

It was shown both analytically and experimentally that zero-power control can be applied to parallel magnetic suspension systems. In the treated suspension systems, the coils are connected in series. The analytical study showed that both current-controlled and voltage-controlled parallel magnetic suspension systems incorporating zero-power control can assign the poles arbitrary if the original parallel suspension systems are controllable; when the closed-loop systems are stable, the coil current converges to zero. The zero-power controller was implemented in the actual current-controlled parallel magnetic suspension system with two floators. It was demonstrated experimantally that the coil current converged to zero even in the presence of external force acting on the floator.

As to the voltage-controlled parallel magnetic suspension, the actual implementation of state feedback and zero-power control is still under way.

The zero-power controlled parallel magnetic suspension has several advantages:

- the number of amplifiers is reduced, which leads to lower cost,
- the energy consumption is reduced to zero in the steady states.

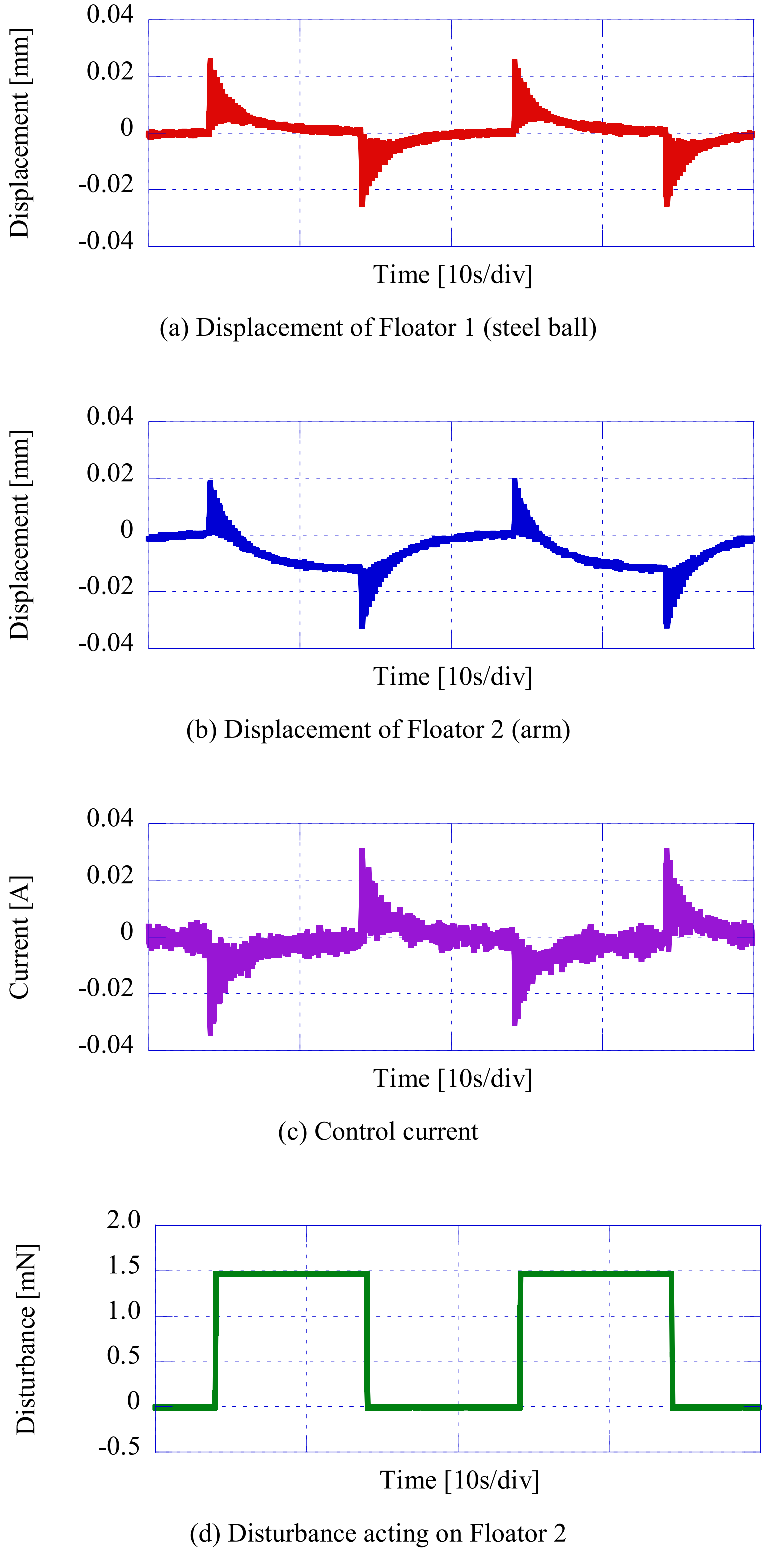

Fig.7 Response of the zero-power parallel suspension system to stepwise disturbance acting on Floator 2 (arm).

However, it has also disadvantatges:

- Interfereces among the floators exist (force acting on one floator causes the other flotors to move),
- Steady-state error remains in displacement when disturbance acts on a floator.

Therefore, possible applications will be in the fields where high-accuracy positioning is unnecessary.

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