New Results on Single-Step Power Control (SSPC) System in Finite State Markov Channel: Power Control Error Modelling and SSPC Modelling

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Abstract—The analysis regarding the impact of the single-step power control (SSPC) scheme on the system performance such as bit error rate, packet error rate, and queueing variation is hard to access. In this work, we propose an approach to analyze that impact when being in finite state Markov channel (FSMC). To achieve the understanding of the relationship between SSPC, FSMC, and the queueing variation, power control error (PCE), error rate, and the PDF of queueing length are first calculated. PCE variation is modelled, which then benefits the evaluation of packet error rate (PER) and bit error rate (BER). Furthermore, based on these analysis, a SSPC model is proposed to simplify the overall system. The behavior of SSPC in FSMC can be better understood with the help of the proposed SSPC model. Simulation results have shown the validation of the proposed PCE variation model and SSPC model.

I. INTRODUCTION

To construct a suitable multimedia system and achieve higher system performance, the interactions between different layers must be understood. This understanding also helps devise a suitable mechanism to jointly adjust two or more layers. In this work, we mainly investigate the impact of single-step power control (SSPC) scheme on the queueing performance of a link in the MAC layer.

In the wireless transmission environment, especially in CDMA-based systems, power control is a significant mechanism to improve system performance and overcome impairment due to channel variation. Authors in [1] proposed a SIR-based power control scheme known as SSPC. In their work, the transmission power is constantly updated by one step size to let the received SNR able to meet the target SNR as possible. This scheme has been widely adopted in many CDMA systems [2]. In many previous works, authors focused on analyzing BER and PER performance of SSPC in different transmission environments, [3] and [4], or evaluating power spectrum density (PSD) of PCE, [5] and [6], which have shown a great impact on the performance of SSPC. However, few authors proposed a method to calculate PCE which dominates the performance of SSPC.

Markov-based channel has been applied to analyze the wireless networks frequently, in [7] and [8]. Markov-based channels can behave with distinct properties in accordance with different application issues. In this paper, we adopt the FSMC proposed in [8] to model the wireless environment. Also to the best of authors’ knowledge, few literatures consider the performance of SSPC on FSMC. In this work, we propose a PCE estimation method based on the properties of FSMC. The BER, PER, and other queueing performance issues can be acquired based on this estimation results. At the same time, we consider the queueing performance issues such as average queueing length, throughput, and the queueing length variation probability. The validation of these results are shown through the simulation results presented in this work. Also in order to have better understanding of the interaction between SSPC and FSMC, we construct a model, which describes how SSPC behaves in FSMC. This model is a two-dimensional Markov chain, and such model can describe how the SSPC adapts itself to the channel variation of FSMC.

II. SYSTEM MODEL

The system of interest in this paper is shown in Fig. 1. The detailed description of the blocks in Fig. 1 are given below.

A. FSMC

The variation of the received SNR is due to the channel fading gain and AWGN, and this behavior is modeled as FSMC with $K$ states. Some studies has shown that the PDF of the received SNR is distributed exponentially in this environment, so that the steady state probability of each state is evaluated as

$$\pi_k = \exp\left(-\frac{\Gamma_k}{\gamma_0}\right) - \exp\left(-\frac{\Gamma_{k+1}}{\gamma_0}\right),$$

$$\pi = [\pi_1, \pi_2, \cdots, \pi_K]^T,$$

where $[\Gamma_1, \Gamma_2, \cdots, \Gamma_{K+1}]$ is the SNR boundaries of the $K$ states with increased order and $\gamma_0$ is the average received SNR. Furthermore, the transition probabilities in state $k$ are shown as

$$p_{k,k+1} \approx \frac{N\Gamma_{k+1}T_p}{\pi_k}$$

and

$$p_{k,k-1} \approx \frac{N\Gamma_kT_p}{\pi_k},$$

where $N_{\Gamma_k}$ is the average number of times per unit interval that a fading signal crosses a given signal level $\Gamma_k$, which is related to the product of Doppler frequency and packet duration, $T_p$.

In FSMC, there are two main properties. The first property is that the transition only occurs to the adjacent two states or remains in the original state. The second property is that the fading gain keeps identically in a state duration. While the packet duration is equal to the state duration, it causes the SNR and BER in a packet duration to be identical.

Fig. 1. System block diagram
B. Reliable data transfer, queueing and arrival processes

In this system, we employ simple-ARQ protocol to ensure each packet transmitted to the receiver successfully. A first-in-first-out (FIFO) system with the finite length buffer is assumed. The arrival process of incoming packets is modeled as Poisson random process with arrival rate, $\lambda$.

C. Power control process

In order to maintain a quality transmission environment, SSPC scheme is utilized in this system. In this process, there is a predefined target SNR $\Gamma_T$ and a step size $\Delta$ of power level, where the target SNR is a goal for keeping the received SNR. By comparing against the previous received SNR at the target SNR, the current transmitted power is update as

$$P(n) = P(n-1) - cmd \cdot \Delta,$$

here the above parameters are represented in decibels, and $cmd$ is the feedback command for the transmitter, that is

$$cmd = \begin{cases} 1, & \text{if } e(n-1) \geq 0 \\ -1, & \text{if } e(n-1) < 0 \end{cases},$$

(1)

where $e(n) = \gamma_{rec}(n) - \Gamma_T$ is the error difference between received SNR, $\gamma_{rec}(n)$, and target SNR, $\Gamma_T$, in decibels. Note that, error-free transmission is assumed in the backward channel.

III. PERFORMANCE DERIVATION

In this section, we first propose a method for estimating PCE in SSPC process. Based on the PCE estimated result, BER and PER with SSPC in FSMC are evaluated. Finally, with the obtained PER result, we can access the queueing performance.

A. PCE estimation method

Before estimating PCE, it is worthwhile to establish the relationship between these variables, received SNR, $\gamma_{rec}$, target SNR, $\Gamma_T$, average SNR, $\gamma_0$, channel fading gain $g$, and the power control compensation $c$. To consider those variables in decibels, the received SNR can be expressed as

$$\gamma_{rec} = \gamma_0 + g + c = \gamma + c,$$

where $\gamma = \gamma_0 + g$ is the variation of the received SNR caused by the channel fading gain, and this term is modeled as a finite-state Markov process. While giving a target SNR, $\Gamma_T$, in the SSPC process, PCE can be expressed as $e = \gamma_{rec} - \Gamma_T$ in definition. From this relationship, the received SNR can be rewritten as

$$\gamma_{rec} = \Gamma_T + e \quad \text{and} \quad e = (\gamma_0 + g) - (\Gamma_T - c).$$

At present, $e$ can be regarded as the difference between two processes: one is $(\gamma_0 + g)$, and the other is $(\Gamma_T - c)$. Note that, $\gamma_0$ and $\Gamma_T$ are deterministic values, and $g$ is a random variable corresponding to FSMC. With the variation of $g$, SSPC process provides a power level of $c$ to compensate the influence of $g$. Thereby, the term $e$ can be viewed as the error which is generated while the process $(\Gamma_T - c)$ is tracking $(\gamma_0 + g)$ with a single-step variation, which is similar to delta modulation (DM). The motivation of SSPC is letting $\gamma_{rec}$ approach $\Gamma_T$ and $e$ tend to zero. The reason of the error term $e$ occurs is the variation of the channel varies more rapidly than the

SSPC process can response. With different channel variation scenarios, we classify PCE into two types: one is the slope overload PCE, and the other is the granular PCE. The slope overload PCE occurs when $\Delta < \frac{|dg|}{dt}$, where $\Delta$ is the power control step size, and $dg/dt$ means the varying rate of the time-variant channel fading gain. Because the step size is not large enough to track the variation of the channel, PCE occurs and accumulates to a considerable value. The other type of PCE is called the granular PCE. This type PCE occurs when the channel fading gain varies quickly around some certain states and this variation causes the $cmd$ being switching back and forth between 1 and -1. The value of PCE in this scenario is always small. Before we present the PCE estimation method, we make some assumptions to simplify the evaluation of the performance. First, we assume that granular PCE is zero mean and can be ignored without affecting the performance. Thus, in the proposed approach, we only consider the slope overload PCE. Further, we assume that the initial PCE condition is zero. In other words, the tracking process is supposed to be perfect initially.

To consider the slope overload PCE, we discuss PCE with three cases. Let $e_{ij}$ be the PCE while the channel transition is from $i$th state to $j$th state. The first case is that the channel state transits from lower state to higher state shown in Fig. 2(a) and Fig. 3(a). We suppose that the initial channel state is in $i$th state, which is $\gamma(n) \in i$th state. After $(j - i)$ packet durations, the destination state is in $j$th state, which is $\gamma(n + j - i) \in j$th state, where $i < j$. Note that, $\gamma(n) \in i$th state means that the summation of average SNR, $\gamma_0$, and current channel fading gain, $g(n)$, belongs to $i$th state. This scenario represents the quickest transition from $i$th state to $j$th state,
and the difference of channel gain between these two states is 
\( \gamma(n + j - i) - \gamma(n) \). However, the power control process just can track or compensate \((j - i - 1) \Delta\), then PCE in this case can be obtained as 
\[ e_{ij} = \gamma(n + j - i) - \gamma(n) - (j - i - 1) \Delta, \]
for \( \gamma(n) < \gamma(n + j - i) \). Note that the value of PCE is positive in this case. Next, we need the associated probability in this case. We have assumed that the initial PCE is zero. To achieve this situation, the transition before the initial state must not be in \((i - 1)\)th state. Otherwise, the initial PCE would be larger than zero. In fact, the transition from \((i - 1)\)th has been discussed in the case \(e_{(i-1)j}\), and it can be excluded. Therefore, the probability in this case is given by 
\[ P_{e_{ij}} = \prod_{k=i}^{j-1} P_{k,k+1}(k) \left[ P_{k,k}(i) \pi_i + P_{k,k-1}(1 + i) \pi_i + 1 \right], \]
when \( \gamma(n) < \gamma(n + j - i) \). While giving \( \gamma(n + j - i) \in j\)th and \( j = 2, 3, \ldots, K \), further, \( \gamma(n) \in \text{ith and } i = 1, 2, \ldots, j - 1 \), then there are \( k(k - 1)/2 \) cases in this part. Similarly, when the channel state transition is from higher state to lower state, we have third case, \( \gamma(n) > \gamma(n + j - i) \). In this case, \( e_{ij} \) is negative. The probability in this case is shown below:
\[ P_{e_{ij}} = \prod_{k=j}^{i+1} P_{k,k-1}(k) \left[ P_{k,k}(i) \pi_i + P_{k,k-1}(1 + i) \pi_i + 1 \right], \]
when \( \gamma(n) > \gamma(n + j - i) \). There are \( k(k - 1)/2 \) cases in this part. In addition to these two cases, we have third case, which accounts for the granular PCE denoted by \( e_{ij} \) when being in \( j\)th state. Thus, we have 
\[ P_{e_{ij}} = 1 - \sum_{i=1}^{K} P_{e_{ij}}, \]
where \( e_{ij} \approx 0 \).

To derive the BER and PER performance, we first base on the estimated PCE to obtain the received SNR followed by substituting the results into the BPSK’s BER formula and let

\[ \theta_i = Q \left( \sqrt{2 \cdot 10^{(\frac{e_{ij} + \mu}{10})}} \right), \]

\[ \phi_i = 1 - \sum_{l=0}^{E} \left( \frac{N_p}{l} \right) \theta_{ij}^l (1 - \theta_{ij})^{(N_p - l)}, \]

where \( \theta_i \) and \( \phi_i \) are functions of \( \gamma(n) \) and \( \gamma(n + j - i) \). These two variables are related in \( \text{ith and } j\)th state of FSMC, respectively. Therefore we average \( \theta_i \) and \( \phi_i \) and obtain

\[ \tilde{\theta}_{ij} = \frac{1}{U_i} \int_{U_i} \theta_{ij} \cdot p(\alpha | \alpha = \text{ith state}) \cdot p(\beta | \beta \in \text{jth state}) d\beta d\alpha, \]

\[ \tilde{\phi}_{ij} = \frac{1}{U_i} \int_{U_i} \phi_{ij} \cdot p(\alpha | \alpha = \text{ith state}) \cdot p(\beta | \beta \in \text{jth state}) d\beta d\alpha. \]

According to these results, the average BER and average PER of state \( k \) with SSPC can be obtained by

\[ \rho_k^{SSPC} = \sum_{i=1}^{K} P_{e_{ik}} \tilde{\theta}_{ik} \quad \text{and} \quad \sigma_k^{SSPC} = \sum_{i=1}^{K} P_{e_{ik}} \tilde{\phi}_{ik}. \]

Then we can combine the state BER and PER with the steady state probability to obtain the average BER and PER, which are

\[ P_{\text{e,bit}}^{SSPC} = \rho_k^{SSPC} \cdot \pi \quad \text{and} \quad P_{\text{e, pkt}}^{SSPC} = \sigma_k^{SSPC} \cdot \pi, \]

where

\[ \rho_k^{SSPC} = [\rho_1^{SSPC} \rho_2^{SSPC} \ldots \rho_K^{SSPC}]^T, \]

\[ \sigma_k^{SSPC} = [\sigma_1^{SSPC} \sigma_2^{SSPC} \ldots \sigma_K^{SSPC}]^T. \]

B. Queueing performance

We discuss the interaction between the transmission and the queueing condition in this subsection. In this work, we assume a lost transmission packet always gets retransmitted until transmission is done successfully. Under this scenario, the transmission times is a Geometric random variable. While giving a Bernoulli trial rate \( r \) (i.e. \( r \) means the successful rate), the mean of the Geometric random variable is \((1 - r)/r\). According to this result, the average transmission times can be obtained as

\[ P_{\text{e, bit}}^{SSPC} + 1 = \frac{1}{1 - P_{\text{e, pkt}}^{SSPC}}, \]

where we let \( r = 1 - P_{\text{e, pkt}}^{SSPC} \). Define the service rate \( \mu \) as the inverse of the transmission times, which is equal to \( \mu = 1 - P_{\text{e, pkt}}^{SSPC} \). Then the average throughput can be obtained as \( \mu/\lambda \), where \( \lambda \) is the arrival rate. We have known that the queueing length accumulates because of the multiple arrivals and the packet re-transmission. Therefore, the pdf of the queueing length within an observation in packets, \( \Omega \), can be expressed as

\[ f_Q(q) = Pr\{q | q \geq 0\} = \frac{\sum_{n=0} f_U(u) f_V(v) \sum_{q=0} f_U(u) f_V(v) \}{\sum_{n=0} f_U(u) f_V(v) \sum_{q=0} f_U(u) f_V(v) \}, \]

where

\[ f_U(u) = (\frac{\lambda \Omega}{u!}) \exp(-\lambda \Omega \mu^u), \]

\[ f_V(v) = \left( \frac{\Omega}{v} \right) (1 - \mu)^{\Omega - v} \mu^v, \]

are the arrival pdf and the service pdf, respectively.

The average queueing length during this observation can be shown as follows.

\[ \overline{q} = \mathbb{E}\{q | \omega\}, \quad \omega = 1, 2, \ldots, \Omega. \]

Next, we present the probability of the queueing variation. Three scenarios are considered in this work.

1) Scenario I: When there is no arrival and the current packet gets transmitted successfully, the queueing length will decrease by one. The probability in this scenario is shown as

\[ P_{Q,q-1} = e^{-\lambda} \mu. \]
2) Scenario II: The queueing length remains the same. Two cases will lead to this. One case is when the current packet gets transmitted successfully and only one packet arrives at the same time. The other case is when the current packet gets lost in between and no packet arrives. Thus, the probability of this scenario can be expressed as

\[ P_{Q,Q} = e^{-\lambda}(1 - \mu) + \lambda e^{-\lambda}. \]

3) Scenario III: The queueing length increases by \( \ell \). Two cases can cause this variation. First case is when the current packet gets transmitted successfully and \( \ell + 1 \) packets arrives at this moment. The other case is when the current packet fails to transmit and there are \( \ell \) packets arriving. Combining these two cases, the probability can be obtained as

\[ P_{Q,Q+\ell} = \frac{\lambda^\ell e^{-\lambda}(1 - \mu)}{\ell!} + \frac{\lambda^{\ell+1} e^{-\lambda}}{(\ell + 1)!}. \]

IV. MODELING SSPC IN FSMC

In this work, we model the operation of SSPC in FSMC as a two-dimensional Markov chain. In such as chain, the state is defined by two parameters, total power compensation up to now, \( c \), and the SNR, \( \gamma \), which is from FSMC. The relationship between adjacent two chains of SSPC model is shown in Fig. 4. To understand the behavior of the SSPC model shown in Fig. 4, we divide the transition of the SSPC model into three cases.

A. The transition occurs within a single chain

In this case, the transition occurs only within chain-I or chain-II. The channel state remains the same throughout the entire transition in this case. When the state transits to the state of its right, it means the current power compensation cannot have the received SNR meet the required SNR and transmission power should be increased by one \( \Delta \). On the other hand, when the state transits to the state of its left, the current transmission power is more than needed and it results in the decrease in transmission power by a \( \Delta \).

B. The transition occurs from chain-I to chain-II

When the transition occurs from chain-I to chain-II, it means the channel transits from the lower state to the higher state. Two situations occurs. One is when the current received SNR is larger than the required SNR. In this case, the transmission power should be decreased by one \( \Delta \). As shown in Fig. 4, the transition from state \((c, \gamma \in i)\) to \((c - \Delta, \gamma \in i + 1)\) belongs to this category. The other is when the current received SNR is smaller than the required SNR. In this case, we should increase the transmission power by one \( \Delta \). As shown in Fig. 4, the transition from state \((c, \gamma \in i)\) to \((c + \Delta, \gamma \in i + 1)\) belongs to this category.

C. The transition occurs from chain-II to chain-I

When the transition occurs from chain-II to chain-I, it means the channel transits from the higher state to the lower state. Two situations occurs. One is when the current received SNR is larger than the required SNR. In this case, the transmission power should be decreased by one \( \Delta \). As shown in Fig. 4, the transition from state \((c, \gamma \in i + 1)\) to \((c - \Delta, \gamma \in i)\) belongs to this category. The other is when the current received SNR is smaller than the required SNR. In this case, we should increase the transmission power by one \( \Delta \). As shown in Fig. 4, the transition from state \((c, \gamma \in i + 1)\) to \((c + \Delta, \gamma \in i)\) belongs to this category.

Next, we have to determine the transition probabilities for these three cases. The rule for the power control compensation in SSPC is rewritten as

\[ c + \gamma - \Gamma_T \geq 0, \]

Given a state in SSPC model, we have 6 transition probabilities associated with this state. We have transition probabilities, \( p_1 \), \( p_2 \), and \( p_3 \), which correspond to the probabilities that transition occurs and transmission power is increased by one \( \Delta \). Likewise, the transition probabilities, \( q_1 \), \( q_2 \), and \( q_3 \), correspond to the probabilities that transition occurs and transmission power is decreased by one \( \Delta \).

We show the derivation of how to obtain \( p_1 \) as example. \( p_1 \) is the probability that the channel remains in the FSMC state and the power control compensation increases a power step.
From this, $p_1$ can be evaluated by

$$p_1 = Pr\{c + \gamma - \Gamma_T < 0|\gamma \in i\} P_{k,k}(i)$$
$$= \int_{\Gamma_i}^{\Gamma_{i+1}} 0.5(1 - \text{sgn}(c + \gamma - \Gamma_T))Pr(\gamma)d\gamma P_{k,k}(i).$$

Similarly, the rest of transition probabilities can be obtained via the following equations.

$$p_2 = Pr\{c + \gamma - \Gamma_T < 0|\gamma \in i\} P_{k,k-1}(i)$$
$$p_3 = Pr\{c + \gamma - \Gamma_T < 0|\gamma \in i\} P_{k,k+1}(i)$$
$$q_1 = Pr\{c + \gamma - \Gamma_T > 0|\gamma \in i\} P_{k,k}(i)$$
$$q_2 = Pr\{c + \gamma - \Gamma_T > 0|\gamma \in i\} P_{k,k-1}(i)$$
$$q_3 = Pr\{c + \gamma - \Gamma_T > 0|\gamma \in i\} P_{k,k+1}(i)$$

Fig. 5 and Fig. 6 are illustrative examples for SSPC model. The six transition probabilities sampled at different time instant are shown in Fig. 5. The power control compensation and the channel state, which are associated to these transition probabilities, are shown in Fig. 6.

V. SIMULATION RESULTS

This system was simulated in the environment with following parameters: the packet size $T_p$ is 3ms, the transmission rate is 32kbps, the error correcting capability is 3 bits per packet, the number of FSMC states $K$ is 11, the product of Doppler frequency and packet size is 0.0338, the target SNRs of power control process are 0 to 25dB, the step size $\Delta$ of SSPC is 1dB, and the simulation duration is 300s.

Fig. 7 shows the results of BER and PER, where the numerical results are obtained from simulation, the theoretical results are derived in this proposal, and the modelling results are product by the SSPC model. Observing this figure, the numerical and the modelling results are almost overlapping, and this fact validates the SSPC model is able to describe the effect of the SSPC on FSMC with high accuracy. Furthermore, there is a difference between the numerical and theoretical results with around 1dB, and it is caused by the PCE estimation error. However, the tendency of these two curves are quite similar. Even so, we find that this difference actually does not affect the derivation of other performances in higher target SNR.

Fig. 8 presents the results of the average throughput. The two curves are almost matched except in low target SNR. This inaccuracy and the crossover phenomenon is due to the error of the PER derivation, because the average throughput is a function of the PER. However, the error can be ignored in the higher target SNR scenarios and does not impact the outcomes.

In addition, the PDF of the queueing length variation under different target SNRs is shown in Fig. 9. No matter in low or high target SNR, our evaluation performs well in the most of cases. And the results are used to build the queueing model.

REFERENCES