

Foundations for Simple Menus of Contracts in Cost-Based Procurement*

Daniel Garrett

Department of Economics
Northwestern University
2001 Sheridan Rd., Evanston
IL 60208
dfgarrett@gmail.com

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Abstract

This paper considers a model of cost-based procurement in which the principal faces ambiguity about the agent's preferences for effort to reduce costs. It evaluates the performance of simple and commonly used incentive schemes whereby the agent chooses between a fixed-price contract and a cost-reimbursement contract. Calculation of the optimal simple scheme requires knowledge only of the cost saving from efficient effort and the distribution of the *innate cost* – the agent's cost of production without effort. The paper argues that the optimal simple scheme can be a “robust” choice given only this information. Two criteria are considered: (1) whether the scheme minimizes the maximum expected payment, and (2) whether the scheme is undominated. Whilst there is always a simple scheme that solves the minimax problem, the question of (weak) dominance is more delicate. It depends on the principal's view about whether the agent's preferences for effort depend on his monetary rewards. If the principal believes that they might, then the optimal simple scheme is undominated.

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1 Introduction

Laffont and Tirole (1986) study a model of cost-based procurement or regulation in which the agent's preferences over production costs are determined firstly by his *innate cost*, the cost that would be realized without effort, and secondly by a *disutility function* that maps his cost-saving effort to the dollar value of his disutility. Although the principal in their model is uncertain as to the agent's innate cost, she has perfect knowledge of his disutility function. Such precise knowledge seems implausible. This paper therefore considers the principal's problem when the form of the disutility function is ambiguous.

The starting point for the analysis is Rogerson's (2003) paper, in which he suggests a simpler, although sub-optimal, solution to Laffont and Tirole's model. In what he terms a Fixed Price Cost Reimbursement (FPCR) menu, the agent is offered two contracts between which he can choose: a fixed-price contract, and a cost-reimbursement contract whereby he is paid the realized cost. Rogerson argues that FPCR menus are very common in procurement and regulation. He also points out that they can be calculated with knowledge only of the distribution of innate costs and a summary measure of the disutility function. This measure is the difference between the agent's innate cost and the lowest price that he would be willing to accept to produce the good. The first-best solution to the principal's problem is to offer this price, and so the difference is termed the *first-best cost saving*.

Since the expected payment under the FPCR menu depends on the first-best cost saving, but not on the other details of the disutility function, we might expect the FPCR menu to be in some sense "robust" to variation in these details. To investigate this claim, two criteria for making a robust choice under ambiguity about the details of the disutility function are considered: firstly, that the mechanism solve the problem of minimizing the principal's maximum expected payment (the *minimax problem*); and secondly, that the mechanism not be *weakly dominated* – i.e., that no mechanism exist which performs better for some realization of the ambiguity, and no worse for all others.

Requiring the mechanism to be undominated seems uncontroversial, since a regulator or procurement officer would wish to take advantage of an alternative mechanism if she had nothing to lose. The minimax criterion reflects a very conservative approach to decision making, but this seems quite plausible in regulatory settings and in public procurement, where contracting officers are caretakers of public funds. Indeed, public managers are often thought more likely to avoid risk than their private-sector counterparts.¹ From a more normative perspective, the analysis provides an objective way to determine incentive contracts based on information that could well be available.

The central result of the paper is that FPCR menus solve the minimax problem. We might not find this too surprising. Since the expected payment under an FPCR menu is known in spite of the ambiguity, it solves the minimax problem provided that it is not *strictly dominated* – i.e., if there is no alternative mechanism that leads to a lower expected payment for all possible realizations of the ambiguity. We may think of the principal as playing a game against nature in which nature tries to make the expected payment as high as possible. The model considered allows nature considerable flexibility to choose disutility functions, subject to the first-best cost saving that the principal knows. Thus nature is able to thwart any attempt by the principal to design a mechanism that yields a lower expected payment regardless of the information that she does not know.²

However, FPCR menus may be weakly dominated. If the principal knows that the agent's preferences for effort do not depend on his monetary rewards (i.e., under the quasi-linearity assumption of Laffont and Tirole's model), then a particularly clean characterization of when this happens is possible. The optimal FPCR menu is undominated if and only if the agent accepts the fixed-price option of this menu for every value of the innate cost. If, on the other hand, the principal is uninformed about how the agent's preferences for effort depend on his monetary rewards, then the optimal FPCR menu is

¹Boyne (2002) reviews the empirical evidence. There appears to be some, although not unequivocal, support for this view.

²Since I do impose *some* natural restrictions on the agent's preference for effort, care is still needed to establish the result.

always undominated. The two results thus provide foundations for use of FPCR menus, according to both the minimax and weak dominance criteria, in distinct settings.

Alternative theories have been put forward to explain the prevalence of simple incentive schemes in regulation and procurement. Rogerson (2003) and Chu and Sappington (2007a) take the view that simple schemes are preferred because the additional complexity of fully-optimal schemes yields only small improvements, at least in a broad range of parametric settings. Another possible explanation is that the principal faces some simple and binding constraint. For example, Innes (1990) and Poblete and Spulber (2009) consider a moral hazard problem in which the agent has limited liability and the principal is unable to commit not to sabotage output. To prevent sabotage, mechanisms must ensure that the principal's returns are non-decreasing in the realized output. This constraint typically binds, and so the corner solution is very often a simple debt contract.

A contribution of this paper is to provide an alternative theory that explains simplicity as arising from natural assumptions about the principal's available information, together with fully-optimal worst-case decision making. Section 2.2 shows that the principal's key information – the distribution of innate costs and the first-best cost saving – could be obtained from past experience if both cost-reimbursement and fixed-price schemes had been offered. This was the situation, for instance, in the French urban transport industry, as studied by Gagnepain and Ivaldi (2002).

The idea that the minimax criterion might lead to optimal selection of simple mechanisms has been observed in two quite different environments by Chung and Ely (2007) and Kocherlakota and Phelan (forthcoming).³ Chung and Ely provide a foundation for dominant-strategy mechanisms in private-value auctions based on the max-min expected payoff criterion.⁴ They assume that whilst the auctioneer knows the distribution of bidder

³There is also a broader literature, examining the properties of mechanisms that are optimal under minimax and minimax regret criteria. A pertinent early paper is by Lopez-Cunat (2000). He considers a model that can be applied directly to Laffont and Tirole's (1986) model of regulation. In this context, the model allows ambiguity over the distribution of innate costs, whilst the disutility function is known with certainty.

⁴The criterion is the same as in this paper, although I refer to the objective of minimizing payments rather than maximizing payoffs.

valuations, she faces ambiguity about the bidders' beliefs about their opponents' valuations. Kocherlakota and Phelan provide a rationalization for a laissez-faire tax system, also using the max-min criterion.

The rest of the paper proceeds as follows. Section 2 introduces the environment, first introducing the full-information model, and then relaxing the principal's knowledge of the disutility function. Section 3 shows that an FPCR menu is an optimal solution to the minimax problem in the class of all deterministic mechanisms. Section 4 addresses the question of when the optimal FPCR menu is weakly dominated, assuming the principal knows that the agent's preferences over effort depend on his monetary rewards. Section 5 shows that if the dependence of effort preferences on money is ambiguous, then the optimal FPCR menu is undominated. Section 6 concludes. All formal proofs are provided in the Appendix.

2 Environment

2.1 The full-information model

The principal is required to purchase one unit of a good from the agent. The agent's realized cost x is observable and verifiable. The agent has some innate cost $\beta \in [\underline{\beta}, \bar{\beta}]$, at which he can produce the good without effort. Innate costs are distributed according to a distribution function F , with a density f that is strictly positive on $[\underline{\beta}, \bar{\beta}]$. By exerting effort e , the agent can reduce the realized cost to $\beta - e$.

Adopting the convention introduced by Laffont and Tirole (1986), the agent's *total payment* is divided into two components: the realized cost x and an additional transfer y . Supposing that the realized cost is always reimbursed, the agent's payoff is $y - \psi(e)$, where ψ is the disutility function. Thus, for now, the agent's preferences over money and effort are independent – a more general relationship is considered in Section 5. The agent's outside option has payoff zero.

The disutility function ψ satisfies the following conditions:

- M (Monotonicity): ψ is non-decreasing.
- NEC (Non-positive Effort is Costless): For each $e \leq 0$, $\psi(e) = 0$.
- PEC (Positive Effort is Costly): For each $e > 0$, $\psi(e) > 0$.
- UB (Upper Bound on cost-saving effort): There exists a number \bar{e} such that $\psi(e) \geq \bar{\beta} - \underline{\beta} + e$ for any $e \geq \bar{e}$.
- LSC (Lower Semi-Continuity): ψ is lower semi-continuous.⁵

The environment is a simple generalization of that studied by Laffont and Tirole (1986), except that negative effort is permitted.⁶ The restrictions on ψ are weaker: whilst M and LSC are satisfied in Laffont and Tirole’s environment, the present model does not require convexity or differentiability. PEC implies that the agent must be given some incentive if he is to exert effort.⁷ UB provides a bound for the set of effort levels at which the agent can receive a positive payoff when being paid no more than $\bar{\beta}$.⁸ It is consistent with an Inada condition such as suggested by Laffont and Tirole’s (1993, chapter 1.2) restatement of their model.

UB and LSC together guarantee that if the agent is offered a transfer scheme that is upper semi-continuous as a function of the realized cost, and if the associated total payment is never greater than $\bar{\beta}$, then a level of effort that maximizes his payoff exists. This is important when we suppose ψ is unknown. Without these assumptions, many transfer schemes would leave the agent with no utility-maximizing realized cost for *some* disutility function, severely restricting the set of possible incentive-compatible schemes.

⁵That is, $\{e \in \mathbb{R} : \psi(e) > \alpha\}$ is an open set for any $\alpha \in \mathbb{R}$.

⁶Although negative effort is not permitted in Laffont and Tirole’s model, Chu and Sappington (2007b) point out that allowing it and assuming NEC would not affect their analysis.

⁷This gives the intended meaning to the term “innate cost” (the term itself is due to Chu and Sappington (2007a)). The innate cost is the lowest incentive-compatible cost realized under a cost-reimbursement contract.

⁸If the agent has innate cost β , exerts effort $e \geq \bar{e}$, and is paid no more than $\bar{\beta}$, then his payoff is no more than $\bar{\beta} - (\beta - e) - (\bar{\beta} - \underline{\beta} + e) = \underline{\beta} - \beta \leq 0$.

2.2 The model with ambiguity

Given a disutility function ψ , the *economic cost* of production is the realized cost $\beta - e$ plus the agent's disutility of effort $\psi(e)$. Given a fixed-price contract, the agent maximizes his payoff by minimizing the economic cost, i.e. by choosing e to maximize $e - \psi(e)$. The maximum value, denoted k , is strictly positive. The agent would accept a fixed-price contract with price p if and only if the price were at least the minimum economic cost $\beta - k$. Therefore, the first-best policy for the principal, knowing β and ψ , would be to offer the fixed-price contract with price $\beta - k$. The principal saves k compared to a cost-reimbursement contract, where the agent's (lowest possible) realized cost is β . Thus, k is the first-best cost saving.

In what follows I suppose that the principal does not know ψ , but has limited information about it. She knows the number k and that ψ satisfies the regularity conditions M, NEC, PEC, UB and LSC. Let $\Psi(k)$ be the set of all possible disutility functions with first-best cost saving k .

The critical information from the principal's perspective is both the distribution of the innate cost F and the number k . As suggested in the introduction, this is information that might plausibly be inferred from past contracting. Data on the realized cost under a cost-reimbursement contract would identify F . If data were also available under a fixed-price contract, then k would also be identified. Suppose the fixed price p had been offered, and that the probability of its acceptance was $Q \in (0, 1)$. Since the agent would have accepted the contract if and only if $\beta \leq p + k$, it must be that

$$F(p + k) = Q,$$

and therefore,

$$k = F^{-1}(Q) - p.$$

3 Optimality of FPCR mechanisms

3.1 FPCR mechanisms

Consider, for the moment, only FPCR menus. These schemes might be thought of firstly as fixed-price contracts, with the cost-reimbursement option acting as a back-up to ensure agent participation. Since the agent receives a payoff of zero from the cost-reimbursement contract, the agent's willingness to accept the fixed-price option is unaffected by the additional option: if the price is p , he accepts if and only if $\beta \leq p + k$. Again, the number k alone determines whether the fixed-price option is preferred.

Rogerson (2003) points out that rather than setting the price, one may consider setting the *threshold* of innate costs β^* , above which the agent prefers the cost-reimbursement contract, and below which he prefers the fixed-price contract. The expected total payment is thus a function of the threshold β^* :

$$P_{FPCR}(\beta^*; F, k) \equiv F(\beta^*)(\beta^* - k) + \int_{\beta^*}^{\bar{\beta}} \tilde{\beta} f(\tilde{\beta}) d\tilde{\beta}.$$

The first term accounts for when the price $\beta^* - k$ is accepted, and the second term accounts for when the cost-reimbursement option is preferred. The function $P_{FPCR}(\cdot; F, k)$ is continuous, and so an optimal threshold exists. Moreover, Rogerson shows that the solution is unique provided the inverse hazard rate $\frac{F}{f}$ is increasing. In this case, the first-order condition informs us that the optimal threshold solves

$$F(\beta^*) = kf(\beta^*)$$

if a solution exists, and is equal to $\bar{\beta}$ otherwise. In case F is the uniform distribution, the optimal threshold is $\min\{\underline{\beta} + k, \bar{\beta}\}$.⁹

Since the principal knows F and k , the expected total payment under an FPCR scheme is unambiguous. However, considering general incentive schemes, this will not typically be

⁹See Rogerson's paper for a proof of these results.

the case. We now turn to a formal definition of the principal's problem in this environment.

3.2 General mechanisms

We will allow the principal to offer any deterministic incentive-compatible mechanism that ensures agent participation. Without any loss to the principal, we may consider mechanisms whereby she offers a transfer function $t : \mathfrak{R} \rightarrow \mathfrak{R}_+$, and then prescribes the agent an incentive-compatible realized cost $X(\beta, \psi)$ if he reports his preference to be given by $(\beta, \psi) \in [\underline{\beta}, \bar{\beta}] \times \Psi(k)$.¹⁰ Equivalently, we may think of the agent as observing the transfer function t and then giving the principal a list of realized costs that he would be willing to produce at and among which she can choose. If the agent's preferred realized cost is unique, then the list is degenerate. The restriction to non-negative transfers is without loss since the principal is required to obtain the good with certainty. It is convenient and also without loss to further restrict attention to mechanisms where the agent's total payment $t(x) + x$ is less than $\bar{\beta}$ whenever the realized cost x is less than $\bar{\beta}$, and where $t(\bar{\beta}) = 0$.

The transfer function that corresponds to an FPCR menu with price p is $t(x) = \max\{0, p - x\}$. The natural definition of an *FPCR mechanism* is any mechanism with the given transfer function that prescribes the realized cost in an incentive-compatible and cost-minimizing way.

The principal is concerned about the expected total payment

$$P(X, t; F, \psi) \equiv \int_{\underline{\beta}}^{\bar{\beta}} \left(X(\psi, \tilde{\beta}) + t(X(\psi, \tilde{\beta})) \right) f(\tilde{\beta}) d\tilde{\beta}$$

that arises when she chooses mechanism (X, t) and the agent's disutility function is ψ . To ensure this is well-defined, we will restrict attention to mechanisms such that

¹⁰The implementation as a transfer scheme and contingent prescription of realized cost, in preference to the fully-direct mechanism, is useful for focussing on the transfer scheme, which is the object of interest. This is without any loss of generality due to the taxation principle, as described by Guesnerie and Laffont (1984). In essence, this says that a mechanism designed for only one agent is incentive compatible if and only if it can be implemented via an appropriate transfer scheme.

$X(\psi, \cdot) + t(X(\psi, \cdot))$ is a measurable function for each $\psi \in \Psi(k)$.¹¹

Since ψ is unknown, the value of $P(X, t; F, \psi)$ is ambiguous to the principal. One way for her to select mechanisms given this ambiguity is to use the minimax criterion. The minimax problem is to minimize by choice of mechanism (X, t) the worst-case expected total payment

$$S(X, t; F, k) \equiv \sup_{\psi \in \Psi(k)} P(X, t; F, \psi).$$

Before stating formally that an FPCR mechanism solves this problem, it may help understanding to consider the performance of another simple mechanism under the minimax criterion. Chu and Sappington (2007a) consider menus consisting of a cost-reimbursement contract and a “linear cost-sharing” contract, whereby the agent is reimbursed a fraction $\alpha \in (0, 1)$ of all cost savings below a certain level, say x^* . The corresponding transfer function is $t(x) = \max\{0, \alpha(x^* - x)\}$. At a sufficiently low realized cost \bar{x} , the total payment $\bar{x} + t(\bar{x})$ is below $\underline{\beta} - k$, the lowest possible economic cost of production.

Define $\bar{e} = \underline{\beta} - \bar{x}$ to be the effort required to realize \bar{x} when the agent has innate cost $\underline{\beta}$. Suppose that the agent’s disutility function were characterized by a fixed disutility for effort above zero, no additional disutility for effort up to \bar{e} , and an arbitrarily large disutility thereafter.¹² Given this disutility function, and given that the transfer function is non-increasing, the agent would either be willing to use effort \bar{e} , or only be willing to use zero effort.¹³

¹¹This will be the case, for example, if t is upper semi-continuous and the principal chooses appropriately among incentive-compatible cost prescriptions. This follows because (due to the upper semi-continuity of the agent’s objective, the restriction to total payments below $\bar{\beta}$ and assumption UB) the mapping from innate costs to the agent’s incentive-compatible realized costs is an upper hemi-continuous correspondence. By Theorem 7.6 of Stokey and Lucas (1989), there exists a measurable selection from such a correspondence. If the principal chooses a measurable selection of realized costs, then the total payment must also be measurable by the upper semi-continuity of the transfer function.

¹²A wide range of disutility functions would suit the present purpose, and could be used to prove Proposition 1 below. I am grateful to Bill Rogerson for suggesting the proposed construction, which seems the simplest.

¹³Strictly speaking, he may also be willing to use negative effort. However, we may ignore this possibility for the purposes of discussion.

Consistency with first-best cost saving k requires the economic cost of production for an agent with innate cost $\underline{\beta}$ using effort \bar{e} to be $\underline{\beta} - k$. Since the total payment $\bar{x} + t(\bar{x})$ is less than $\underline{\beta} - k$, the agent strictly prefers zero effort. That t is non-increasing implies that the agent would not willingly exert effort for any other innate cost either. Therefore, for this disutility function, a cost-reimbursement contract performs just as well. Since the expected total payment under a cost-reimbursement contract, which provides no incentives for effort, does not depend on the disutility function, it must perform at least as well as Chu and Sappington's proposed menu according to the minimax criterion.

More generally, we must allow the principal to use transfer schemes such that the lowest total payment is above $\underline{\beta} - k$. Considering all possible mechanisms, the following proposition shows that the principal can do at least as well according to the minimax criterion by choosing an FPCR mechanism.

Proposition 1 *Let F be a distribution function on $[\underline{\beta}, \bar{\beta}]$ and $k > 0$ the first-best cost saving. For any mechanism (X, t) , there exists an FPCR mechanism such that the principal has expected total payment no greater than $S(X, t; F, k)$ for any $\psi \in \Psi(k)$. In particular, any optimal FPCR mechanism solves the minimax problem.*

The proof is most easily understood by considering the example of a mechanism (X, t) for which t is non-increasing and continuous, and for which $l(t)$, the minimum total payment under t , is attained. To contrast with the situation above, suppose that $l(t)$ exceeds $\underline{\beta} - k$.¹⁴

Suppose that the minimum total payment occurs at \bar{x} and consider the fictitious environment in which the agent is forced to choose between using zero effort and using the effort required for realized cost \bar{x} . If the agent has an innate cost above $l(t) + k$, then given first-best cost saving k , he will strictly prefer zero effort. This is the same outcome as for an FPCR mechanism with price $l(t)$.

¹⁴The formal proof extends the argument below to all cases, including situations in which t fails to be non-increasing or is discontinuous.

To build on this insight, consider the agent's actual decision problem when his disutility function takes the form, introduced above, where there is a fixed disutility for positive effort up to some level \bar{e} . As above, the agent is either willing to use effort \bar{e} or strictly prefers no effort. Let $\varepsilon > 0$ and suppose the agent's innate cost is $l(t) + k + \varepsilon$. Define $\bar{e} = \bar{x} - (l(t) + k + \varepsilon)$ to be the effort required for the agent to realize \bar{x} . Because the first-best cost saving is k , the economic cost of producing with effort \bar{e} must be $l(t) + \varepsilon$. Since the total payment at \bar{x} is only $l(t)$, the agent prefers zero effort. That t is non-increasing implies that the agent prefers zero effort when his innate cost is above $l(t) + k + \varepsilon$ as well.

We have seen, then, that under (X, t) , and for a particular possible realization of the disutility function, the agent prefers zero effort for any innate cost above $l(t) + k + \varepsilon$. On the other hand, the total payment is never less than $l(t)$. Therefore, only for innate costs in the interval $(l(t) + k, l(t) + k + \varepsilon)$ could the FPCR mechanism with price $l(t)$ have a higher total payment. The probability of these innate costs vanishes as ε is taken to zero, and hence the worst-case expected total payment, i.e. the supremum over all possible disutility functions in $\Psi(k)$, must be at least that under the FPCR mechanism with price $l(t)$.

3.3 Further ambiguity

The result of Proposition 1 does not require the principal to know the first-best cost saving k exactly. Instead, she might only know that the first-best cost saving is an element of some interval $\mathcal{K} = [\underline{k}, \bar{k}]$. That is, the principal may wish to consider disutility functions for which the first-best cost saving can take a range of values. The modified problem is to minimize by choice of the mechanism (X, t)

$$\begin{aligned} \hat{S}(X, t; F, \mathcal{K}) &\equiv \sup_{\{\psi \in \Psi(k): k \in \mathcal{K}\}} \int_{\underline{\beta}}^{\bar{\beta}} X(\psi, \tilde{\beta}) + t(X(\psi, \tilde{\beta})) dF(\tilde{\beta}) \\ &= \sup_{k \in \mathcal{K}} S(X, t; F, k). \end{aligned}$$

Consider the modified problem in which the principal is told k and can minimize $S(X, t; F, k)$ given this information. Clearly, the expected total payment is largest for $k = \underline{k}$. Therefore $\hat{S}(X, t; F, \mathcal{K})$ must be minimized by any policy that minimizes $S(X, t; F, \underline{k})$. The result is summarized in Corollary 1.

Corollary 1 *Let F be a distribution function on $[\underline{\beta}, \bar{\beta}]$ and suppose $\mathcal{K} = [\underline{k}, \bar{k}]$ is an interval of possible first-best cost savings. The FPCR mechanism that is optimal for $k = \underline{k}$ solves the modified minimax problem in which it is only known that $k \in \mathcal{K}$.*

The principal might also face ambiguity over the distribution of innate costs. Again, an FPCR mechanism may solve the minimax problem. Suppose that the principal knows only that $k \in \mathcal{K}$ and that the distribution function comes from some set \mathcal{F} . Suppose, in addition, that there exists a distribution $G \in \mathcal{F}$ such that G is first-order stochastically dominated by every $F \in \mathcal{F}$. Then the above argument implies that an FPCR mechanism is optimal, where the price is calculated with reference to first-best cost saving \underline{k} and innate cost distribution G .

4 Weak dominance of FPCR mechanisms

Although the optimal FPCR mechanism solves the minimax problem, it may be weakly dominated. This is concerning for a theory of fully-optimal simple mechanisms – a regulator or procurement officer might feel justified in choosing more complex schemes if she stood only to gain from doing so.

Consider an optimal FPCR mechanism with innate-cost threshold β^* . To show that this mechanism is weakly dominated by another, we must compare the expected total payment under the FPCR mechanism (which does not depend on the disutility function) to the expected total payment under the alternative mechanism for each possible disutility function.

To show that the FPCR mechanism is weakly dominated, it turns out we can consider

alternative mechanisms with transfer function

$$t_{\alpha,\nu,\beta^*}(x) = \max\{0, \alpha(\beta^* - k + \nu - x), \beta^* - k - x\}. \quad (1)$$

The transfer function t_{α,ν,β^*} is the upper envelope of a menu of three contracts: a cost-reimbursement contract, a linear cost-sharing contract that makes the agent bear α of any increment of the realized cost and yields a positive transfer if and only if the realized cost is less than $\beta^* - k + \nu$, and a fixed-price contract with price $\beta^* - k$.

Any total payment above $\beta^* - k + \nu$ must arise from the cost-reimbursement contract. Innate costs above $\beta^* + \nu$ thus face no additional incentives to exert effort compared to the FPCR transfer function. However, for innate costs in the interval $(\beta^*, \beta^* + \nu)$, the agent may be induced to exert effort, depending on the disutility function. This represents a possible advantage over the FPCR mechanism.

We can fully specify the mechanism associated with t_{α,ν,β^*} by letting $X_{\alpha,\nu,\beta^*}(\beta, \psi)$ assign the minimum incentive-compatible realized cost for each agent preference (β, ψ) .¹⁵ Since the total payment under t_{α,ν,β^*} is non-decreasing, this minimizes the expected total payment and ensures it is well-defined.¹⁶ These mechanisms are used to show the following result.

Proposition 2 *Suppose that the optimal FPCR mechanism is unique and has innate-cost threshold β^* . The optimal FPCR mechanism is undominated if and only if $\beta^* = \bar{\beta}$.*

The result states that the optimal FPCR mechanism is weakly dominated whenever it fails to provide incentives for positive effort for all innate costs (i.e., when $\beta^* < \bar{\beta}$). The

¹⁵Minima exist because, for any possible agent preference (β, ψ) , the set of incentive-compatible realized costs is compact. This follows from UB, LSC and the continuity of t_{α,ν,β^*} .

¹⁶The agent's prescribed effort is non-increasing in his innate cost for each disutility function. Therefore his effort, and hence total payment, are measurable functions. To see that the prescribed effort is non-increasing, note that, because t_{α,ν,β^*} is (weakly) convex, the agent's payoff $t_{\alpha,\nu,\beta^*}(\beta - e) - \psi(e)$ exhibits (weakly) increasing differences in $(\beta, -e)$. Since under t_{α,ν,β^*} there must always be an incentive-compatible realized cost less than or equal to the innate cost, we may restrict attention to non-negative efforts. Applying monotone comparative statics (see Topkis (1998) and Milgrom and Shannon (1994)) for the lattice $[\underline{\beta}, \bar{\beta}] \times \mathfrak{R}_-$, we have that the minimum of $\arg \max_e \{t_{\alpha,\nu}(\beta - e) - \psi(e)\}$ is non-increasing in β .

proof shows that whenever this is the case, there exists a transfer function of the form specified in (1) that can induce effort for innate costs above β^* for some disutility function. Although the agent then also receives a higher total payment for some innate costs below β^* , this detriment does not fully offset the benefit of additional effort for innate costs above β^* , provided that ν is sufficiently small.

On the other hand, if the optimal FPCR mechanism elicits effort for all innate costs (i.e. $\beta^* = \bar{\beta}$), then there is no advantage to additional incentives. The optimal FPCR mechanism is therefore undominated. This means that any mechanism that solves the minimax problem must have the same expected total payment as for the optimal FPCR mechanism, regardless of the disutility function $\psi \in \Psi(k)$. In this sense, the FPCR mechanism is essentially the unique solution to the minimax problem when $\beta^* = \bar{\beta}$.¹⁷ This will also be the case for the undominated mechanisms considered in Section 5.

If innate costs are uniformly distributed, the two cases can be distinguished simply as follows.

Corollary 2 *Suppose the first-best cost saving is k and that innate costs are uniformly distributed on $[\underline{\beta}, \bar{\beta}]$. The optimal FPCR mechanism is undominated if and only if $k \geq \bar{\beta} - \underline{\beta}$.*

When the FPCR mechanism is weakly dominated, it would be natural to consider more complex mechanisms that also solve the minimax problem. Note, however, that not all mechanisms that weakly dominate the optimal FPCR mechanism need be undominated. Indeed, characterizing the set of undominated solutions to the minimax problem appears to be a difficult task.¹⁸

¹⁷Mechanisms may vary in the realized cost prescription compared to the optimal FPCR mechanism without affecting the expected total payment. This can occur either because there is more than one realized cost that minimizes the total payment, or because differences in the total payment arise only on a set of measure zero.

¹⁸An example, available from the author, shows a) that not every mechanism that weakly dominates an optimal FPCR mechanism is itself undominated, and b) that there may exist a range of undominated mechanisms (with distinct transfer functions) that solve the minimax problem.

5 Ambiguity about dependence of effort preferences on money

Optimal FPCR mechanisms may also be undominated if the principal faces the possibility that the agent's preferences for effort depend on the transfer he receives; in other words, if she wishes to admit the possibility that preferences are not quasi-linear in the transfer. Although quasi-linearity might be an appropriate assumption in some circumstances, there are many reasons why the principal may not believe that it holds. A firm's management may be more willing to exert effort when the profit margin is small, perhaps because of an increase in the perceived risk of termination or takeover. On the other hand, it may be less inclined to exert effort if it views the available rewards that the principal has offered as too small – perhaps purely for reasons of reciprocity, in the sense suggested by Akerlof (1982). In fact, it seems there would rarely be objective information precluding quite arbitrary dependence of the agent's preferences over effort on the transfer received.

To see that allowing more general dependence can rule out the possibility of weak dominance, consider preferences for which the disutility of effort can depend on the transfer as follows. Let the agent's payoff for an arbitrary pair $(e, y) \in \mathbb{R}^2$ be $y - \psi(e, y)$. Assume that $y - \psi(e, y)$ is strictly increasing in y . Assume, analogously to the quasi-linear environment, that the possible disutility functions satisfy:

- M' (Monotonicity'): For each y , $\psi(\cdot, y)$ is non-decreasing.
- NEC' (Non-positive Effort is Costless'): For each y and each $e \leq 0$, $\psi(e, y) = 0$.
- PEC' (Positive Effort is Costly'): For each y and each $e > 0$, $\psi(e, y) > 0$.
- UB' (Upper Bound on cost-saving effort'): There exists a number \bar{e} such that $\psi(e, y) \geq \bar{\beta} - \underline{\beta} + e$ for any $e \geq \bar{e}$ and any y .
- LSC' (Lower Semi-Continuity'): The function ψ is lower semi-continuous in (e, y) .

The first-best cost saving k is now given by $\max\{e - y : y - \psi(e, y) \geq 0\}$. The following result then applies when the principal's view of possible disutility functions is governed

by the above assumptions.

Proposition 3 *If ψ can depend on the transfer, then any optimal FPCR mechanism is undominated.*

The proof builds on that of Proposition 1. For an overview of the proof, consider a uniquely optimal FPCR mechanism and an alternative mechanism (X, t) for which t is non-increasing and the minimum total payment $l(t)$ is attained (the formal proof considers all possible alternatives). If (X, t) were to weakly dominate the FPCR mechanism, we have seen that a) necessarily $l(t)$ would be equal to the price of the FPCR mechanism, and b) it would offer a higher total payment than the FPCR mechanism for some values of realized cost (in order to elicit effort when innate costs are above $l(t) + k$). It would be enough to show that there exists a disutility function for which the higher total payments are preferred for innate costs below $l(t) + k$, whilst the agent still prefers zero effort for innate costs above $l(t) + k$. This is essentially the approach taken. The key step is to make the disutility small for appropriate effort levels, conditional on receiving a sufficiently large transfer. This violation of quasi-linearity means that there are incentives for choosing realized costs for which the total payment exceeds $l(t)$ for innate costs below $l(t) + k$.

6 Conclusions

This paper has provided a foundation for the use of FPCR menus based on optimal worst-case decision making. If the principal faces sufficient ambiguity, including ambiguity about how the agent's preferences for effort depend on monetary transfers, then the optimal FPCR menu fulfils both the minimax and weak dominance criteria.

This result may be viewed as a possible explanation for observed behavior – FPCR menus, whether explicitly offered as two-contract menus or as fixed-price contracts with an implicit limited-liability provision, are very common in practice. The result may also provide guidance to regulators and policy makers. Under assumptions they may find

plausible to make, and information that might be obtained, FPCR menus are an optimal worst-case choice.

The paper also provides an alternative approach for empirically determining the possible gains from implementing optimal procurement or regulatory mechanisms as studied, for instance, by Gagnepain and Ivaldi (2002) and by Gasmi, Laffont and Sharkey (1999). Both papers make functional form assumptions on the disutility of effort that are motivated by tractability, but not necessarily realism. The alternative approach, the details of which must depend on the situation at hand, would be to make informed assumptions about the possible forms of the disutility function (perhaps those considered here), and then calculate the solution to the minimax problem. The solution would provide a sharp lower bound on the gains from implementing the optimal mechanism, given the information that the researcher possesses.

The approach to modeling, however, raises two natural concerns. Firstly, what information precisely is available to regulators and procurement officers – how much ambiguity do they face? Secondly, what is an appropriate view of their decision criteria? The minimax criterion might well be “too conservative”. Future research might address these questions and further explore the trade-offs involved in considering different available information and decision criteria.

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Appendix

This Appendix provides formal proofs of all results.

Proof of Proposition 1. Let $\widehat{t}(x) = \sup_{x' \geq x} t(x')$ and define

$$l(t) = \max \{ \inf \{ \widehat{t}(x) + x \}, \underline{\beta} - k \}.$$

The number $l(t)$ is a lower bound on the agent's incentive-compatible total payments. Firstly, it is never incentive compatible to pay the agent less than $\underline{\beta} - k$, since this gives him a negative payoff regardless of his innate cost and disutility function. Secondly, the infimum of $\widehat{t}(x) + x$ over all realized costs is no greater than the infimum of $t(x) + x$ over only incentive-compatible ones. This follows because a necessary condition for x to be incentive compatible is that $t(x) = \widehat{t}(x)$. If $t(x) < \widehat{t}(x)$, then by definition of \widehat{t} , there must be an $x' > x$ with $t(x') > t(x)$. The agent prefers x' because it both gives a higher transfer and requires less effort.

In contrast to the situation considered in the discussion, there may be no value x such that $\widehat{t}(x) + x = l(t)$. Therefore, let $\varepsilon > 0$ and define x_ε to be any realized cost such that $\widehat{t}(x_\varepsilon) + x_\varepsilon < l(t) + \varepsilon$. Following the construction in the discussion, define the disutility function $\psi_{x_\varepsilon, \varepsilon} \in \Psi(k)$ by

$$\psi_{x_\varepsilon, \varepsilon}(e) = \begin{cases} 0 & \text{if } e \leq 0 \\ l(t) + \varepsilon - x_\varepsilon & \text{if } 0 < e \leq l(t) + k + \varepsilon - x_\varepsilon \\ \bar{\beta} - \underline{\beta} + e & \text{if } e > l(t) + k + \varepsilon - x_\varepsilon \end{cases}.$$

Suppose that $\beta \geq l(t) + k + \varepsilon$. Then, at effort $e \in (0, l(t) + k + \varepsilon - x_\varepsilon]$, the agent

realizes some cost $x \in [x_\varepsilon, \beta)$. The agent's payoff is then

$$\begin{aligned} t(x) - (l(t) + \varepsilon - x_\varepsilon) &\leq \hat{t}(x) - (l(t) + \varepsilon - x_\varepsilon) \\ &\leq \hat{t}(x_\varepsilon) + x_\varepsilon - (l(t) + \varepsilon) \\ &< 0. \end{aligned}$$

The first inequality follows because \hat{t} is everywhere at least t , the second because $x_\varepsilon \leq x$ and because \hat{t} is non-increasing, and the third by choice of x_ε . Therefore effort prescriptions in the interval $(0, l(t) + k + \varepsilon - x_\varepsilon]$ are not incentive compatible. This implies that the agent's total payment must be at least his innate cost β if he is to have a non-negative payoff.¹⁹

An FPCR mechanism with price $l(t)$ implies a total payment of $l(t)$ for innate costs no greater than $l(t) + k$, regardless of the disutility function. It follows that the FPCR mechanism with fixed price $l(t)$ implies a higher total payment for at most innate costs $\beta \in (l(t) + k, l(t) + k + \varepsilon)$. The probability of such innate costs converges to zero as ε tends to zero. ■

Proof of Proposition 2. Sufficiency. Suppose $\beta^* < \bar{\beta} - k$. Let $\nu \in (0, \bar{\beta} - \beta^*]$ and $\alpha \in (0, 1)$. Consider the mechanism $(X_{\alpha, \nu, \beta^*}, t_{\alpha, \nu, \beta^*})$ defined in the text, with t_{α, ν, β^*} given by (1). Denote by d^* the threshold level of innate cost, dependent on the agent's disutility function, above which he is prescribed at least his innate cost, and below which he is prescribed less than his innate cost. As discussed in the text, that $\nu < \bar{\beta} - \beta^*$ implies $d^* < \bar{\beta}$ for any disutility function. Since the threshold under the FPCR mechanism is β^* , $\delta = d^* - \beta^*$ is the measure of innate costs for which the agent exerts positive effort under $(X_{\alpha, \nu, \beta^*}, t_{\alpha, \nu, \beta^*})$, but not the FPCR mechanism.

When $\delta = 0$, the total payment is the same as under the FPCR mechanism for each

¹⁹The other effort level to consider is $e > l(t) + k + \varepsilon - x_\varepsilon$. In this case, the agent receives a non-negative payoff only if the transfer is at least $\bar{\beta} - \underline{\beta} + e$. For innate cost β , the payment is then at least $\beta - e + (\bar{\beta} - \underline{\beta} + e) \geq \bar{\beta}$. However, as noted above, we need not consider mechanisms that imply a total payment at least $\bar{\beta}$ for innate costs other than $\bar{\beta}$ (since they are weakly dominated).

innate cost. The important case to consider is therefore $\delta > 0$. This occurs if the disutility of low effort levels is sufficiently small. For example, let $\varepsilon > 0$ and define $\psi_\varepsilon \in \Psi(k)$ by

$$\psi_\varepsilon(e) = \begin{cases} 0 & \text{if } e \leq 0 \\ \varepsilon & \text{if } 0 < e \leq k + \varepsilon \\ \bar{\beta} - \underline{\beta} + e & \text{if } e > k + \varepsilon \end{cases} .$$

For $\varepsilon < \alpha\nu$, the agent with innate cost β^* can get a strictly positive payoff under the transfer function t_{α,ν,β^*} . Indeed, if he uses effort $e = k$, then his payoff under the cost-sharing contract is $\alpha\nu - \varepsilon > 0$. Since ψ_ε is continuous, there exists an interval of innate costs above β^* for which the payoff is also positive, and thus for which positive effort is preferred.

For innate costs in the interval $(\beta^*, d^*]$, the agent must prefer the cost-sharing contract over the fixed-price contract. Under the cost-sharing contract, if the agent has innate cost β less than d^* , then he appropriates α of the associated cost savings. His payoff is therefore $\alpha(d^* - \beta)$. On the other hand, his payoff from efficient effort under the fixed-price contract is $\beta^* - \beta$. The fixed-price contract is (weakly) preferred by the agent if and only if $\beta^* - \beta \geq \alpha(d^* - \beta)$, or equivalently, $\beta \leq \beta^* - \frac{\alpha}{1-\alpha}\delta$.

The mechanism therefore prescribes the cost-sharing contract for innate costs in the interval $(\beta^* - \frac{\alpha}{1-\alpha}\delta, \beta^* + \delta]$. The principal benefits relative to the FPCR mechanism when $\beta > \beta^*$, saving no less than $k - \nu$, and makes a total payment higher by no more than ν when $\beta \leq \beta^*$. The expected additional benefit of using $(X_{\alpha,\nu,\beta^*}, t_{\alpha,\nu,\beta^*})$ instead of the optimal FPCR mechanism is therefore no less than

$$h(\delta) = (k - \nu) (F(\beta^* + \delta) - F(\beta^*)) - \nu \left(F(\beta^*) - F\left(\beta^* - \frac{\alpha}{1-\alpha}\delta\right) \right).$$

Note that $\delta < \nu$. For any $\eta > 0$, the definition of the derivative implies that there is

$\nu > 0$ sufficiently small that

$$\frac{F(\beta^* + \delta) - F(\beta^*)}{\delta} - f(\beta^*) > -\eta,$$

and

$$\frac{F(\beta^*) - F\left(\beta^* - \frac{\alpha}{1-\alpha}\delta\right)}{\delta} - \frac{\alpha}{1-\alpha}f(\beta^*) < \eta.$$

Therefore,

$$\begin{aligned} h(\delta) &> \delta(k - \nu)(f(\beta^*) - \eta) - \nu\delta\left(\frac{\alpha}{1-\alpha}f(\beta^*) + \eta\right) \\ &= \delta\left(f(\beta^*)\left(k - \frac{\nu}{1-\alpha}\right) - \eta k\right). \end{aligned}$$

The quantity

$$f(\beta^*)\left(k - \frac{\nu}{1-\alpha}\right) - \eta k$$

can be taken arbitrarily close to $f(\beta^*)k > 0$ by choosing η and ν sufficiently small. For these choices, we have $h(\delta) > 0$.

Necessity. If $\beta^* = \bar{\beta}$, then the total payment is $\bar{\beta} - k$ for every $\beta \in [\underline{\beta}, \bar{\beta}]$ and every $\psi \in \Psi(k)$. Suppose that (X, t) were a mechanism with a lower total payment for some preference (β, ψ) . There must then be a realized cost x that is incentive compatible for (β, ψ) and such that $t(x) + x < \bar{\beta} - k$. As argued in the proof of Proposition 1, incentive compatibility of x implies $t(x) = \hat{t}(x)$. This implies that $l(t) \equiv \max\{\inf\{\hat{t}(x) + x\}, \underline{\beta} - k\} < \bar{\beta} - k$. As seen in the proof of Proposition 1, (X, t) has an expected total payment that, for some $\psi \in \Psi(k)$, is less than that of an FPCR mechanism with price $l(t)$ (which is suboptimal) by no more than an arbitrarily small amount. Thus, there must exist $\psi \in \Psi(k)$ such that the expected total payment is strictly greater than that under the optimal FPCR mechanism, which has price $\bar{\beta} - k$. ■

Proof of Proposition 3. It is enough to consider an alternative mechanism (X, t) that solves the minimax problem. Let $l(t) = \inf \{\hat{t}(x) + x\}$. The proof of Proposition 1 shows $l(t) = \beta^* - k$, where β^* is the threshold of an optimal FPCR mechanism. We may also assume t has a higher total payment than the optimal FPCR mechanism at some realized cost.

Define $\tilde{t}(x) = \sup_{x' > x} t(x')$. In order to provide an outline of the proof, consider a disutility function $\psi(e) = \tilde{t}(\beta_\varepsilon^* - e) + \varepsilon$ (not an element of $\Psi(k)$), where β_ε^* is greater than β^* but converges to it as the positive number ε is taken to zero. For innate cost $\beta > \beta_\varepsilon^*$ and disutility function ψ , the agent's payoff is no more than $-\varepsilon$.

This construction need not imply a higher expected total payment under (X, t) than for the FPCR mechanism. What is needed is a way to make the agent prefer realized costs for which the total payment is above $l(t)$ for at least a positive measure of innate costs below β^* . To this end, we will choose a particular realized cost z_ε for which $t(z_\varepsilon) + z_\varepsilon > l(t)$ (more below).

For innate costs less than β^* , effort no greater than $e^*(z_\varepsilon) = \beta^* - z_\varepsilon$ is needed to attain z_ε . We may then modify the disutility function so that the agent suffers only small disutility (say ε) from choosing effort up to $e^*(z_\varepsilon)$ whenever he would receive a non-negative payoff under the original disutility function (this is a violation of quasi-linearity). Further, we may choose z_ε so that this level of realized cost implies a non-negative payoff under the original disutility function for all innate costs up to just below β^* . For such innate costs, provided they are sufficiently close to β^* , the payoff that results from choosing z_ε can only be attained from a total payment that is close to $t(z_\varepsilon) + z_\varepsilon$, i.e. above $l(t)$. A lower bound on the additional total payment above $l(t)$, together with a measure of innate costs that must receive the additional payment, can be chosen so as not to vanish as ε is taken to zero. On the other hand, the set of innate costs above β^* for which the agent chooses positive effort does vanish. This means that there exists a disutility function for which the expected total payment under (X, t) is higher than the FPCR mechanism.

The formal proof proceeds in four steps. Step 1 establishes properties of \tilde{t} required to

guarantee that the realized cost z_ε can be chosen appropriately. Given $\varepsilon > 0$, z_ε is chosen in Step 2 so that the associated transfer exceeds the FPCR transfer (by at least some amount that does not depend on ε). It is also chosen so that the agent has a positive payoff from this realized cost if his innate cost is below β^* (by at least some amount that shrinks to zero as ε is taken to zero), and if his disutility function is $\psi(e) = \tilde{t}(\beta_\varepsilon^* - e) + \varepsilon$. Step 3 shows that for innate costs above β_ε^* , the agent must prefer non-positive effort given disutility function ψ . Step 4 constructs the modified disutility function as described above and completes the argument.

Step 1 *There exist \hat{x} and $\gamma, \delta > 0$ such that each $x \in (\hat{x} - \delta, \hat{x})$ satisfies both (i) $\tilde{t}(x) > \tilde{t}(\hat{x}) + \gamma(\hat{x} - x)$ and (ii) $\tilde{t}(x) > \max\{0, l(t) - x\} + \delta$.*

Proof. There must exist \tilde{x} and $\delta > 0$ such that $\tilde{t}(\tilde{x}) - \max\{0, l(t) - \tilde{x}\} \geq 3\delta$. Let $U = 2\bar{\beta} - \tilde{x}$ and define $\gamma = \frac{\max\{0, l(t) - \tilde{x}\} + 2\delta}{U - \tilde{x}}$ and $g(x) = \gamma(U - x)$. Since we can restrict attention to transfers for which $\tilde{t}(\bar{\beta}) = 0$, the set $\{x \in [\tilde{x}, \bar{\beta}] : \tilde{t}(x) - g(x) \leq 0\}$ is non-empty. Since $\tilde{t} - g$ is lower semi-continuous, this set is compact and so its minimum, denoted \hat{x} , exists.

By construction, $\tilde{t}(x) > g(x)$ for any $x \in [\tilde{x}, \hat{x})$. This is also true for any $x \in (\hat{x} - \delta, \hat{x})$, since for such x ,

$$\begin{aligned}
\tilde{t}(x) - g(x) &\geq \tilde{t}(\tilde{x}) - g(\tilde{x}) - \gamma(\tilde{x} - x) \\
&> \tilde{t}(\tilde{x}) - g(\tilde{x}) - \delta \\
&= \tilde{t}(\tilde{x}) - \max\{0, l(t) - \tilde{x}\} - 3\delta \\
&\geq 0.
\end{aligned}$$

The first inequality follows since \tilde{t} is non-increasing and the second follows by choice of x

and because $\gamma < 1$. This implies property (i) since, for any $x \in (\tilde{x} - \delta, \hat{x})$,

$$\begin{aligned}\tilde{t}(x) &> g(x) \\ &= g(\hat{x}) + \gamma(\hat{x} - x) \\ &\geq \tilde{t}(\hat{x}) + \gamma(\hat{x} - x).\end{aligned}$$

To see that property (ii) holds, note that g is the line passing through $\max\{0, l(t) - \tilde{x}\} + 2\delta$ at \tilde{x} and $\frac{1}{2}(\max\{0, l(t) - \tilde{x}\} + 2\delta)$ at $\bar{\beta}$. Therefore, $g(x) \geq \max\{0, l(t) - x\} + \delta$ for $x \in [\tilde{x}, \bar{\beta}]$, and so $\tilde{t}(x) > \max\{0, l(t) - x\} + \delta$ for $x \in [\tilde{x}, \hat{x})$. That \tilde{t} is non-increasing guarantees this for $x \in (\tilde{x} - \delta, \tilde{x})$ as well. ■

Step 2 Let γ and δ be defined as in Step 1, and let $\varepsilon \in (0, \frac{\gamma\delta}{2})$ and $\beta_\varepsilon^* > \beta^*$. There exists z_ε such that (i) $t(z_\varepsilon) > \max\{0, l(t) - z_\varepsilon\} + \frac{\delta}{2}$ and (ii) for each $\beta \leq \beta^* - \frac{2\varepsilon}{\gamma}$, $t(z_\varepsilon) > \tilde{t}(\beta_\varepsilon^* - \beta + z_\varepsilon) + \varepsilon$.

Proof. Let $\eta < \min\{\varepsilon, \frac{\delta}{2}\}$ and choose $z_\varepsilon \in (\hat{x} - \frac{2\varepsilon}{\gamma}, \hat{x})$ such that $t(z_\varepsilon) > \tilde{t}(\hat{x} - \frac{2\varepsilon}{\gamma}) - \eta$. This is possible by definition of \tilde{t} and by (i) of Step 1. Property (i) then follows from (ii) of Step 1, which guarantees that $\tilde{t}(z_\varepsilon) > \max\{0, l(t) - z_\varepsilon\} + \delta$, and the fact that $t(z_\varepsilon) > \tilde{t}(z_\varepsilon) - \eta$ (which is true by choice of z_ε , since \tilde{t} is non-increasing).

For property (ii), we have that for any $\beta \leq \beta^* - \frac{2\varepsilon}{\gamma}$,

$$\begin{aligned}t(z_\varepsilon) - \tilde{t}(\beta_\varepsilon^* - \beta + z_\varepsilon) - \varepsilon &> \tilde{t}\left(\hat{x} - \frac{2\varepsilon}{\gamma}\right) - \eta \\ &\quad - \tilde{t}\left(\beta_\varepsilon^* - \beta - \frac{2\varepsilon}{\gamma} + \hat{x}\right) - \varepsilon \\ &\geq \tilde{t}\left(\hat{x} - \frac{2\varepsilon}{\gamma}\right) - \eta - \tilde{t}(\hat{x}) - \varepsilon \\ &> \varepsilon - \eta \\ &> 0.\end{aligned}$$

The first inequality follows by choice of z_ε and because \tilde{t} is non-increasing. The second

inequality also follows because \tilde{t} is non-increasing. The third follows by property (i) of Step 1, since $\frac{2\varepsilon}{\gamma} < \delta$. ■

Step 3 *If $\beta \geq \beta_\varepsilon^* + \varepsilon$, then $t(x) < \tilde{t}(\beta_\varepsilon^* - \beta + x) + \varepsilon$ for any x .*

Proof. This follows because for any realized cost x ,

$$\begin{aligned} t(x) - \tilde{t}(\beta_\varepsilon^* - \beta + x) - \varepsilon &\leq \tilde{t}(x - \varepsilon) - \tilde{t}(\beta_\varepsilon^* - \beta + x) - \varepsilon \\ &\leq -\varepsilon < 0. \end{aligned}$$

The first inequality is immediate from the definition of \tilde{t} , and the second follows because \tilde{t} is non-increasing. ■

Step 4 *There exists a disutility function with first-best cost saving k such that t implies a higher expected total payment than the FPCR mechanism with threshold β^* .*

Proof. By definition of $l(t)$, we may choose $\underline{x} < z_\varepsilon$ such that

$$m(\underline{x}) = \min_{x \geq \underline{x}} \tilde{t}(x) + x < l(t) + \varepsilon.$$

Put $\omega_\varepsilon = m(\underline{x}) - l(t)$ and put $\beta_\varepsilon^* = \beta^* + \varepsilon + \omega_\varepsilon$. Define $\bar{e} = \beta_\varepsilon^* - \underline{x}$ and $e^*(z_\varepsilon) = \beta^* - z_\varepsilon$, and note that $\bar{e} > e^*(z_\varepsilon)$. Define the following disutility function:

$$\psi_{\varepsilon, z_\varepsilon}(e, y) = \begin{cases} 0 & \text{if } e \leq 0; \\ \varepsilon & \text{if } e \in (0, e^*(z_\varepsilon)] \\ & \text{and } y \geq \tilde{t}(\beta_\varepsilon^* - e) + \varepsilon; \\ \tilde{t}(\beta_\varepsilon^* - e) + \varepsilon & \text{if } e \in (e^*(z_\varepsilon), \bar{e}], \\ & \text{or if } e \in (0, e^*(z_\varepsilon)] \\ & \text{and } y < \tilde{t}(\beta_\varepsilon^* - e) + \varepsilon; \\ \bar{\beta} - \underline{\beta} + e & \text{if } e > \bar{e}. \end{cases} .$$

Note that for $e \in (0, \bar{e}]$, $y - \psi_{\varepsilon, z_\varepsilon}(e, y) \geq 0$ if and only if $y \geq \tilde{t}(\beta_\varepsilon^* - e) + \varepsilon$. The maximum first-best cost saving is therefore

$$\begin{aligned} \max_{e \leq \bar{e}} \{e - \tilde{t}(\beta_\varepsilon^* - e) - \varepsilon\} &= \max_{x \geq \underline{x}} \{\beta^* + \omega_\varepsilon - x - \tilde{t}(x)\} \\ &= \beta^* - l(t) \\ &= k. \end{aligned}$$

It is straightforward to verify that $y - \psi_{\varepsilon, z_\varepsilon}(e, y)$ is increasing in y and satisfies the principal's assumptions (M', NEC', PEC', UB' and LSC').

By property (ii) of Step 2, if $\beta \leq \beta^* - \frac{2\varepsilon}{\gamma}$ and the agent's realized cost is z_ε , then his disutility from effort is ε . By (i) of Step 2, his transfer from realized cost z_ε is more than $\max\{0, l(t) - z_\varepsilon\} + \frac{\delta}{2}$ and so his payoff is more than $\max\{0, l(t) - z_\varepsilon\} + \frac{\delta}{2} - \varepsilon$. If the agent has innate cost $\beta \in (\beta^* - \frac{\delta}{4}, \beta^* - \frac{2\varepsilon}{\gamma}]$, then to secure this payoff would require a total payment more than $\frac{\delta}{4} - \varepsilon$ more than $l(t)$, regardless of the realized cost.²⁰ The set $(\beta^* - \frac{\delta}{4}, \beta^* - \frac{2\varepsilon}{\gamma}]$ is non-empty for ε sufficiently small, and then has positive probability since F has full support. Moreover, the lower bound on possible total payments for these innate costs converges to $l(t) + \frac{\delta}{4}$ as ε is taken to zero. By Step 3, for innate costs above $\beta_\varepsilon^* + \varepsilon$, incentive compatibility requires the agent to realize no less than his innate cost. The only innate costs for which the payment might be lower than the FPCR mechanism are therefore those in the interval $(\beta^*, \beta_\varepsilon^* + \varepsilon)$. The probability of these converges to zero as ε tends to zero. ■ ■

²⁰If the agent uses effort $e \leq e^*(z_\varepsilon)$, then he must receive a transfer at least $\max\{0, l(t) - z_\varepsilon\} + \frac{\delta}{2} - \varepsilon$ to get this payoff. His realized cost is greater than $z_\varepsilon - \frac{\delta}{4}$, and so the payment is greater than $\max\{z_\varepsilon, l(t)\} + \frac{\delta}{4} - \varepsilon \geq l(t) + \frac{\delta}{4} - \varepsilon$. If he uses $e > e^*(z_\varepsilon)$, at best he produces the good at a total economic cost of $\beta^* - k - \frac{\delta}{4} = l(t) - \frac{\delta}{4}$. He therefore requires a payment of more than $l(t) + \frac{\delta}{4} - \varepsilon$ to obtain the desired payoff.