Harmonic Analysis of Aortic Pressure Pulses in the Dog

By Robert L. Farrow, Ph.D., and Ralph W. Stacy, Ph.D.

ONE OF the most useful tools of modern arterial dynamics is the Fourier analysis of complex pulse waves into their simple sinusoidal harmonic components. These harmonics are simple functions described by relatively simple mathematics, so that calculations on them are feasible. Lessen and Peterson[1] have pointed out that the principle of superposition can be expected to apply in arteries; hence calculations performed on individual harmonics can be followed by summation of these harmonics into a new complex wave. In this work, we consider the arterial pulse complex as one of a train of such complexes, i.e., the pulse wave is considered as a periodic phenomenon.

Fourier analysis, as a procedure, is tedious and time-consuming when made by hand, and there are many sites of possible errors. Partly for this reason and partly because of lack of appreciation of the value of such analyses, the collection of data on individual harmonics of arterial-pressure pulse waves has been limited.

Conclusions drawn from Fourier analysis of a single arterial pulse are open to considerable criticism, since (a) no single pulse is exactly like the adjacent pulses, and (b) higher harmonics are rendered inaccurate by the relatively low signal-to-noise ratio. Both of these factors are minimized, however, by considering a large number of pulses. The averaging process tends to eliminate variations of pulse-wave form over the respiratory cycle, periodic variations in peripheral resistance, etc. Also, the use of large numbers of pulses cancels random noise variations in the measurement process, and relatively small harmonic component magnitudes become meaningful.

The work described herein utilizes the speed and accuracy of the digital computer to carry out Fourier analyses on a large group of pulses, recorded at two sites a short distance apart in the abdominal aorta, and recorded with the subject in normal condition, and after administration of drugs, etc. that alter the peripheral resistance. The size of the sampling in this study is sufficient to allow some statistical analysis, with a description of the spread of values. To have accomplished this study by hand calculations would have taken an inordinate amount of time and effort.

From the Division of Biophysics, Department of Physiology, Ohio State University, Columbus, Ohio.

Supported by grants from the Central Ohio Heart Association and the American Heart Institute, National Institutes of Health, U. S. Public Health Service.

Received for publication October 28, 1960.

Methods

The pulses used in this study were recorded from six mongrel dogs, male and female, all weighing more than 20 Kg. The animals were anesthetized with pentobarbital sodium (30 mg./Kg.).

The left renal artery and the inferior mesenteric artery were cannulated with cannulas constructed from 3-inch no. 18 hypodermic needles. These were matched individually to Statham P-23-D pressure transducers, and their lengths adjusted so that frequency response and phase-shift characteristics of the two pick-up assemblies were identical up to 30 c.p.s. The cannulas were placed into the branch arteries and advanced until they lay just at the junction of these arteries with the abdominal aorta. The sites of measurement were selected because of the absence of collateral branches between them.

Transducers were energized and monitored by carrier amplifiers operating at a carrier frequency of 20,000 c.p.s., and recordings made with a Hathaway optical oscillograph using high-frequency, low-sensitivity gavanometers. Tests showed that the frequency response of the entire recording assembly was flat to more than 100 c.p.s., easily exceeding three times the highest harmonic frequency encountered. In the range 0 to 30 c.p.s., phase shifts were less than 2 degrees and were identical in both recording assemblies.

The systems were calibrated before and after each recording by applying pressures measured with a mercury manometer. These calibration records were examined for linearity and tracking error in each case.

Table 1

Absolute Values for the Modulus and Phase Angle of the First Eleven Harmonics (Average of 343 Pulses)

Harmonic	Renal pulse Mod. mm. Hg	SEM	Renal pulse Phase degrees	SEM	Mesenteric pulse Mod. mm. Hg	SEM	Mesenteric pulse Phase degrees	SEM
1	19.2 ±	0.45	153.2 ±	8.2	23.8 ±	2.17	198.9 ±	115.0
2	9.2 ±	0.24	290.4 ±	4.1	11.1 ±	0.34	284.6 ±	2.1
3	4.6 ±	0.50	290.7 ±	1.8	7.1 ±	1.32	255.2 ±	1.7
4	3.5 ±	0.67	287.8 ±	2.8	6.2 ±	1.52	256.0 ±	2.4
5	2.8 ±	0.48	283.1 ±	1.7	4.2 ±	0.86	233.7 ±	1.9
6	1.7 ±	0.04	250.5 ±	2.7	2.0 ±	0.32	211.9 ±	2.6
7	1.7 ±	0.48	259.1 ±	2.1	2.0 ±	0.37	198.6 ±	2.4
8	1.6 ±	0.68	246.2 ±	2.4	1.4 ±	0.34	168.8 ±	2.8
9	1.4 ±	0.48	241.0 ±	3.2	1.8 ±	0.41	163.7 ±	3.5
10	0.8 ±	0.03	232.6 ±	3.0	1.1 ±	0.28	146.9 ±	3.7
11	1.2 ±	0.48	215.7 ±	4.2	2.0 ±	0.58	148.7 ±	4.2
Constant term	130.6 ±	1.30	—	—	132.7 ±	1.77	—	—

Recordings were made under four sets of conditions in each animal. These were: (a) animal normal; (b) after intra-arterial injection of norepinephrine (0.0015 mg./Kg. body weight); (c) after intra-arterial injection of acetylcholine chloride (0.005 mg./Kg. body weight); and (d) after manual bilateral occlusion of the femoral arteries in the region of Scarpa's triangle.

The intra-arterial injections were made through a metal catheter introduced through the left carotid artery and advanced as far as the junction with the thoracic aorta. All recordings were made at the peak of effects resulting from these maneuvers.

The photographic records were developed in the usual manner, and dried in a continuous strip photographic drier to minimize errors caused by paper shrinkage. Pulses were then selected for analysis.

The pulse period (i.e., duration of the single pulse) was divided into 24 equal intervals, and pressure values were determined at each of these 24 ordinates. The zeroth ordinate on the upstream pulse was used as the zeroth ordinate for the downstream pulse also. All measurements were made to the nearest 0.1 mm. excursion, which was equivalent to about 1 mm. Hg pressure. The data so obtained were then punched onto IBM cards, and the cards fed into the computer (IBM 650) along the Fourier analysis program. The computer output consisted of the sine and cosine coefficients for each of the first 11 harmonics of the pulse, plus the values for modulus and phase angle. Actual computer time for analysis of one pulse on this machine was about one minute for each set of data.

Also calculated in this program was the apparent phase velocity, obtained from the relationship:

$$APV = \frac{S}{\frac{\theta M_1 - \theta M_2}{360}\ T}$$

where (θM_1) and (θM_2) are phase angles of the M^{th} harmonic of the renal and mesenteric pulses respectively, (S) is the distance between points of measurement, and (T) is the period of the pulse. (S) was measured, at autopsy, as the distance between cannula tips. The average value for (S) was 10 cm.

Lastly, the standard error of the mean was calculated for all data groups, using standard statistical methods. This calculation was also done on the computer, requiring a total of eight hours of machine time.

Results

Table 1 presents average values for modulus and phase angle of each of the first 11 harmonics of renal pulses and inferior mesenteric pulses. Each value is the average of 343 values, since a total of 343 pulse pairs was analyzed. The average fundamental frequency for these pulses was 2.62 c.p.s. (157.4 beats per minute).

This table reveals a marked increase in all harmonics (except the eighth) as the pulse travels toward the periphery. The largest increase is in the fourth harmonic, which is 77 per cent greater at the inferior mesenteric level than at the renal level.

Table 2

Comparison of Forty-five Control Pulses with the Effects of Norepinephrine, Bilateral Femoral Occlusion, and Acetylcholine

Harmonic	Control Mod. mm. Hg	Control APV cm./sec.	Norepinephrine Mod. mm. Hg	Norepinephrine APV cm./sec.	Femoral occlusion Mod. mm. Hg	Femoral occlusion APV cm./sec.	Acetylcholine Mod. mm. Hg	Acetylcholine APV cm./sec.
1	13.5	1524.1	24.7	—3640.8	11.9	—17788.6	22.7	14834.9
2	8.8	676.7	11.8	6985.3	8.7	2345.8	10.1	1149.1
3	3.7	300.9	9.3	508.2	1.3	389.0	2.9	412.6
4	1.7	277.5	5.3	388.3	1.3	6435.8	2.7	401.6
5	1.6	228.4	2.9	282.6	1.1	271.4	2.6	500.0
6	1.4	126.9	2.3	426.4	0.5	143.9	1.8	261.2
Mean Pressure	136.6 ± 1.8		156.1 ± 8.1		149.9 ± 6.6		145.6 ± 12.0	

In accordance with Parseval's theorem, energy is roughly proportional to the square of the pressure components. If one takes this literally, about 90 per cent of the pressure energy is contained in the fundamental and the second harmonic of these pulses. There is an increase in the harmonic energy of the pulse as it travels peripherally, amounting to 19.8 per cent per 10 cm. distance.

The data included in the values of table 1 include, in addition to normal pulses, those made with high and low "peripheral resistance" conditions, and it is to be expected that a wide range of values would be encountered. This is manifest in the relatively large values of the standard errors of means shown. In particular, this is true of phase-angle values, indicating the considerable dependence of the phase angle on the variables included in this study. It is interesting that the greatest variability is to be seen in the phase angle of the fundamental.

Table 2 is a restatement of these data, this time broken down into groups according to the physiological conditions included. The modulus values given are averages of renal and mesenteric pulse data for all pulses included in each group; the apparent phase velocity (APV) values are calculated from the differences of phase between renal and mesenteric pulses. It is obviously not possible to calculate the average APV from the average phase difference, because the relation between APV and phase difference is nonlinear. The mean of APV values calculated from individual phase differences is not equal to an APV value calculated from the mean phase difference.

It may be seen that, at the time of maximal drug effects, the pulses are richer in harmonic content than are the control pulses. In the fundamentals, the modulus is nearly doubled by norepinephrine and acetylcholine. The effect of occlusion is much less pronounced.

The influence of these experimental maneuvers on the APV is most striking, particularly in the case of the fundamental. The two methods by which "peripheral resistance" is increased (i.e., norepinephrine and manual occlusion) both cause shifts of phase angle of the fundamental great enough that negative phase velocities are seen. It should be pointed out that at the frequencies and velocities involved, the phase lag of a sine wave uncomplicated by reflections along a 10-cm. stretch is very small (some 2 degrees), and it takes very little change in the relative phase angle of the mesenteric and renal pulses to cause a shift to negative values for the APV.

The administration of intra-arterial acetylcholine (a decrease in peripheral resistance) causes the fundamental phase angle of the mesenteric pulse to be advanced, again giving a large positive value of APV. APV values for the second harmonics are higher in the "maneuver" pulses than in the control pulses and the differences are statistically significant. At higher frequencies, similar changes are seen, but these are not statistically significant.

Mean pressure did not change greatly dur-

Table 3

Comparison Between the Group Velocity as Measured from the Pulse-Wave Foot and the Apparent Phase Velocity of the First Three Harmonics, on Nine Representative Pulses

Measured group velocity (cm./sec.)	Apparent phase velocity of the first three harmonics (cm./sec.)		
	C_1	C_2	C_3
233.6	—19162.2	276.2	190.1
402.1	— 7435.4	286.6	178.5
501.3	— 1954.7	239.4	224.8
709.3	— 686.7	—14.0	1556.7
793.1	— 25.5	172.6	199.1
522.1	3320.4	—14.9	175.3
2241.3	8424.3	1047.5	635.8
2173.4	13761.4	1919.7	626.5
2733.1	28744.6	2200.4	765.6

ing the experimental maneuvers, owing to the fact that injections were made intra-arterially and influenced only the resistance of the caudal part of the system; reflex compensation in other areas prevented large changes. Since this mean pressure was calculated from 24 ordinate values, it represents a more accurate determination than is usually encountered. It is the DC component calculated from the Fourier series.

In addition to the analysis just listed, some of the pulses were examined for "group velocity," which is the velocity calculated from measurements of time lag between points of pressure inflection (foot-of-the-wave) on renal and mesenteric pulses. The pulses were arranged in ascending order of group velocities. The values for these pulses are presented in table 3, along with values for the APV of the first three harmonics.

The most significant figures are seen in the comparison of group velocities with the APV of the fundamentals. At low group-velocity values, the APV of the fundamental shows a high negative value; as the group velocity increases, the fundamental APV decreases. There is little correlation between APV values of the second and third harmonics and group velocity.

Lastly, it is of interest to plot the APV values calculated on the first nine harmonics of the entire 343 pulse pairs versus the frequency of these harmonics. Frequency values range from 2.62 c.p.s. to 23.58 c.p.s. The plot is shown in figure 1. A series of maxima and minima occurs, with minima falling at the third, sixth, and ninth harmonics (7.86, 15.72, and 23.58 c.p.s. respectively). This phenomenon will be related to transmission phenomena in later discussion.

Discussion

The transmission of waves in the arterial system is a relatively complex phenomenon. There are changes occurring in the wave contour as it passes down the artery, and these changes are attributable to a number of mechanical phenomena. For example, an amplification may occur owing to tapering of the wall of the vessel;[2] there may be frequency-dependent damping of the wave by viscoelastic elements of the wall;[3] there are almost certainly reflections of energy from junctions and discontinuities in the arterial tree; and there are probably variations in the propagation velocities of various harmonics, associated with their frequencies.[4]

Thus, it is hardly likely that any description of a single phenomenon will give complete insight into the behavior of this system. On the other hand, each such description contributes toward ultimate understanding. In the case of harmonic analysis, we take a step toward simplification, aimed at making elegant description possible.

Landowne[5] used Fourier analysis on arterial pulses, and Womersley[2] and Karreman[6] have utilized the concepts associated with such analysis extensively. Müller[7] and Morgan and Ferranty[8] have also used the analysis in arterial studies. These and subsequent studies have established the usefulness of the consideration of single harmonics.

At this point, it is of interest to examine the relative magnitude of the harmonics studies. The significance of such harmonics depends on the accuracy of recording and reading of pressures, the number of ordinates used in the Fourier analysis, and the extent to which the analyzed pulse can be considered as representative of other pulses in the periodic chain. It is generally considered that a Fourier analysis provides significant results for harmonic numbers up to one-half the number of intervals selected. Thus, if one analyzed a curve with a 24-ordinate system, and if the fundamental frequency were 3 c.p.s., significant results should be obtained to 36 c.p.s.

This would be true, however, only if there were no error of measurement involved. In the pulses studied here, the random error is at least 1 mm. Hg. Thus, some of the small values obtained for the magnitudes of higher harmonics are of the same order of magnitude as the error of measurement.

Taken singly, this fact would obviate the significance of results of Fourier analysis for higher harmonics. However, averaging of the results of analysis of a number of pulses reduces the effect of measurement error, as well as the error resulting from failure to select a perfectly representative pulse. It is difficult to assess this reduction of error mathematically, but we estimate that the errors involved in our data must be 10^{-2} to 10^{-3} times those in single pulse analysis.

Even so, we would be reluctant to accept conclusions based on data describing harmonics greater than the fourth or fifth. Higher harmonic data are included in the tables only for what they may be worth.

The greater amount of harmonic energy found in the more peripheral pulses in this study strongly suggests the existence of retrograde transmission of reflected pressure waves. This reflection is quite analogous to the reflection of electrical energy in a transmission line when the terminal impedance does not exactly match the characteristic impedance of the line. The reflected pressures are added vectorially to the incident pressures, thus producing shifts in both the amplitude and the phase of the harmonics. Since the ratio of terminal impedance to the line impedance varies with the frequency of the wave, it is to be expected that such reflection (hence the resulting wave changes) will be frequency-dependent.

In the electrical transmission-line systems, it is possible to delineate maxima and minima

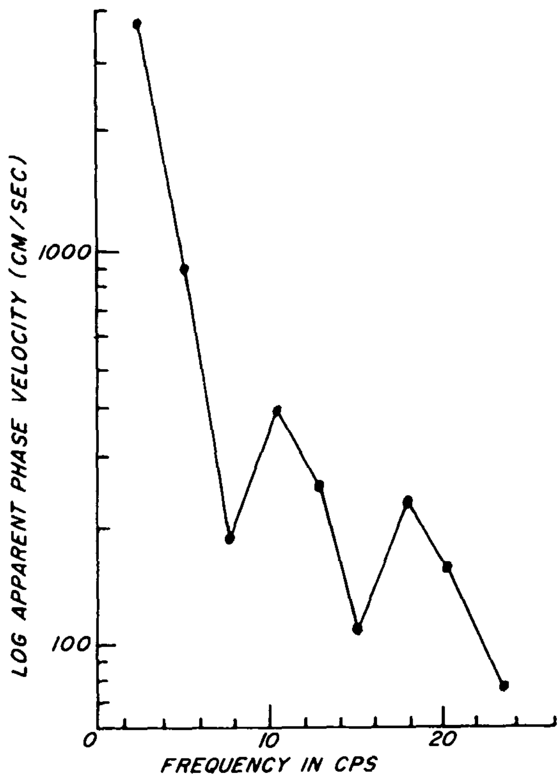

Figure 1

Graph of the apparent phase velocity of the first nine harmonics versus frequency, for 343 pulses. Note minima at third, sixth, and ninth harmonics. The average fundamental frequency is 2.62 c.p.s.

associated with the existence of standing waves; these maxima and minima occur at one-fourth wavelength intervals. Such a demonstration is not feasible in the arterial system, since, at a frequency of 1 c.p.s. and a velocity of 4 M./second, the wavelength is 4 M., and nodes and antinodes would be 1 M. apart. Such distances are not available. At higher harmonics, however, one-fourth wavelength values are of practical magnitude. Thus, a harmonic analysis provides data that can be useful in such thinking.

Unfortunately, the damping of pressure waves increases as the frequency increases, so that attenuation of high-frequency reflections obviates large reflection effects. Thus one finds major effects of changes of terminal impedance only in the lower-frequency harmonics (particularly the fundamental). This agrees with the observations of Porje.[4]

In the studies described herein, marked changes in phase and amplitude of the fundamental are seen when the terminal resistance is grossly altered by the administration of a vasoconstrictor or vasodilator. It should be pointed out that, in the short distance over which the wave transmission is observed (some 10 cm.), the total phase shift of a simple wave at the frequency of the fundamental is only about 2 degrees. Thus, a small shift of absolute phase of either or both of the recorded waves resulting from superposition of a reflected wave can cause great variations in this phase value. Furthermore, as the phase shift occurring between the points of measurement approaches zero, the APV approaches infinity. Further, slight shifts will result in very large negative values for APV. It is interesting that such negative values do occur in the fundamental, under conditions of major mismatch of terminal and line impedance where reflection coefficients are greatest.

Table 3 reveals a very interesting correlation between what we have called "group velocity" and the APV of the fundamental. This group velocity is that which is termed "pulse-wave velocity" in the physiological literature, and is measured from the time decrement of passage of the foot of the pulse wave past two points in the system. The exact position of the foot of the pulse wave in the pulse pattern depends on the harmonic structure of the pulse wave, so it is not surprising to find it closely correlated with phase of the fundamental.

This group velocity has been used regularly in such descriptions as Moen's equation for the determination of the elastic characteristics of the arterial wall. Now, we come to a realization that the "velocity" that should be used in such equations is a "true velocity," which should be the measured pulse-wave velocity corrected for the change in phase angle that occurs as a result of reflections originating at the site of the terminal impedance. Not even the APV of the harmonic, as obtained by the Fourier analysis performed in our studies, would be adequate for this. In order to obtain the true velocity called for in the wave equation and its modifications, it will be necessary to measure the phase velocity of an incident wave entirely unaltered by the presence of a reflected component. This situation calls for a complete re-evaluation of the measurement of pulse-wave velocity, and such is a major research effort.

It is significant that these considerations do not imply inadequacy of Moen's equation; rather, they call attention to the fact that foot-of-the-wave pulse-wave velocity values are not accurate measures of the "true" pulse-wave velocity. The authors believe that Moen's equation may be accurately descriptive of pulse-wave relations, provided the velocity used is obtained in such a manner that reflection effects are eliminated.

The work of Taylor[9] on analysis of the pulse wave using transmission-line theory substantiates many of our findings. Taylor has shown that the apparent phase velocity reaches minima at points that are multiples of one-fourth wavelength distant from the point of reflection. As we see from figure 1, this actually occurs at our measurement site in the third, sixth, and ninth harmonics. In spite of our reluctance to use higher harmonic data, the pattern shown is indicative of the significance of our results, so that the curve of figure 1 is probably meaningful.

Now we can make a rather interesting calculation from these data, by making a few "measurable" assumptions. If the true velocity of the wave is 10 M. per second, and the third harmonic frequency is 7.86 c.p.s., the wavelength of this harmonic is 127.2 cm., so that one-fourth wavelength is 32 cm. From a point midway between our measurement sites, this distance brings us to an apparent reflection point that is in the peripheral one-third of the thighs of our animals.

This apparent reflection point is, of course, merely a useful artifact of such calculation, for we are aware that reflection actually occurs from many points distributed throughout the entire peripheral arterial bed. Considering a single harmonic, though, these many reflections would themselves be added vectorially to produce a single sine wave transmitted in a retrograde direction, so that, for practical purposes, the reflection site may be considered as discrete. Immediately central to the limits of the distributed reflection bed, the reflected wave would be quite as discrete as though the reflection were from a single point.

Thus, it is in order to analyze reflections in the arterial system in terms of a specific reflection site, reflection coefficient, and phase angle of reflection. This is true, of course, so long as single harmonics are being considered. Such an analysis should be useful in the physiological studies of the system.

The use of the Fourier series as a tool concept is naturally always open to scrutiny with regard to its applicability to a specific system. In the arterial system, we feel we are fully justified in its application. Since pressures are mechanical forces and are periodic, with values of amplitude and direction, they are directly additive. Furthermore, the arterial system exists as a second-order system, since it does contain elements of inertia, resistance, and elasticity. Whether this system is handled as a lumped system or a distributed system, the principle of superposition must apply. The behavior of a single harmonic can be described with reasonable success; the behavior of the entire complex wave is many times more difficult to describe.

The use of the computer to perform Fourier analysis speeds the relatively lengthy and tedious process to the point of its becoming a practical procedure for routine application in cardiovascular studies. The error-eliminating feature of the handling of large numbers of pulses makes Fourier analysis more significant physiologically.

Summary

Aortic pulse waves were recorded in dogs at the junctions of the left renal artery and the inferior mesenteric artery with the aorta, under normal conditions, after administration of norepinephrine and acetylcholine, and after manual blocking of the femoral arteries. These pulses (343) were subjected to Fourier analysis, and apparent phase velocities and foot-of-the-wave velocities were calculated. The results strongly indicate the presence of reflected waves, and it is shown that foot-of-the-wave velocities provide erroneous information. The arterial system is such that in the arteries per se, the reflected wave is a discrete and definable quantity; its origin can be shown to be an apparent site of reflection located in the peripheral one-third of the thighs of the dog.

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Circulation Research
JOURNAL OF THE AMERICAN HEART ASSOCIATION

American Heart Association

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Circ Res. 1961;9:395-401
doi: 10.1161/01.RES.9.2.395
Circulation Research is published by the American Heart Association, 7272 Greenville Avenue, Dallas, TX 75231

Print ISSN: 0009-7330. Online ISSN: 1524-4571