PARACATADIOPTRIC CAMERA CALIBRATION USING SPHERE IMAGES

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ABSTRACT

The problem of calibrating paracatadioptric camera from sphere images is still open. In this paper, we propose a calibration method for paracatadioptric camera based on spheres. We notice that, under central catadioptric camera, a sphere is projected to two conics on the image plane, which can also be seen as the projections of two parallel circles on the viewing sphere by a virtue camera. These two conics are called a pair of antipodal sphere images. Firstly, we study properties of $K (K \geq 3)$ pairs of antipodal sphere images under paracatadioptric camera. Then, they can be estimated using these properties. Finally, paracatadioptric camera can be calibrated by three pairs of antipodal sphere images or more. This method only requires the projected contour of parabolic mirror is visible on the image plane in one view. Experimental results on both simulated and real image data have demonstrated the effectiveness of our method.

Index Terms— paracatadioptric camera, sphere images, antipodal sphere images

1. INTRODUCTION

Many applications in computer vision, such as robot navigation and virtual reality, a camera with a quite large field of view is required. Combining a camera with mirrors, referred to as catadioptric camera, can enhance the field of view of the camera. Catadioptric camera can be classified into two groups, central and noncentral [1]. People have already proposed many calibration methods for central catadioptric camera and they can be classed into four categories. The first category [2] only used point correspondences in multiple views. The second category [3, 4] required a 3D/2D calibration pattern with control points. The third category [5-8] only made use of the properties of line images. The fourth category [9, 10] only used the properties of sphere images.

In case of paracatadioptric camera, the calibration methods [9, 10] using spheres are degenerate and can not be used to determine the camera intrinsic parameters. In this paper, we propose a calibration method for paracatadioptric cameras based on spheres, which requires the projected contour of parabolic mirror is visible on the image plane in one view. Under central catadioptric camera, we notice that a sphere is projected to two conics, which can be seen as the projections of two parallel circles on the viewing sphere by a virtue camera and are called a pair of antipodal sphere images. Therefore, if three pairs of antipodal sphere images or more are known, central catadioptric camera can be calibrated by the method proposed in [11]. Based on the idea, firstly, we study the properties of $K (K \geq 3)$ pairs of antipodal sphere images under paracatadioptric camera. Next, these properties are used to estimate sphere images and their antipodal sphere images. Then, paracatadioptric camera can be calibrated by three pairs of antipodal sphere images or more. Extensive experiments have shown the effectiveness of our method.

This paper is organized as follows: Section 2 reviews the unified sphere model introduced by Geyer and Daniilidis [5]. Section 3 studies the properties of $K (K \geq 3)$ pairs of antipodal sphere images. In section 4, the calibration algorithm for paracatadioptric camera from sphere images is described in detail. Experimental results are shown in Section 5. Finally, Section 6 presents some concluding remarks.

2. CENTRAL CATADIOPTRIC CAMERA

A bold letter denotes a vector or a matrix. Without special explanation, a vector is homogenous coordinates. In the following, we briefly review the image formation for central catadioptric camera introduced in [5](see Fig.1):

Under the viewing sphere coordinate system $O−X_sY_sZ_s$, a 3D point $X = (x,y,z)^T$ is projected to a point $X_s = (x_s,y_s,z_s)^T$ on the unit sphere centered at the viewpoint $O$; Secondly, the point $X_s$ is projected to a point $m$ on the image plane II by a pinhole camera through the perspective center $O_c$. The image plane is perpendicular to the line going through the viewpoints $O$ and $O_c$. Let the intrinsic parameter matrix of the pinhole camera be

$$K_c = \begin{pmatrix} r_c f_c & s & u_0 \\ 0 & f_c & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $r_c$ is the aspect ratio, $f_c$ is the effective focal length, $(u_0,v_0,1)^T$ denoted as $p$ is the principal point, and $s$ is the skew factor. The imaging process of a space point $X$ to $m$
can be described as:

\[ \alpha \mathbf{m} = \mathbf{K}_c \left( \mathbf{R} \mathbf{X} + \mathbf{t} \right) + \xi \mathbf{e} \].

(1)

where \( \alpha \) is a scalar, \( \mathbf{R} \) is a \( 3 \times 3 \) rotation matrix, \( \mathbf{t} \) is a 3-vector of translation, \( \| \| \) denotes the norm of vector in it, \( \mathbf{e} = (0, 0, 1)^T \), and \( \xi \) is the mirror parameter that is the distance from \( \mathbf{O} \) to \( \mathbf{O}_c \). The mirror is a paraboloid if \( \xi = 1 \), an ellipsoid or hyperboloid if \( 0 < \xi < 1 \), and a plane if \( \xi = 0 \).

Fig. 1. The image formation of a point.

3. PROPERTIES OF \( K \) (\( K \geq 3 \)) PAIRS OF ANTIPODAL SPHERE IMAGES UNDER PARACATADIOPTRIC CAMERA

The projection of a sphere under central catadioptric camera is two conics, which are defined as a pair of antipodal sphere images. One conic, being visible on the image plane, is called the sphere image. And the other conic, being invisible on the image plane, is called the antipodal sphere image. In addition, from the image formation, we notice that the projection of a sphere is two parallel circles on the viewing sphere, because the unite normal vectors of the base planes containing them are parallel. Thus, the projection of a sphere can be seen as the projections of two parallel circles on the viewing sphere by a virtue camera. Under paracatadioptric camera, the representations of two conics, being the projection of a sphere (See Fig.2), are [9]:

\[ \mathbf{C}_\pm = \mathbf{K}_c^{-T} \mathbf{C}'_\pm \mathbf{K}_c^{-1} \]

(2)

where

\[
\mathbf{C}'_\pm = \begin{pmatrix}
(\pm d_0 - n_z^2) & 0 & (\pm d_0 - n_z)n_x \\
0 & (\pm d_0 - n_z^2) & (\pm d_0 - n_z)n_y \\
(\pm d_0 - n_z)n_x & (\pm d_0 - n_z)n_y & (d_0^2 - n_z^2)
\end{pmatrix}.
\]

\( (n_x, n_y, n_z)^T \) is the unite normal vector of the base plane containing circle on the viewing sphere, and \( d_0 \) is the distance from the origin \( \mathbf{O} \) to the base plane. \( \mathbf{C}_+ \) represents the equation of the sphere image and \( \mathbf{C}_- \) represents the equation of the antipodal sphere image.

From \( K \) (\( K \geq 3 \)) viewing points, a sphere in space is projected to \( K \) pairs of conic curves \( \mathbf{C}_{\pm i} \)

\[ \mathbf{C}_{\pm i} = \begin{pmatrix}
a_{\pm i} & b_{\pm i} & d_{\pm i} \\
b_{\pm i} & c_{\pm i} & e_{\pm i} \\
d_{\pm i} & e_{\pm i} & f_{\pm i}
\end{pmatrix}, \quad i = 1, 2, 3, ..., K.
\]

(3)

Expanding the right side of (2), we obtain

\[
\begin{align*}
b_{\pm i} &= \left(\frac{d_0}{n_z^2} \pm a_{\pm i} \right) \xi \\
c_{\pm i} &= \left(\frac{d_0}{n_z^2} + \frac{1}{n_z^2} n_x^2 \right) a_{\pm i} + \frac{1}{n_z^2} d_0 n_x \\
d_{\pm i} &= \left(\frac{d_0}{n_z^2} + \frac{1}{n_z^2} n_y^2 \right) a_{\pm i} + \frac{1}{n_z^2} d_0 n_y \\
e_{\pm i} &= \left(\frac{d_0}{n_z^2} \pm \frac{n_x}{n_z^2} n_x \right) a_{\pm i} + \frac{1}{n_z^2} d_0 n_x \\
f_{\pm i} &= \left(\frac{d_0}{n_z^2} \pm \frac{n_y}{n_z^2} n_y \right) a_{\pm i} + \frac{1}{n_z^2} d_0 n_y
\end{align*}
\]

(4)

where

\[
\begin{align*}
k_1 &= \frac{1}{r_c^2} \\
k_2 &= -\frac{r_c}{r_c^2} \\
k_3 &= \frac{r_c^2}{r_c^2} - \frac{u_0}{v_c^2} \\
k_4 &= -\frac{3}{r_c^2}
\end{align*}
\]

(5)

From the first two expressions in (4), we know that

\[
\begin{align*}
b_{\pm 1} &= b_{\pm 2} = \cdot \cdot \cdot = b_{\pm K} = k_2 \\
c_{\pm 1} &= c_{\pm 2} = \cdot \cdot \cdot = c_{\pm K} = k_2 \\
d_{\pm 1} &= d_{\pm 2} = \cdot \cdot \cdot = d_{\pm K} = k_2 \\
e_{\pm 1} &= e_{\pm 2} = \cdot \cdot \cdot = e_{\pm K} = k_2 \\
f_{\pm 1} &= f_{\pm 2} = \cdot \cdot \cdot = f_{\pm K} = k_2
\end{align*}
\]

(6)

Then, from (6) follows that

\[
\begin{align*}
\alpha_{\pm i} &= a_{\pm i} b_{\pm i} - a_{\pm i} b_{\pm i} = 0 \quad i = 2, 3, ..., K \\
\beta_{\pm i} &= a_{\pm i} c_{\pm i} - a_{\pm i} c_{\pm i} = 0 \quad i = 2, 3, ..., K
\end{align*}
\]

(7)

Next, from the last three expressions in (4) and (5), after some manipulations, we have

\[
\begin{align*}
d_{0i} &= \frac{v_i - v_{i+1}}{a_{i+1} v_{i+1}^2 - p_{i+1}^2 C_{i+1} \mathbf{p}^T C_{i+1} \mathbf{p}} \\
d_{0i} &= \frac{v_i - v_{i+1}}{a_{i+1} v_{i+1}^2 - p_{i+1}^2 C_{i+1} \mathbf{p}^T C_{i+1} \mathbf{p}}
\end{align*}
\]

(8)

where \( v_{i+1} = (-a_{i+1} b_{i+1}, d_{i+1}) \), \( v_{i+1}^2 = (0, a_{i+1} c_{i+1} - b_{i+1}^2) \), \( a_{i+1} = b_{i+1} + d_{i+1} \).

Therefore, in each view, from the first three expressions in (8), we obtain that

\[
\begin{align*}
\eta_i &= \frac{1}{a_{i+1}} \frac{v_i - v_{i+1}}{v_{i+1}^2} - \frac{1}{a_{i+1}} \frac{v_i - v_{i+1}}{v_{i+1}^2} = 0 \\
\chi_i &= \frac{1}{a_{i+1}} \frac{v_i - v_{i+1}}{v_{i+1}^2} - \frac{1}{a_{i+1}} \frac{v_i - v_{i+1}}{v_{i+1}^2} = 0 \\
\end{align*}
\]

(9)

\[ i = 1, 2, 3, ..., K \]
In different views, from the last expression in (8), we have
\[
\nu_i = \frac{1}{a_{i+1}a_{i-1}} p^T C_{i+1} p p^T C_{i-1} p - \frac{1}{a_{i+1}a_{i-1}} p^T C_{i+1} p p^T C_{i-1} p = 0, \quad i = 2, 3, ..., K.
\]
If a set of \( K \) conic curves corresponds to the paracatadioptric projection of a sphere in \( K \) views, then (7), (9) and (10) are true. Therefore, they can be used to optimize the equations of the antipodal sphere images.

### 4. CALIBRATION ALGORITHM FOR PARACATADIOPTRIC CAMERAS

It is well known that a pinhole camera can be calibrated from two parallel circles [11]. Therefore, if \( K \) (\( K \geq 3 \)) pairs of antipodal sphere images or more are known, the paracatadioptric camera can be calibrated. In this section, using the properties of antipodal sphere images, we propose a calibration method for paracatadioptric camera based on spheres.

#### 4.1. Estimation of \( K \) (\( K \geq 3 \)) pairs of antipodal sphere images under paracatadioptric camera

Consider the projections of a sphere under paracatadioptric camera in \( K \) different views. To each sphere image \( C_{i+1} \) in (3) corresponds a set of image points \( m_j \), with \( j = 1, 2, 3, ..., N_i \) and \( N_i \geq 5 \). It is obvious the conics \( C_{i+1} \), being visible on the image plane, can be estimated by the least squares method. In the following, we determine the equations \( C_{i-1} \) of the antipodal sphere images.

Firstly, we have an initialization of the equations \( C_{i-1} \). Because the projected contour \( C \) (a \( 3 \times 3 \) matrix) of the parabolic mirror is visible on the image plane. Then, the initial values of \( r_c, s, u_0, v_0, f_c \) can be obtained [9]:

\[
\begin{align*}
\text{Step 1:} & \quad r_c = \sqrt{\frac{C(1,2)^2 + C(2,2)}{C(1,1)}} \\
\text{Step 2:} & \quad s = \frac{C(1,2)}{C(1,1) - C(1,2)C(2,1)} \\
\text{Step 3:} & \quad u_0 = \frac{C(1,1)C(2,2) - C(2,1)^2}{C(1,2)(C(2,2) - C(2,1)C(1,3))} \\
\text{Step 4:} & \quad v_0 = \frac{C(1,1)C(1,3) - C(1,2)^2}{C(1,2)(C(2,2) - C(2,1)C(1,3))} \\
\text{Step 5:} & \quad f_c = (u_0, v_0, 1)^T C(u_0, v_0, 1)
\end{align*}
\]

By the property of antipodal image points shown in [8], we can compute the antipodal image points \( m_j \) of \( m_j \) using the obtained intrinsic parameters in (11), \( j = 1, 2, 3, ..., N_i \). Then, estimate \( C_{i-1} \) using \( m_j \), \( j = 1, 2, ..., N_i \) by the least squares method.

Next, we optimize the equations \( C_{i-1} \) of the antipodal sphere images through minimizing the following objection function:

\[
F = \sum_{i=1}^{K} (\eta_i^2 + \chi_i^2) + \sum_{i=2}^{K} (\alpha_i^2 + \beta_i^2 + \nu_i^2)
\]

(12)

where \( \alpha_i, \beta_i, \nu_i \) are as in (7), \( \eta_i, \chi_i \) are as in (9) and \( \nu_i \) is as in (10).

#### 4.2. The calibration method for paracatadioptric camera based on spheres

Our algorithm to calibrate paracatadioptric from the projections of a sphere in \( K \) (\( K \geq 3 \)) views is shown as follows:

**Step 1:** Extract the pixels of sphere images in \( K \) views and estimate their equations \( C_{i+1} \) by least squares method, \( i = 1, 2, 3, ..., K \);

**Step 2:** In some view, extract the pixels of projected contour of parabolic mirror and determine the equation \( C \) by the least squares method;

**Step 3:** By the estimation method presented in section 4.1, determine the equations \( C_{i-1} \) of the antipodal sphere images in \( K \) views, \( i = 1, 2, 3, ..., K \);

**Step 4:** Calibrate the intrinsic parameters of paracatadioptric camera by the calibration method proposed in [11].

### 5. EXPERIMENTS

In this section, we perform a number of experiments with simulated and real images to evaluate the performance of our calibration algorithm.

#### 5.1. Experimental results with simulated data

The simulated camera has the following intrinsic parameter matrix:

\[
K_c = \begin{pmatrix}
610 & 0.8 & 500 \\
0 & 600 & 350 \\
0 & 0 & 1
\end{pmatrix}
\]

(13)

The projections of a sphere in three views are generated to calibrate the paracatadioptric camera. One of them is shown in Fig.3. The biggest conic is the projected contour of parabolic mirror. The projected contour and each sphere image are consisted of 100 points respectively. Gaussian noise with mean 0 and standard deviation \( \sigma \) ranging from 0 to 3 is directly added to each of the points on the sphere images and the project contour of parabolic mirror.

![Fig. 3. A simulated sphere image](source)

We calibrate intrinsic parameters for paracatadioptric camera using the method proposed in section 4.2. At each noise level, we perform 100 independent trails. The means and the standard deviations of intrinsic parameters are computed and shown in Fig.4. Since the performances of \( u_0 \) and \( v_0 \) are very similar, the estimated results for \( v_0 \) are not shown.
here. From Fig.4, the results illustrate the proposed method is quite correct and stable to the intrinsic parameters.

**Fig. 4.** The simulated results for paracatadioptric camera. (a)∼(d) The means and standard deviations of \( r_c, s, f_c \) and \( u_0 \) respectively.

5.2. Experimental results with real image

Three real images of a ping-pong are used for the real experiment. They are captured by a CANON A640 with a hyperboloid mirror designed by the Center for Machine Perception, Czech technical University. The mirror parameter \( \xi = 0.966 \) that is close to 1. Here, we approximately regarded it as 1. One of them is shown in Fig.5(a). The sphere images are extracted using Canny’s edge detector. Then, applying the proposed calibration algorithm proposed in section 4.2, the intrinsic parameters can be estimated. We use the estimated intrinsic parameters to rectify the test image shown in Fig.5(b). The rectified results are shown in Fig.5(c). It can be seen that the calibration method is very effective.

**Fig. 5.** (a) is one of the sphere images used in the real experiments. (b) is the test image. (c) is the rectified result.

6. CONCLUSION

We study properties of antipodal sphere images under a paracatadioptric camera. Then based on them, a calibration method for the camera is proposed. This solves the degeneracy problem in [9, 10] and makes sphere based calibrations for central catadioptric cameras complete. Simulated and real experiments both validate the effectiveness of our method.

7. REFERENCES