Measuring the Strength of Networks of Teams: Metrics and Properties

Ali Assarpour*, Saman Farhat*, Ou Liu*, Alexey Nikolaev*,
Amotz Bar-Noy*, Prithwish Basu†, Saad Mneimneh*, Ram Ramanathan†
*City University of New York, amotz@sci.brooklyn.cuny.edu
†Raytheon BBN Technologies, {pbasu, ramanath}@bbn.com

***Abstract*—A Network of Teams (NoT) is a set of overlapping teams working toward a common goal. NoTs arise in several contexts such as large software projects, sensors cooperating for target tracking, and terrorist cell networks. Modeling a NoT as a *simplicial complex*, we consider the problem of quantifying the *strength* of a NoT using metrics based purely on its structure. While the design of a metric clearly depends on the context in question, most applications require the metric to be *monotonic*. We define three kinds of monotonicity – *weak*, *team*, and *structural* – depending upon whether we add/remove nodes, entire teams, or facets, respectively We propose seven metrics and analyze their monotonicity, both mathematically and experimentally. Specifically, we prove that some metrics or monotonic, and show that some or not by providing counter examples. For non-monotonic metrics, we generate random simplicial complexes using a number of models, and estimate the fraction of cases for which a metric is not monotonic.**

I. Introduction

A Network of Teams (NoT) is a collection of teams that cooperate to achieve a common goal. Each team is composed of a set of tightly-knit nodes to accomplish a specific task. The higher level network of teams (participants may belong to multiple teams) serves to form an overall loosely-knit but connected network facilitating information flow and coordination between the teams. A NoT arises in a variety of contexts such as social, communication, distributed computation, transportation, etc. Examples include large software projects where each team is responsible for a module that is then integrated into a larger system [1], [2]; a network of sensor teams, each team tracking a moving object at a particular range of locations [3]; an insurgent cell network [4]. In each case, the overlap may be via nodes working as part of multiple teams, a specific case of which is a leader or a communication hub.

In the design and management of networks of teams, a common question that arises is: which structure is better for performance? While this has been the subject of some study in social and organizational science, much of the work focuses on structure *within* a team, and is based on empirical analysis. In this paper, we focus on a *network* of fully-connected teams, and take an analytical approach to develop metrics that can quantify the strength of a NoT. Given such a "strength function", one can compare NoTs quantitatively, and quantify the effects of addition/removal of nodes or teams.

The metric is clearly dependent on the context, that is, there is unlikely to be a universal metric that is best across different problem domains. On the other hand, any metric should possess desirable properties, for example, monotonicity – i.e., assuming nodes are homogeneous, if we add nodes or teams to a NoT, its strength should not decrease[1]. Accordingly, we define a number of metrics for measuring strength, and investigate their monotonicity analytically and experimentally.

Collaboration networks are traditionally modeled by graphs, e.g., [5], [6]. However, such graph representations do not capture collaboration as a *group*. For example, consider a complete graph on four vertices, each vertex representing a person. This graph could represent a single team with four members, four 3-member teams, or six 2-member teams. Each case represents a significantly different interaction pattern and performance, and graphs do not bring out this difference as they only capture cardinality-2 relationships. Thus, we need an abstraction where higher-order aggregations can be represented distinctly from the union of pair-wise interactions.

In this paper, we use the *abstract simplicial complex* to represent and analyze networks of teams. An abstract simplicial complex consists of a set V and a set of subsets of V (a subset is called a *simplex* or *face*) "closed" under the subset operation. A simplicial complex is a generalization of a graph and therefore admits any analysis or metric based on graphs. Additionally it provides analytical possibilities not possible with a graph-based representation. In section II-A we provide a brief background on simplicial complexes as necessary for understanding this paper. Prior works [8], [9], [10] have established the usefulness of simplicial complexes for analyzing collaboration networks[2].

Using simplicial complexes to model NoTs, we present seven metrics for assessing the strength of NoTs. We define three kinds of monotonicity: *weak*, *team*, and *structural*,

Research was sponsored by the Army Research Laboratory and was accomplished under Cooperative Agreement Number W911NF-09-2-0053 (the ARL Network Science CTA). The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U.S. Government. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation here on.

[1]While one may argue that in real life performance sometimes decreases with increasing size, this depends on the definition of strength. We shall discuss this further in section II-C.

[2]The *hypergraph* [11] is another possible abstraction, but as argued in [9], a simplicial complex is a better fit as it is closed under subsets, capturing the subset closure property of the collaboration relationship.

978-1-4673-7131-5/15/$31.00 ©2015 IEEE

based on the removal, respectively, of a vertex, team or facet of the simplicial complex representing the NoT. That is, a metric is vertex (team) (facet) monotonic if upon removal of any vertex (team) (facet), in any simplicial complex, the strength of the resultant complex is no greater than that of the original. We formalize these notions is section II. We begin by mathematical analysis and provide proofs of monotonicity or counter examples thereof. Since most cases are non-monotonic, we further examine if some metrics are *almost always* monotonic using two approaches: exhaustive enumeration of small simplicial complexes; and by generating random simplicial complexes using six generative models.

Related work. The topic of measuring team performance has been studied from a sociology perspective [12], [13], [14], [15]. A simple measure of the effectiveness of a team as the graph density is proposed in [12]. Correlation between performance measures and structure in several settings have been studied in [13], [14]. More recently, in [16], a measure was proposed for the robustness of social networks. These works focused on teams and the structure within, whereas we focus on networks of teams (NoT).

Well established in mathematics, in particular algebraic topology [17], simplicial complexes have been used as a part of Q-analysis in the 1970s to analyze general structure [8], and have been applied into specific social network problems [18]. The application of simplicial complex concepts to collaboration networks appears to varying extents in [9], [10].

To our knowledge, this is the first work that formulates and studies the problem of strength metrics for a Network of Teams (NoT) and their monotonicity analysis using a combination of theoretical and experimental approaches.

II. Preliminaries

A. Simplicial Complex

An *abstract simplicial complex*, or *simplicial complex* for brevity, is denoted by $\Delta = (V, S)$, where V is a set of vertices, and $S = \{S_i | S_i \subseteq V, S_j \in S \forall S_j \subseteq S_i\}$ is a non-empty set of subsets of V "closed" under the subset operation. $S_i \in S$ is called a *simplex* or a *face*. The dimension of a simplex S_i is $dim_{S_i} = |S_i| - 1$, and of Δ is $dim_\Delta = \max\{dim_{S_i} | S_i \in S\}$. A *facet* is any simplex in a complex that is not a face of any larger simplex.

The *facet degree* of a node v, denoted by $d(v)$ is the number of facets v is a member of. The *open neighborhood* of a node denoted by $N(v)$ is set of nodes that have pairwise relation with v, and the *closed neighborhood* of a node denoted by $N[v]$ is $N(v) \cup v$. Furthermore a complex will be denoted by the list of facets it contains with the understanding that all sub-faces of the facet belong to the complex as well. Figure 1 shows an example complex $\{\{1, 2\}, \{2, 3, 4\}\}$.

We have only given the minimum background required for understanding the rest of the paper. Readers interested in learning more about simplicial complexes and algebraic topology in general are referred to [17].

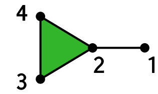


Fig. 1: Example Simplicial Complex. The dimension of the complex is determined by the facet $\{2, 3, 4\}$ which has dimension 2. Node 2 has facet degree 2; all other nodes have facet degree 1.

B. A NoT as a Simplicial Complex

Let T be the set of overlapping teams. Then we represent the network of teams as a simplicial complex $\Delta = (V, S)$, where each $v \in V$ corresponds to a node (person, sensor etc.), and each $s \in S$ corresponds to a subset of a team. In particular, each team $t \in T$ corresponds to a facet in S. We assume implicitly that each team is a fully connected network, that is, the all nodes collaborate with each other within the team. This follows from our assumption of a closely-knit relationship within a team. Note that real-world teams that are not fully connected can be represented as multiple overlapping teams, each of which is fully connected.

C. Strength of a NoT

We define strength $\Psi_m : \Delta \mapsto \mathbb{R}^+$ as a function that estimates a positive real value for a given simplicial complex representing a NoT. The subscript m refers to different metrics, or different strength functions. The goal of this paper is to propose and study different Ψ_m. In assessing the strength of any network, one may take two possible approaches. In the first, that we call *manifested strength*, interactions happen along all of the links and all nodes are involved, that is, one is not allowed to exclude any nodes or edges.

Definition II.1. *Given a NoT $\Delta = (V, S)$, the* manifested strength $\Psi^M(\Delta)$ *is a measure of the performance output of Δ when the actors in V interact in the way prescribed by Δ's structure.*

In the second, that we call *inherent strength*, the network operates in the best possible mode given the resources, and may exclude participation by nodes or along edges if that were deemed necessary to maximize the performance.

Definition II.2. *Given a NoT $\Delta = (V, S)$, the inherent strength $\Psi^I(\Delta) = \max_S \Psi^M(S)$, where S is a sub-complex of Δ.*

The inherent strength of a network is thus the best possible operating point of the set of resources available, constrained to interact along the simplices present in the complex representing the NoT. This might be the result, for example, of the participants or some entity figuring out a way to exclude interactions that spoil it. We believe that the inherent strength is closer to how teams and network thereof converge to operating, and therefore we will use it.

Observation II.1. *The inherent strength function $\Psi^I(\Delta)$ is monotonically non-decreasing with increasing size of Δ.*

By definition, since the inherent strength captures the maximum of all possible sub-interactions, addition of resources can never lower it. It follows that any metric that we come up with to measure inherent strength of a NoT is ideally monotonic. Surprisingly, balancing monotonicity with intuition is not an easy task, and in fact quite an interesting problem, as the remainder of the paper will illustrate.

D. Monotonicity of Strength Functions

When a face is removed from a simplicial complex, all its supersets are also automatically removed from the complex in order to preserve the subset closure property. Since all facets are faces, and each individual node is a singleton face, we can consider the following four operations:

Node removal. When a node is removed, all the faces that contain it are also removed. This corresponds to the situation when a person is removed from the group, and it is expected that the group becomes weaker. For example, consider $\Delta = \{\{1,2,3\},\{2,3,4\},\{1,5,6\}\}$; after deleting node 1, it becomes $\Delta' = \{\{2,3,4\},\{5,6\}\}$. Note that the face $\{2,3\}$ that was created after deleting node 1 from the facet $\{1,2,3\}$ was "absorbed" by the facet $\{2,3,4\}$ and the face $\{5,6\}$ is the result of deleteing node 1 from the facet $\{1,5,6\}$.

Team removal. If facets are interpreted as teams, the removal of a team is the removal of a facet together with all its smaller faces that are not faces of any other facet, keeping only the singleton faces of the removed team. For example, consider the simplicial complex $\Delta = \{\{1,2,3,4\},\{3,4,5\}\}$: removing the team $\{1,2,3,4\}$ creates the complex $\Delta' = \{\{1\},\{2\},\{3,4,5\}\}$. In a realistic setting, it may correspond to the removal of the communication media between the people of a group, so they cannot collaborate anymore, unless there is another method of communication between them. Another interpretation of team removal is the cancelation of a task the team is working on.

Facet removal is the removal of a single facet keeping all its faces. Facet removal is the simplest possible reduction of a simplicial complex, since only single simplex is deleted. Thus mathematically, it is the simplest possible operation. Any node removal or team removal operation can be implemented by a sequence of facet removals. For example, consider the simplicial complex $\Delta = \{\{1,2,3\},\{2,3,4\}\}$ after removing the facet $\{1,2,3\}$ it becomes $\Delta' = \{\{1,2\},\{1,3\},\{2,3,4\}\}$. Note that the face $\{2,3\}$ that was created after deleting the facet was "absorbed" by the facet $\{2,3,4\}\}$.

We say that a metric Ψ_m is monotonic if it does not increase when a node (a team, or a facet) is removed. Particularly, we are interested in the following monotonicity properties:

Definition II.3. *A metric m is weakly monotonic if for every Δ, deleting (adding) a node causes $\Psi_m(\Delta)$ to go down (up) or remain the same.*

Definition II.4. *A metric m is team-monotonic if for every Δ, deleting (adding) a "team" causes $\Psi_m(\Delta)$ to go down (up) or remain the same.*

Definition II.5. *A metric m is structurally monotonic if for every Δ, deleting (adding) a facet causes $\Psi_m(\Delta)$ to go down (up) or remain the same.*

Observation II.2. *Functions that are structurally monotonic are also weakly monotonic.*

Notice in case of structural monotonicity, in order to delete a node, first all facets containing the node must be removed, then the node itself becomes a facet and can be deleted.

Observation II.3. *Functions that are structurally monotonic are also "team"-monotonic.*

Observation II.4. *Weak monotonicity and "team"-monotonicity do not imply each other.*

III. Strength Metrics and their Monotonicity

The following metrics are based mainly on the following three factors: (i) the number of facets, (ii) their size, (iii) and the facet degree of nodes, denoted by $d(v)$. The intuition here is that on one hand a NoT is stronger with larger facets but on the other hand individuals would have to divide their time according to some criteria among the tasks in which they are concurrently involved. The following naive function is the most natural way to capture the above intuition, In this function the strength contribution of node v is equally divided, $1/d(v)$, among all the facets that contains it.

$$\Psi(\Delta) = \sum_{f} \left(\sum_{v \in f} \frac{1}{d(v)} \right) \tag{1}$$

That is, the strength of an Δ is the sum over all facets f of the Δ, and each facet gets $1/d(v)$ from each of its members. The resulting strength of the complex is obtained as the sum of all individual contributions to all facets. This metric is simply equal to $|V|$, the number of nodes in the complex, hence it is trivially structurally monotonic. However, it is not very useful, since it does not let us differentiate complexes with the same number of nodes.

In what follows, we propose several modifications to this metric. We converged on seven metrics, which we find interesting and sufficiently motivated.

The following metric is weakly monotonic, and monotonic w.r.t. "team" removal, as it will be shown later. Here, the summations in the naive function (equation 1) are replaced by products and $1/d(v)$ is replaced by $1 + 1/d(v)$ to avoid products of small numbers. Unfortunately, the size of the facets do not influence the function due to its simplified version.

$$\Psi_1(\Delta) = \prod_{f} \prod_{v \in f} \left(1 + \frac{1}{d(v)}\right) = \prod_{v} \left(1 + \frac{1}{d(v)}\right)^{d(v)} \tag{M1}$$

Also we consider a variant of the previous metric, where each vertex v contributes $2 \leq (1 + \frac{1}{d(v)})^{|f|} < e$ to the strength of each facet it belongs to, the contributions of the vertices are averaged for each facet, and the results are multiplied.

$$\Psi_2(\Delta) = \prod_{f} \frac{1}{|f|} \cdot \sum_{v \in f} \left(1 + \frac{1}{d(v)}\right)^{|f|} \tag{M2}$$

Another metric similar to the metric (M1) is as follows. Here, instead of focusing on the individuals, we compute the strength of each facet and multiply them.

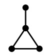
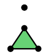
Metric						
1	20.25	18.96	22.78	25.63	24.0	16.0
2	21.97	24.11	22.78	25.63	26.36	16.0
3	13.72	14.55	29.1	35.53	36.91	4.74
4	1.25	1.0	0.75	1.0	0.92	2.0
5	6.0	6.0	5.5	6.0	6.0	6.5
6	7.83	7.46	7.0	8.0	7.79	10.0
7	2.94	3.16	3.12	2.83	2.96	2.73
Metric						
1	20.25	22.78	36.0	48.0	106.55	91.13
2	24.17	24.89	41.71	55.88	181.57	118.85
3	6.19	14.83	5.96	37.61	42.62	15.41
4	1.0	1.0	2.0	1.25	0.74	1.5
5	7.33	6.67	9.17	8.28	11.0	11.0
6	11.66	9.83	14.66	11.94	16.93	17.49
7	2.45	2.64	3.0	3.36	3.89	3.67

TABLE I: The value of the seven metrics for some small examples of simplicial complexes.

$$\Psi_3(\Delta) = \prod_f \left(1 + \frac{1}{\sum_{v\in f} d(v)}\right)^{\sum_{v\in f} d(v)} \tag{M3}$$

The following function is motivated as a probabilistic metric, where the product $\prod_v (1/d(v))$ is the probability that all the team members get together.

$$\Psi_4(\Delta) = \sum_f \left((|f|-1)\cdot \prod_{v\in f} \frac{1}{d(v)}\right) \tag{M4}$$

We can improve the metric defined by the equation (1) if we make the strength of each facet grow with the number of nodes in it. One way to do that is to make each facet f contribute $g(|f|)\sum_{v\in f}\frac{1}{d(v)}$ to the total strength of the complex, where $g(\cdot)$ is an increasing sublinear function. We choose it to be equal to the harmonic number $g(n) = H_n = \sum_{k=1}^{n} 1/k$. This metric works correcly in the pathological case, when $\Delta = \{\{1, 2, \ldots n\}\}$: if the facet is removed, the result is a complex with n facets of size $n-1$.

$$\Psi_5(\Delta) = \sum_f \left(H_{|f|} \sum_{v\in f} \frac{1}{d(v)}\right) \tag{M5}$$

Better use of time: when an individual is doing many related tasks, they may contribute $1/\sqrt{d(v)}$ to each, then their total contribution is ≥ 1, and the metric is:

$$\Psi_6(\Delta) = \sum_f \left(\sum_{v\in f} \frac{1}{\sqrt{d(v)}}\right)^2 \tag{M6}$$

Conversely, less effective use of time is made possibly due to many unrelated tasks, contributing $1/d(v)^2$ to each task:

$$\Psi_7(\Delta) = \sum_f \sqrt{\sum_{v\in f} \frac{1}{d(v)^2}} \tag{M7}$$

In Table I we record the strength of all seven metrics on some small examples.

In the remainder of this section we provide results for the monotonicity of some of these metrics for SCs and for the special case of graphs.

Theorem III.1. *For $\beta > 1$ and $\alpha = 1/\beta$, the metric $\Psi(\Delta) = \sum_{f\in F}\left(\delta + \sum_{v\in f}\left(\frac{1}{d(v)}\right)^{\alpha}\right)^{\beta}$ is not monotonic.*

Proof: When $\alpha = 1/\beta$, $\beta > 1$ and any constant δ a generic counter example to monotonicity can be constructed with form $\Delta = \{\{1, 2, 3, \ldots, k\}, \{1, k+1\}\}$ whose aggregate strength improves when node $k+1$ is removed. □

Theorem III.2. *Metric 7 is weakly monotonic on graph SCs, but not monotonic on general SCs.*

The weak monotonicity on dimension 1 SC's (graphs) is based on the following lemma, proof omitted due to space constraints.

Lemma III.1. *If $\forall i$: $x_i > y_i > 0$, and $\alpha > 1$, then*

$$(\sum_{1\leq i\leq n} x_i)^{\frac{1}{\alpha}} - (\sum_{1\leq i\leq n} y_i)^{\frac{1}{\alpha}} \leq \sum_{1\leq i\leq n} (x_i^{\frac{1}{\alpha}}) - \sum_{1\leq i\leq n} (y_i^{\frac{1}{\alpha}}).$$

Theorem III.3. *Metric 7 is weakly monotonic on a graph simplicial complex.*

The main idea is to consider how the degrees of the vertices change when a node is removed, and apply Lemma III.1. We have omitted the proof due to space constraints.

Definition III.1 (Degeneration)**.** *Degeneration occurs when removing a node u from an Simplicial Complex Δ, if there exists a facet $S_1 = \{u, v_1, v_2, ..., v_k\}$, $S_1 \in \Delta$, $k \geq 2$ (i.e. $|S_1| \geq 3$) and $S_2 = \{v_1, v_2, ..., v_k, w_1, ..., w_m\}, m \geq 1$. In this case after the removal of u, the S_1 is absorbed by S_2.*

Theorem III.4. *If degeneration does not happen, metric 7 is weakly monotonic on a simplicial complex. (Proof omitted).*

Theorem III.5. *If degeneration happens, metric 7 is not weakly monotonic on a simplicial complex.*

The proof goes by constructing a counterexample simplicial complex. A basic gadget is $\{\{u, v_1, v_2\}, \{v_1, v_2, w\}\}$, notice removal of node u from the gadget will cause a degeneration to happen. We expand this gadget in steps: We increase the degree of w to a large number $N+1$ by adding a large number of edges $(w, o_1), \ldots, (w, o_N)$ to the gadget. Now the gadget is $\{\{u, v_1, v_2\}, \{v_1, v_2, w\}, \{w, o_1\}, \ldots, \{w, o_N\}\}$. Then we add one neighbour s to v_1 and one neighbour t to v_2, besides we increase the degrees of s and t to a large number $N+1$ by adding a large number of edges $(s, s_1), \ldots, (s, s_N), \ldots, (t, t_N)$. After that, we repeat this gadget N times and glue them together like a star with u at the center. It can be shown that the removal of the vertex u causes the metric to increase.

Theorem III.6. *Metric 1 is weakly and team monotonic.*

Proof: The function $g(d) = (1 + 1/d)^d$ is an increasing function when the argument is positive, and for $d \geq 1$ its values are bounded between $2 \leq g(d) < e$.

When removing a node from a Δ, the number of facets does not increase, some of them can only disappear, and so $d(v)$ cannot increase, $g(d(v))$ goes down or remains the same, and so does the product of them. Therefore the metric is weakly monotonic. Similarly, when removing a "team", although the total number of facets may increase, the individual $d(v)$ only decrease or remain the same, and so does the metric. It follows that it is monotonic w.r.t. "team" removal. □

Following the same argument, we can also prove that the metric 3 is weakly monotonic too.

IV. Exhaustive monotonicity tests on small simplicial complexes.

Since many of the proposed metrics cannot be analytically proven to be monotonic, in this section we evaluate them on some small SCs to determine if some of them are "almost monotonic". We enumerate all possible complexes for $n \in \{3, 4, 5, 6\}$ (called SC3, SC4, SC5, and SC6, respectively) and evaluate the metrics on them. Our findings are summarized

Metrics	Weak monotonicity counterexamples
4	$\{\{1,2\},\{1,3\},\{2,3\}\} \xrightarrow{\text{delete } 3} \{\{1,2\}\}$
5	$\{\{1,6\},\{2,6\},\{3,6\},\{4,6\},\{1,2,3,4,5\}\} \xrightarrow{\text{delete } 6} \{\{1,2,3,4,5\}\}$
6	$\{\{1,5\},\{2,5\},\{1,2,3,4\}\} \xrightarrow{\text{delete } 5} \{\{1,2,3,4\}\}$
Metrics	**Monotonicity w.r.t. team removal counterexamples**
2	$\{\{1,6\},\{1,2,3\},\{2,3,4\},\{2,3,5\}\} \xrightarrow{\text{delete } \{1,6\}} \{\{6\},\{1,2,3\},\{2,3,4\},\{2,3,5\}\}$
3	$\{\{1,2,3\}\} \xrightarrow{\text{delete } \{1,2,3\}} \{\{1\},\{2\},\{3\}\}$
4 and 7	$\{\{1,4\},\{1,2,3\}\} \xrightarrow{\text{delete } \{1,4\}} \{\{4\},\{1,2,3\}\}$
6	$\{\{1,5\},\{1,2,3,4\}\} \xrightarrow{\text{delete } \{1,5\}} \{\{5\},\{1,2,3,4\}\}$
Metrics	**Structural monotonicity counterexamples**
1, 2, 3	$\{\{1,2,3\}\} \xrightarrow{\text{remove } \{1,2,3\}} \{\{1,2\},\{1,3\},\{2,3\}\}$
4 and 7	$\{\{1,2\},\{1,3\},\{2,3\}\} \xrightarrow{\text{remove } \{2,3\}} \{\{1,2\},\{1,3\}\}$
5 and 6	$\{\{1,4\},\{2,4\},\{1,2,3\}\} \xrightarrow{\text{remove } \{2,4\}} \{\{1,2,3\},\{1,4\}\}$

TABLE II: Small counterexamples exhibiting violation of the monotonicity of the metrics 1-7 with respect to removing a node (weak monotonicity), team, and facet (structural monotonicity).

in the Tables II and III, and in Fig 2. We tested 3 types of monotonicity:

Node removal. We try to remove each node from a complex and count the percentage of the removal operations that violated monotonicity. The percentage is computed for each SC in a group, and then they are averaged over all SCs in the group. As shown in the Table III, all metrics except the metric 4 perform very well. And in particular, metrics 1, 2, 3, and 7 always decrease when a node is removed (although metrics 2 and 7 may fail for larger complexes).

The averaging procedure for tesing team removal and facet removal is done the same way as for the node removal.

When removing a team, the metrics 1, 2, 4, 5, and 6 performed well, and metrics 1 and 5 were perfect in this test.

Facet removal. All metrics were making mistakes sometimes, although metrics 1, 2, 3, 5, and 6 are doing relatively well. The Fig. 2 may explain why these metrics passed the test better than metrics 4 and 7.

V. Statistical performance of metrics

In general structural monotonicity does not hold for the functions based on facet degree of nodes. This raises the question of how well do these types of functions perform on average. That is, are the failures due to some small set pathological cases? We study the average behavior of the metrics tested (as usually) for the removal of (a) a random node, (b) a random team, and (c) a random facet, with respect

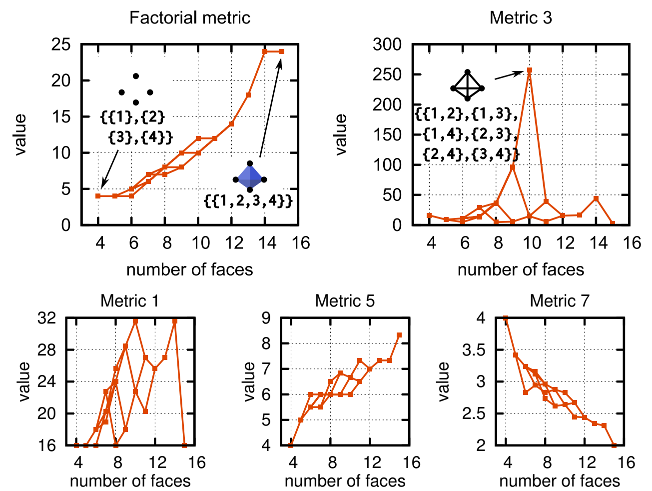


Fig. 2: Structural monotonicity illustrated visually. Metrics are computed on all SC4 complexes. Each **node/point** in the plot **represents a simplicial complex** from SC4, and there is an edge between two nodes if they can can be obtained from one another by adding or removing a face. In particular, consider the top left panel demonstrating the factorial metric $\Psi(\Delta) = \sum_f |f|!$: The complex with only singleton facets $\{\{1\},\{2\},\{3\},\{4\}\}$ has the minimum number of faces (the leftmost point), and the complex $\{\{1,2,3,4\}\}$ has the maximum number of faces (the rightmost point). As expected, the metric is structurally monotonic and therefore it always increases or remains the same when the number of faces increases. Observe that this is not always the case for other metrics: While metrics 5 and 6 (and to some extent 1 and 2) do have this general trend of going up, metric 7 tends to decrease when faces are added. Another interesting feature is that metric 3, which is otherwise quite similar to the metrics 1 and 2, has a special maximum that corresponds to the complex with 6 edges.

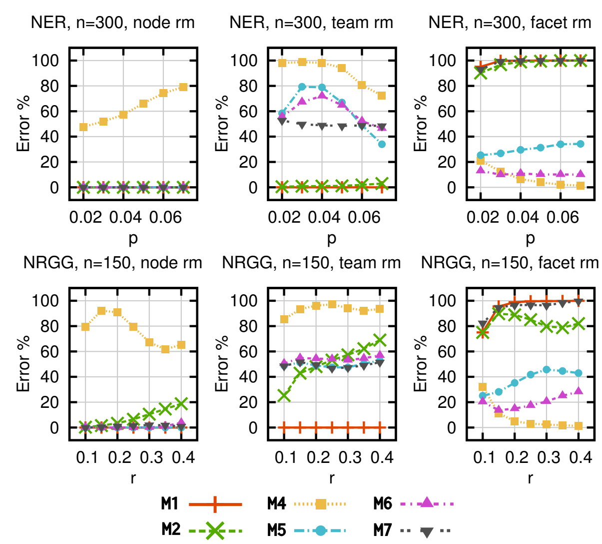


Fig. 3: (Top row): Neighborhood ER for fixed $n = 300$ and varying $p = 0.02, 0.03, \ldots 0.07$. (Bottom row): Neighborhood RGG for fixed $n = 150$ and varying $r = 0.1, 0.15, \ldots 0.4$. Different lines represent different metrics.

removing a node				
Metric	SC3	SC4	SC5	SC6
1	0%	0%	0%	0%
2	0%	0%	0%	0%
3	0%	0%	0%	0%
4	20%	33.75%	68.22%	91.4%
5	0%	0%	0%	0.003%
6	0%	0%	0.78%	0.26%
7	0%	0%	0%	0%

removing a team				
Metric	SC3	SC4	SC5	SC6
1	0%	0%	0%	0%
2	0%	0%	0%	0.009%
3	18.18%	10.77%	3.88%	0.38%
4	0%	4.62%	2.78%	0.24%
5	0%	0%	0%	0%
6	0%	0%	0.11%	0.004%
7	36.36%	27.69%	11.54%	1.25%

removing a facet				
Metric	SC3	SC4	SC5	SC6
1	9.09%	13.85%	18.87%	17.49%
2	9.09%	13.85%	18.65%	19.08%
3	18.18%	18.46%	19.53%	17.51%
4	27.27%	55.39%	79.58%	86.03%
5	0%	9.23%	25.53%	36.15%
6	0%	15.39%	32.19%	42.62%
7	63.64%	72.31%	75.47%	66.48%

TABLE III: Percent of error when removing a node, a team, or a facet. Tested exhaustively on small complexes of size 3, ...6 (SC3, SC4, SC5, and SC6).

to the following random complex generators:

- GA. General SC. Every subset $S \subseteq \{1, \dots n\}$ is added with probability $p^{|S|-1}$.
- GB. General SC. Starting with singletons $\{1\}$, $\dots\{n\}$. With probability p add a face of the dimension k, if all its subfaces of the dimension $k-1$ already exist.
- NER. Neighborhood SC of a random Erdos-Renyi graph. Generate a random graph for the given p and n Then, for every vertex v, its open neighborhood $N(v)$ is a facet.
- CER. Clique SC of a random Erdos-Renyi graph. From a random graph, each maximal clique is a facet.
- NRGG. Neighborhood SC of a random geometric graph, parameterized by the radius r.
- CRGG. Clique SC of a random geometric graph.

Some monotonicity tests for our metrics are shown in Fig 3.

The metrics 1, 2, and 3 are quite similar, and feature good monotonicity properties, and usually they either fare very well (node and team removal) or rather poorly (facet removal, especially when the facets are large, i.e. p and r are large).

Interestingly, metric 4, which is usually quite bad, turns out to be structurally monotonic when the facets are large (that is, when p or r are large). It can be explained as follows. When a big facet is removed, it spawns a large number of smaller facets, and many of these facets happen to be new, and so the number of facets increases significantly. This change, in turn, by the nature of the metric, reduces the strength. In terms of their performance, metric 4 is a direct opposite of metric 1: when one works well, the other does not, and vice versa.

Metrics 5 and 6 can be described as pragmatic – they do not guarantee absolute monotonicity, but demonstrate a consistently good performance overall.

Metric 7 usually does not work well, except for the random node removal tests. A majority of the other metrics also do well on this test.

VI. Concluding Remarks

We have investigated metrics for measuring the inherent strength of a Network of Teams (NoT), and their monotonicity properties of three kinds – weak, team and structural. We have also provided experimental results on the statistics of monotonicity violations. While there is no one metric that is the best in all situations, the following guidelines may be followed: when networks are small and node/team changes are of interest, metrics 1-3 are good; for large networks metric 4 does well for structural monotonicity (facet changes); and when not much is known about expected size and nature of changes, metrics 5-6 are pragmatic reasonable choices. An appropriate choice and use of metric(s) can help determine how best to augment a friendly network or degrade an adversarial network.

Future research directions include finding better metrics that satisfy all three monotonicity properties, matching strength metrics to real world applications, investigating other desirable properties (e.g. sub-additivity), and comparing the metrics to crowd-sourced evaluations of the same SCs.

References

[1] K. Ehrlich and M. Cataldo, “All-for-one and one-for-all?: A multi-level analysis of communication patterns and individual performance in geographically distributed software development,” in *Proceedings of the ACM 2012 Conference on Computer Supported Cooperative Work*, pp. 945–954, 2012.

[2] M. A. Marks, L. A. DeChurch, J. E. Mathieu, F. J. Panzer, and A. Alonso, “Teamwork in multiteam systems.,” *Journal of Applied Psychology*, vol. 90, no. 5, p. 964, 2005.

[3] J. Liu, J. Reich, and F. Zhao, “Collaborative in-network processing for target tracking,” *EURASIP J. Appl. Signal Process.*, vol. 2003, pp. 378–391, January 2003.

[4] V. E. Krebs, “Mapping networks of terrorist cells,” *Connections*, vol. 24, no. 3, pp. 43–52, 2002.

[5] A. Barabâsi, H. Jeong, Z. Néda, E. Ravasz, A. Schubert, and T. Vicsek, “Evolution of the social network of scientific collaborations,” *Physica A: Statistical Mech. and its Applications*, vol. 311, no. 3, pp. 590–614, 2002.

[6] M. E. J. Newman, “The structure of scientific collaboration networks,” *Proc. National Academy of Sciences*, vol. 98, no. 2, pp. 404–409, 2001.

[7] J. J. Ramasco, S. N. Dorogovtsev, and R. Pastor-Satorras, “Self-organization of collaboration networks,” *Phys. Rev. E*, vol. 70, Sep 2004.

[8] R. Atkin, *Mathematical structure in human affairs*. Heinemann Educational London, 1974.

[9] R. Ramanathan, A. Bar-Noy, P. Basu, M. Johnson, W. Ren, A. Swami, and Q. Zhao, “Beyond graphs: Capturing groups in networks,” in *Proc. IEEE NetSciComm*, pp. 870–875, IEEE, 2011.

[10] T. Moore, R. Drost, P. Basu, R. Ramanathan, and A. Swami, “Analyzing collaboration networks using simplicial complexes: A case study,” in *Proc. IEEE NetSciComm*, pp. 238–243, IEEE, 2012.

[11] C. Berge, *Hypergraphs*. North-Holland, 1989.

[12] R. H. Lucius and K. W. Kuhnert, “Using sociometry to predict team performance in the work place,” *The journal of Psychology*, vol. 131, no. 1, pp. 21–32, 1997.

[13] T. T. Baldwin, M. D. Bedell, and J. L. Johnson, “The social fabric of a team-based mba program: Network effects on student satisfaction and performance,” *Academy of Management Journal*, vol. 40, no. 6, pp. 1369–1397, 1997.

[14] J. N. Cummings and R. Cross, “Structural properties of work groups and their consequences for performance,” *Social networks*, vol. 25, no. 3, pp. 197–210, 2003.

[15] H. Oh, M.-H. Chung, and G. Labianca, “Group social capital and group effectiveness: The role of informal socializing ties,” *Academy of Management Journal*, vol. 47, no. 6, pp. 860–875, 2004.

[16] M. Piraveenan, S. Uddin, and K. S. K. Chung, “Measuring topological robustness of networks under sustained targeted attacks,” in *Proceedings of the 2012 International Conference on Advances in Social Networks Analysis and Mining (ASONAM 2012)*, pp. 38–45, 2012.

[17] E. Spanier, *Algebraic topology*, vol. 55. Springer, 1994.

[18] P. Gould and A. Gatrell, “A structural analysis of a game: the Liverpool v Manchester United Cup Final of 1977,” *Social Networks*, vol. 2, no. 3, pp. 253–273, 1980.