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# Signature for Content Distribution with Network Coding

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# Content distribution of large files

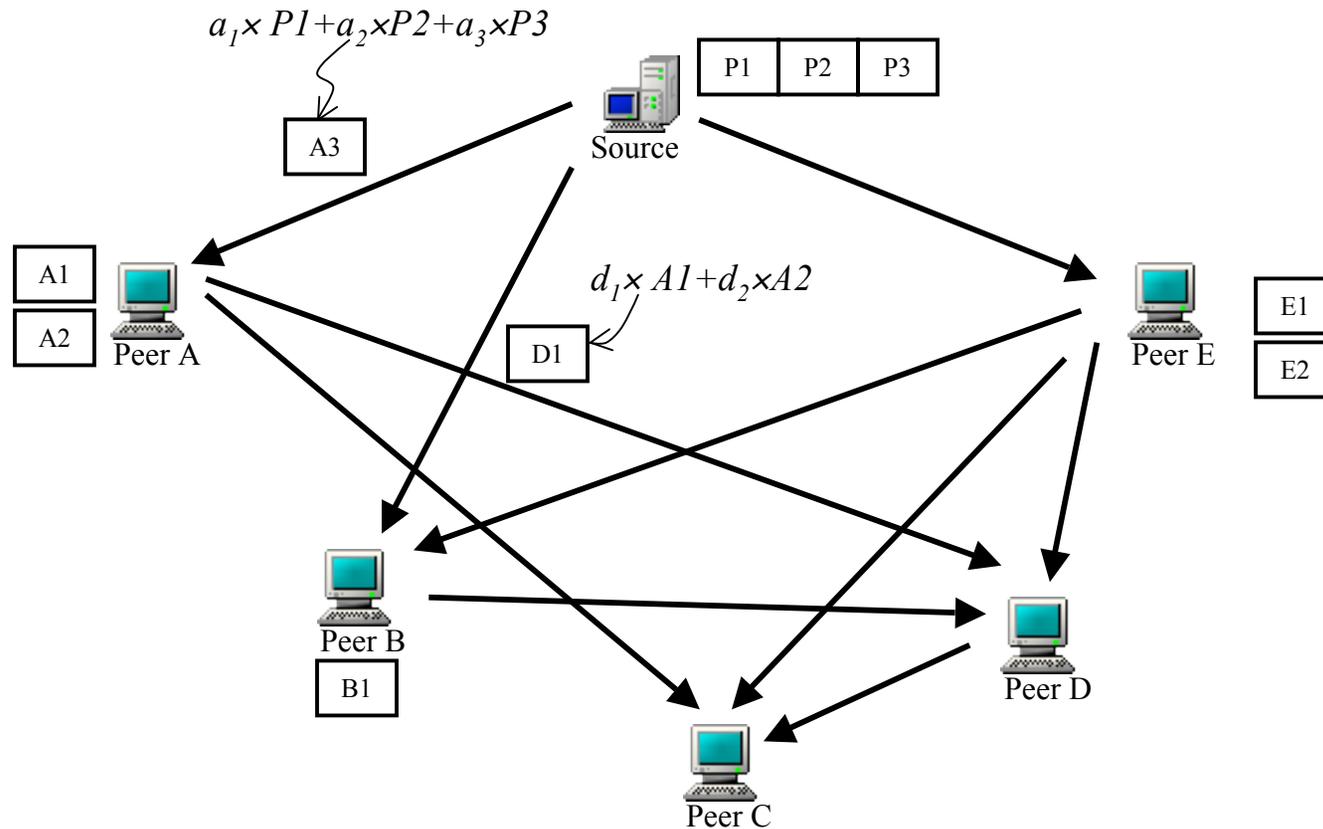
- Distribution of large files to many users.
  - Traditional solutions are based on a client-server model.
  - Alternative technique - P2P swamping.
  - Example - BitTorrent
    - Divide file into many pieces.
    - Client requests different pieces from server(s) or other users.
    - Client becomes server to pieces downloaded.
    - When a client has obtained all pieces, re-construct the whole file.
    - Problem: hard to do optimal scheduling of pieces to nodes.
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# Content distribution using network coding

- Use network coding to increase the efficiency of network coding in a P2P cooperative architecture.
  - Instead of storing pieces on servers, store random linear combination of the pieces on servers.
  - Clients also generate random linear combination of the pieces they have received to send out.
  - When a client has accumulated enough degrees of freedom, decode to obtain the whole file.
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# Content distribution using network coding



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# Security for network coding

- Network coding is vulnerable to pollution attacks by malicious nodes in the network.
  - Malicious user can send packets with valid linear combination in the header, but garbage in the payload.
  - The pollution of packets spreads quickly.
  - Need a homomorphic signature scheme that allows nodes to verify any linear combination of pieces without contacting the original sender.
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# Problem formulation

- A source  $s$  wishes to send a large file to a group of peers,  $T$ .
- View the data to be transmitted as vectors  $\bar{\mathbf{v}}_1, \dots, \bar{\mathbf{v}}_m$  in  $n$ -dimensional vector space  $F_p^n$ , where  $p$  is a prime. The source node augments these vector to  $\mathbf{v}_1, \dots, \mathbf{v}_m$  given by

$$\mathbf{v}_i = (0, \dots, 1, \dots, 0, \bar{v}_{i1}, \dots, \bar{v}_{in})$$

where the first  $m$  elements are zero except the  $i$ -th one is 1, and  $\bar{v}_{ij} \in F_p$ .

- Each packets received by a peer is a linear combination of all the pieces.

$$\mathbf{w} = \sum_{i=1}^m \beta_i \mathbf{v}_i$$

# Signature for network coding

- The vectors  $\mathbf{v}_1, \dots, \mathbf{v}_m$  span a subspace  $\mathbf{V}$  of  $F_p^{m+n}$ .
- A received packet is a valid linear combination if and only if it belongs to  $\mathbf{V}$ .
- Each node verifies the integrity of a received vector  $\mathbf{w}$  by checking the membership of  $\mathbf{w}$  in  $\mathbf{V}$ .
- Our approach has the following ingredients:
  - $q$ : a large prime such that  $p$  is a divisor of  $q - 1$ .
  - $g$ : a generator of the group  $G$  of order  $p$  in  $F_q$ .
  - Private key:  $K_{pr} = \{a_i\}_{i=1, \dots, m+n}$ , a random set of elements in  $F_q^*$ .
  - Public key:  $K_{pu} = \{h_i = g^{a_i}\}_{i=1, \dots, m+n}$ .

# Signature for network coding

- The scheme works as follows:
  -  The source finds a vector  $\mathbf{u}$  that is orthogonal to all vectors in  $\mathbf{V}$ .
  -  The source computes vector  $\mathbf{x} = (u_1 / a_1, \dots, u_{m+n} / a_{m+n})$ .
  -  The source signs  $\mathbf{x}$  with some standard signature scheme and publishes it.
  -  When a node receives a vector  $\mathbf{w}$  and wants to verify that  $\mathbf{w}$  is in  $\mathbf{V}$ , it computes

$$d = \prod_{i=1}^{m+n} h_i^{x_i w_i}$$

and verifies that  $d = 1$ .

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# Discussion

- It can be shown that it is as hard as the  $(p, m, m+n)$  Diffie-Hellman problem
  - Thus, it is as hard as the Discrete Logarithm problem to find new vectors that also satisfy the verification criterion other than those that are in  $V$
  - Overheads
    - Part of the public key  $K_{pu}$  has to be re-generated for each file, otherwise a malicious node can use the information from the previous file
    - Signature vector,  $x$
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# Discussion

- If the file sizes are large, after the initial setup, each additional file distributed only incurs a negligible amount of overhead using our signature scheme.
  - Under our assumptions that
    -  there is no secure side-channel to transfer hash values from the source to all the peer nodes, and;
    -  all peers have full knowledge of the public information of the security scheme,our signature scheme has to be applied on the original file, not on hashes.
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# Conclusions

- Proposed a solution to the security problem in content distribution with network coding.
  - Use a signature vector for each file that can be used to easily check the integrity of all the packets received for this file.
  - This scheme is secure and has low overhead.
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