

Cosmological constraints from the CMB and Ly- α forest revisited

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ABSTRACT

The WMAP team has recently highlighted the usefulness of combining the Ly- α forest constraints with those from the cosmic microwave background (CMB). This combination is particularly powerful as a probe of the primordial shape of the power spectrum. Converting between the Ly- α forest observations and the linear mass power spectrum requires a careful treatment of nuisance parameters and modeling with cosmological simulations. We point out several issues which lead to an expansion of the errors, the two most important being the range of cosmological parameters explored in simulations and the treatment of the mean transmitted flux constraints. We employ a likelihood calculator for the current Ly- α data set based on an extensive 6-dimensional grid of simulations. We show that the current uncertainties in the mean transmission and the flux power spectrum define a degeneracy line in the amplitude-slope plane. The CMB degeneracy due to the primordial power spectrum shape follows a similar relation in this plane. This weakens the statistical significance of the primordial power spectrum shape constraints based on combined CMB+Ly- α forest analysis. Using the current data the simplest $n = 1$ scale invariant model with $dn/d \ln k = 0$ and no tensors has a $\Delta\chi^2 = 4$ compared to the best fitting model in which these 3 parameters are free. Current data therefore do not require relaxing these parameters to improve the fit.

1 INTRODUCTION

Over the past several years we have seen the gradual emergence of a standard cosmological model. We seem to live in a universe dominated by dark energy and dark matter, with a small contribution from baryons. The universe is close to spatially flat and has adiabatic Gaussian initial conditions, consistent with inflation. This picture is confirmed by a range of observations, especially those of the cosmic microwave background (CMB) anisotropies. The latest and most important confirmation has been recently reported by the WMAP satellite (Spergel et al. 2003).

Given that the classical cosmological parameters and the density content of the universe are now gradually being settled the focus is shifting towards other cosmological tests that could reveal important information on the nature of the early universe and its creation. Among these is the amplitude and shape of the power spectrum of the primordial fluctuations. It can be constrained both by the CMB and other probes of large scale structure. Combining different data sets that cover a large range in scale is particularly powerful. Of the current cosmological probes, the Ly- α forest – the absorption observed in quasar spectra by neutral hydrogen in the intergalactic medium (hereafter IGM) – has the potential to give the most precise information on small scales.

For this reason, the WMAP team considered Ly- α forest

constraints in combination with CMB and galaxy clustering constraints to gain better leverage on cosmological parameters. The WMAP data alone are passably well fit by an inflationary CDM model with scale invariant spectrum $n = 1$. When combined with ACBAR, CBI and 2dFGRS the spectrum is pulled slightly towards $n < 1$ at $k = 0.05/\text{Mpc}$ and $dn/d \ln k < 0$. However, the evidence for running of the spectral index is only at the 1.3σ level. When the data are combined with Ly- α forest constraints the statistical significance of both of these results is increased to $2\text{-}\sigma$. The primary reason for this is that in the WMAP analysis of Ly- α forest the preferred slope at $k \sim 0.3 \text{ h/Mpc}$ is $n \sim 0.8$. A running spectral index at the level of $dn/d \ln k = -0.03$ would have significant implications for inflationary models. Here we will argue that the best estimate of the matter power spectrum on Ly- α forest scales is higher in amplitude and slope than the WMAP analysis and that the uncertainty in the amplitude and slope of this power spectrum is larger than their estimate. Both of these changes weaken the case for a running spectral index and indeed make a scale-invariant $n = 1$ model appear to be a good fit to current CMB and Ly- α forest data.

Ly- α forest observations and constraints on cosmology have been explored by several groups in the past after the pioneering work by Croft et al. (1998). On the observational side the two most recent analyses are those by Croft et al.

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(2002), hereafter C02, and McDonald et al. (2000), hereafter M00. The two groups obtain results for the flux power spectrum, $P_F(k)$, in agreement with each other within the errors. In order to compare directly to the WMAP analysis we will only use C02 in this study. Both of these papers also explored theoretical implications. Additional recent theoretical analyses have been performed by Gnedin & Hamilton (2002), hereafter GH, and Zaldarriaga et al. (2001), hereafter ZSH. There are however significant discrepancies among them. For example, using a grid of PM simulations as opposed to direct inversion ZSH show that the errors in C02 and GH may have been underestimated. Another apparent discrepancy is the different scaling of the linear amplitude of the power spectrum with the mean flux in C02 (equation 12) and GH (equation 8): the latter finds almost a factor of 5 weaker dependence relative to the former. Yet another apparent discrepancy is the comparison between the C02 fit and the WMAP reanalysis of the GH results (Verde et al. 2003). While C02 conclude that a scale invariant $n \sim 1$ model fits the data for $\Omega_m \sim 0.3$ and $h \sim 0.7$, the WMAP team uses the GH analysis of the same data (but the C02 normalization uncertainty and expanded small-scale errors) to conclude that the Ly- α forest requires a shallow, $n \sim 0.8$ slope on the scales probed by it (note that we are not referring to the overall slope from the combined analysis of WMAP, just the Ly- α forest portion of it, see figure 1).

We will examine these differences in more detail below, here we highlight the most relevant theoretical issues:

1) Coverage of cosmological models: the relation between $P_F(k)$ and the 3-d linear theory mass power spectrum, $P_L(k)$, is non-linear and model dependent, i.e., it has the completely general form $P_F(k) = f[P_L(k)]$. One must use cosmological simulations to connect the two. One cannot assume that their relation from one cosmological model applies to other models (at the level of accuracy needed for the present data) without verifying it with simulations. A grid of simulations must be built to cover the parameter space of interest. Some of the different scalings found in the literature may be simply due to expansion around a different fiducial model.

2) Treatment of nuisance parameters, such as the temperature-density relation of the gas in the IGM and the mean transmitted flux, $\bar{F}(z) = \langle \exp(-\tau) \rangle$: as emphasized by C02, the inferred amplitude of the matter power spectrum is sensitive to the value of \bar{F} . We will argue below that both the mean value and the error bar on \bar{F} used by C02 were too low. Furthermore, the nuisance parameters couple non-linearly to the power spectrum and cannot be modeled as an overall calibration factor multiplying the inferred $P_L(k)$. Specifically, a change in \bar{F} is not degenerate only with the overall amplitude of $P_L(k)$ but also with its slope. This has been emphasized previously by ZSH and can be seen in figure 16 of C02. However, both C02 and GH model the influence of \bar{F} only as an overall calibration factor multiplying the inferred $P_L(k)$, ignoring its impact on the shape.

3) Care with numerical details: the inference of the matter power spectrum from $P_F(k)$ relies on numerical simulations. Previous investigations include some tests of the validity of the approximate semi-analytic descriptions of the IGM used in these simulations and some investigations of the influence of numerical resolution and box size. However, these tests do not demonstrate numerical convergence

of the results at the level of precision that is of interest, given the quality of the flux power spectrum measurements themselves. The numerical simulations used to interpret $P_F(k)$ in previous analyses were not tested for convergence of the final results with resolution and box-size. Similarly, GH and ZSH use collisionless particle-mesh simulations combined with approximate semi-analytic descriptions of the gas instead of fully hydrodynamic simulations, without quantitative tests of the accuracy of the approximations.

The goal of this paper is to investigate the cosmological ramifications of a more reliable conversion between the measured flux power spectrum and the matter power spectrum. Of the three points mentioned above, all three are significant, but the combination of items one and two proves to be the most important for the current data sample.

2 COMPARISON WITH PREVIOUS ANALYSES

In the standard picture of the Ly- α forest the gas in the IGM is in ionization equilibrium. The rate of ionization by the UV background balances the rate of recombination of protons and electrons. The recombination rate depends on the temperature of the gas, which is a function of the gas density. The temperature-density relation can be parameterized by an amplitude, T_0 , and a slope $\gamma - 1 = d \ln T / d \ln \rho$. The uncertainties in the intensity of the UV background, the mean baryon density, and other parameters that set the normalization of the relation between optical depth and density can be combined into one parameter: the mean transmitted flux, $\bar{F}(z)$. These parameters of the gas model, T_0 , $\gamma - 1$, and \bar{F} , must be marginalized over when computing constraints on cosmology.

In observationally favored models, the Universe is effectively Einstein-de Sitter at $z > 2$, so the cosmology information relevant to the Ly- α forest is completely contained within $P_L(k)$ measured in velocity units. We parameterize the linear power spectrum by its amplitude, $\Delta^2(k_p) \equiv k_p^3 P_L(k_p) / 2\pi^2$, its effective slope, $n_{\text{eff}}(k_p) \equiv d \ln P_L / d \ln k|_{k_p}$, and the effective running of the slope, $dn_{\text{eff}} / d \ln k|_{k_p}$, where $k_p = 0.03$ s/km is the C02 pivot point.

Rather than attempting to invert $P_F(k)$ to obtain the matter power spectrum we compare the theoretical $P_F(k)$ directly to the observed one. The advantage of this approach is that we can compute χ^2 from the data without worrying about complicated window functions and covariances between the data points (there will still be some covariances between the $P_F(k)$ points, which are not given by C02, but simulations indicate that they will not be large on the relevant scales).

For our present analysis we use a likelihood function module based on the C02 data and the library of simulations described in McDonald et al. (2003). These simulations cover the plausibly allowed range of \bar{F} , T_0 , $\gamma - 1$, Δ^2 , n_{eff} , and $dn_{\text{eff}} / d \ln k$. Simulations with several box and grid sizes are used to guarantee convergence. The grid is based on hydro-particle mesh simulations (Gnedin & Hui 1998; hereafter HPM), but these are explicitly calibrated using fully hydrodynamic simulations (Cen et al. 1994, 2002). The simulation results are combined in an interpolation code that

produces $P_F(k)$ for any relatively smooth (CDM-like) input $P_L(k)$, \bar{F} , T_0 , and $\gamma - 1$.

We start by comparing C02, GH, and GH as interpreted by Verde et al. (2003) to each other. The original C02 result for the amplitude and slope of $P_L(k_p = 0.03 \text{ s/km})$ is plotted as the large filled square with error bars in figure 1(a). Our fit to the points in Table 4 of C02 is plotted as the large open square with no error bars (some, but not all, of the discrepancy can be explained by our linearization of the fit using $\ln P$ vs. $\ln k$). C02 used a pure power law in their fit, and chose the pivot point to make the error bars on the slope and amplitude independent. We perform the fit using a Λ CDM transfer function shape, still using the power law free parameters, and plot the surprisingly different result as the open triangle in figure 1(a). We suppress the error bars on the amplitude for this and subsequent fits to the C02 or GH points because they are generally similar in size to the errors on the original C02 point. The difference in the form of the fit begins to explain the smaller n_{eff} implied by the WMAP analysis. Our identical fit to the points in Table 1 of GH produces the still smaller n_{eff} represented by the small open square in figure 1(a). Verde et al. (2003) expanded the errors on the last 3 points to bring C02 and GH into agreement on these points within $1-\sigma$. We show the results of fits using this prescription as the open pentagon and hexagon (for C02 and GH, respectively). Ultimately, the agreement between C02 and GH is not bad, but not perfect.

Our version of the constraint is shown in figure 1(a) by the filled square for the best fitting point and the solid, 68% contour for the error. This fit was performed assuming $\bar{F} = 0.705 \pm 0.012$ to match the C02 value (GH discuss a larger error bar but this only affects the error on the amplitude – WMAP effectively used the C02 error). We will argue in Section 3 that this value and error bar are both too small. Our amplitude is 20% smaller than the GH amplitude, while our slope is very similar. We disagree on the shape of the error contours – the GH errors on Δ^2 and n_{eff} are uncorrelated, while our contour shows significant correlation. The shape of our error contours is in a good agreement with those in ZSH, which also show a significant correlation between the slope and the amplitude. It also appears to agree with figure 16 of C02, although they did not bring this point out in their discussion of the $P_L(k)$ errors. The errors one finds from a direct fit to the C02 or GH points are too small, but the expansion of the errors by Verde et al. (2003) appears to have produced reasonable results.

What is the reason for the disagreement between our central value, GH, and C02? We find that resolution and box size effects go in the right direction to explain the disagreement. GH used 20 Mpc/h simulations with 256^3 particles, while the resolution used by McDonald et al. (2003) is a factor of two better than this (see also McDonald (2001) and Meiksin & White (2001)). When we degrade our resolution to match GH we obtain the filled triangle and dashed contour in figure 1(a), with the increase in inferred slope and amplitude slightly over-shooting the GH result. C02 used simulations with another factor of two poorer mass resolution.

We also show the result of removing the hydrodynamic correction (the difference between hydrodynamic simulations and HPM simulations) that McDonald et al. (2003) (see also Mandelbaum et al. (2003)) apply to the HPM sim-

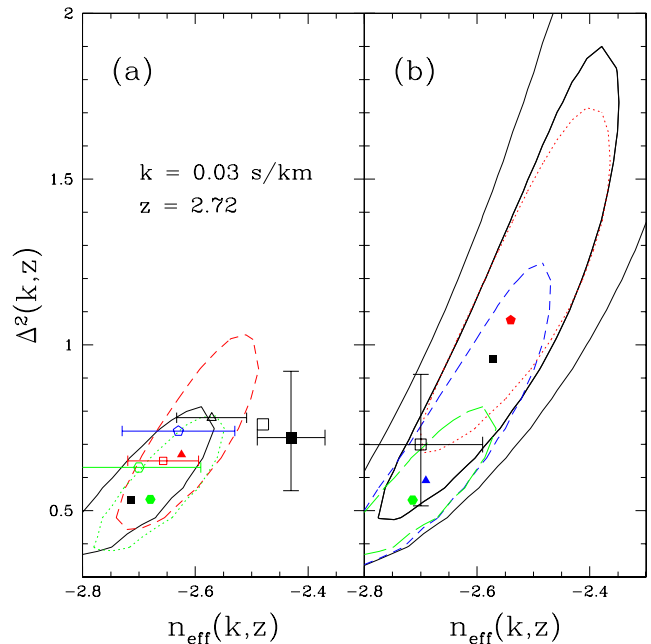


Figure 1. Constraints on effective slope n_{eff} and amplitude Δ^2 at $k_p = 0.03 \text{ s/km}$. (a) The large, solid square with error bars is the original C02 result for a power law fit, while the large empty square is our similar fit to the C02 $P_L(k)$ points. Small empty symbols are our fits to the C02 and GH $P_L(k)$ results using a CDM shape of the transfer function (which differs from a pure power law fit, which does not include the fact that the effective slope is rapidly changing even in the narrow range probed here) – triangle and square: fits to C02 and GH, respectively, using their error bars; pentagon and hexagon: similar with expanded errors on the last 3 points (slope errors are from the fit, amplitude calibration errors are all similar to the original C02 point). Small solid symbols and 68% contours are constraints from our direct fits to the C02 $P_F(k)$ points – square and solid contour: our full procedure using C02 \bar{F} constraints; triangle and dashed contour: similar with degraded simulation resolution; hexagon and dotted contour: degraded resolution and no correction using fully hydrodynamic simulations. (b) The empty square and its associated errors show our estimate of the Ly- α constraint used by WMAP (we apply an 11% amplitude increase over the GH points). Solid symbols and contours are our fits to the C02 $P_F(k)$ points for different assumptions about \bar{F} – hexagon and long dashed contour: $\bar{F} = 0.705 \pm 0.012$; triangle and short dashed contour: $\bar{F} = 0.705 \pm 0.027$; pentagon and dotted contour: $\bar{F} = 0.742 \pm 0.012$; square and solid contour: $\bar{F} = 0.742 \pm 0.027$. The latter is our final result, for which we also show the 95% contour.

ulations (filled hexagon and dotted contour). This has a comparable effect to the resolution and works in the opposite direction (note that this is only true for this specific choice of parameters). In the next section we find that the errors caused by these numerical details are insignificant compared to the current overall uncertainties, but they will become significant with future data sets.

3 MEAN ABSORPTION

There are 3 nuisance parameters we include in our simulations: mean transmitted flux, \bar{F} , mean temperature, T_0 , and the slope of the temperature density relation, $\gamma - 1$ (we note that hydrodynamical simulations give results very close to the power law form). Among the nuisance parameters \bar{F} has the largest effect, while the temperature-density relation leads to effects that can be neglected in the current discussion. There is considerable uncertainty in the value of $\bar{F}(z)$ [often transformed to an effective mean optical depth, $\bar{\tau}_{eff} = -\ln(\bar{F})$]. There are two main approaches in the literature that attempt to determine it directly. The method pioneered by Press et al. (1993), hereafter PRS, uses low resolution spectra and assumes an extrapolation of the quasar continuum from the red side of the Ly- α emission line. They give $\bar{\tau}_{eff} = A(1+z)^{g+1}$ with $g = 2.46 \pm 0.37$ and $A = 0.0175 - 0.0056g \pm 0.0002$. Ignoring the errors one finds $\bar{\tau}_{eff} = 0.35$ (here and in the following all the values are reported at $z = 2.72$, the central value for the C02 data). However, assuming the probability distributions for A and g are Gaussian one finds $\bar{\tau}_{eff} = 0.29 \pm 0.14$. This value and error estimate appears to be inconsistent with their figure 4, so it is not clear which to use. Note that C02 quote $\bar{\tau}_{eff} = 0.35 \pm 0.018$ and GH give $\bar{\tau}_{eff} = 0.35^{+0.051}_{-0.034}$. Both of these, especially the former, have significantly smaller errors than one obtains from the original expression by PRS, but are closer (especially the latter) to a “by eye” estimate from figure 4 of PRS (note, however, that it is unclear how the band in PRS figure 4 was determined).

A more recent analysis by Bernardi et al. (2003) uses a very similar method to PRS and applies it to a much larger SDSS sample, consisting of around 1000 quasars in the relevant redshift range. They find a rather similar result $\bar{\tau}_{eff} \sim 0.35$. While formal error bars are not quoted they appear to be rather small, although the fact that the evolution they find is not very smooth complicates the error estimate, since one must be careful to match the redshift distribution to that of the C02 sample.

The main systematic uncertainty in the PRS and Bernardi et al. (2003) methods is the reliability of the continuum extrapolation: Bernardi et al. (2003) assume that the continuum power-law found on the red side of the Ly- α emission line extends to the blue side, while PRS assume that the formula $A\lambda + B\lambda^{1/2}$ can be used for this extrapolation. In a recent analysis of low redshift HST quasars (for which the absorption is much less significant in the Ly- α forest range) Telfer et al. (2002) find that there is a clear break in the slope close to the Ly- α emission line, $\lambda \sim 1200 - 1300\text{\AA}$. The change in the slope is around one (see their figure 4 and table 1). This appears to produce a *minimum* 0.05 underestimate of \bar{F} in the Ly- α forest region if one uses the PRS or Bernardi et al. (2003) method. In this case instead of $\bar{F} = 0.70$ one should use $\bar{F} = 0.75$.

A different local continuum fitting method has been applied by Rauch et al. (1997) and M00 to high resolution HIRES spectra, which have high signal to noise ratio. In this method one locally estimates the continuum using the regions where there is no apparent absorption. This is done by attempting to identify unabsorbed regions within the forest and connecting them with a smooth curve. The main uncertainty in this method is the reliability of the local con-

tinuum estimation, which would tend to overestimate \bar{F} if there is some absorption everywhere. This method improves at low redshifts where the absorbers become rarer and the voids empty of neutral hydrogen can be identified more easily. However, the number of QSOs analyzed was rather small, so the redshift evolution was only coarsely determined. Interpolating M00 to $z = 2.72$ we find $\bar{F} = 0.742 \pm 0.017$, or $\bar{\tau}_{eff} = 0.298 \pm 0.023$, where the error is statistical only based on bootstrap resampling. This result is higher than the PRS value as given by C02 and GH. This was interpreted by C02 as a sign of deficiency in the local continuum fitting method. On the other hand, the two values come into a close agreement if a 0.05 correction based on the Telfer et al. (2002) results is applied, arguing against the idea that the local continuum fitting method has a significant bias.

Two other tests suggest that there is no large bias in the M00 $\bar{F}(z)$ estimate: First, to test the idea that the local continuum method is biased by a lack of unabsorbed regions, M00 determined the mean value of the minimum absorption in $10h^{-1}\text{Mpc}$ chunks of spectra from a hydrodynamic simulation. The offset, interpolated to $z = 2.72$, was only $\Delta\bar{F} = 0.01$, which should represent an upper limit on the bias. Second, we recently analyzed a sample of 15000 QSO spectra from the Sloan Digital Sky Survey to determine self-consistently the evolution of $\bar{F}(z)$ with redshift, the composite QSO spectrum and the principal components of the deviations from the mean composite (McDonald et al. 2003, in preparation). Since we do not assume anything about the shape of the continuum we can only determine $\bar{F}(z)$ as a function of redshift up to an overall constant, which is degenerate with the QSO shape. We find that the redshift evolution of M00 agrees well with this analysis within the errors, suggesting again that the systematic errors in the local continuum method are not large. If there was a bias from the local continuum fitting, one would expect the error to increase with redshift due to the increase in the mean absorption. That continuum extrapolation method of PRS may lead to an underestimate of \bar{F} has also been pointed out by Kim et al. (2001) and Meiksin et al. (2001).

We should mention that C02 also present the flux filling factor statistic, which supports the lower value of \bar{F} . This is a statistic based on the unsmoothed data and is sensitive to the smallest structure in the forest. As such it is probably sensitive to the resolution and small scale physics details in the simulations, as well as details of the noise distribution. Another promising method to determine the mean flux indirectly may be offered by the bispectrum analysis, which concentrates on the large scales where both simulation resolution and noise properties are less important (Mandelbaum et al. 2003).

Given the current situation it seems clear that using $\bar{F} = 0.705 \pm 0.012$ underestimates the error and probably the amplitude and that a more conservative treatment is needed. Here we will adopt the value $\bar{F} = 0.742 \pm 0.027$. We have slightly expanded the errors relative to M00 to account for the uncertainties in the local continuum method, the redshift evolution, the possibility of imperfect removal of metal absorption by M00, and the effect of damped Ly- α systems. While we believe this value represents a more realistic estimate, we will also show below the results assuming $\bar{F} = 0.705$ but with expanded errors to more realistically account for the current uncertainties.

Figure 1(b) shows the change in the amplitude and slope of the inferred linear power at the pivot point if one uses different values for \bar{F} or for its error. We show both the best fitted value and the corresponding 68% error contours. The main effect comes from changing \bar{F} from 0.705 to 0.742, even if we keep the errors unchanged at 0.012. We see that this changes the best fit amplitude by almost a factor of 2 and the slope by 0.2. The effect on the amplitude is much larger than the GH estimate, but closer to the C02 estimate (we remind the reader here that the error estimates for the mean flux were smaller in C02 than in GH, making their final estimates comparable to each other and significantly smaller than our estimate). As discussed above, a change in \bar{F} does not correspond to a simple rescaling of the overall amplitude of the matter power spectrum. One can see how the error contours really explode in the direction of large Δ^2 and large n_{eff} , probably a consequence of the fluctuations becoming more nonlinear. We note in this context that there are competing effects when one increases the amplitude of fluctuations: while density fluctuations increase so do velocities and the corresponding suppression of power on small scales (“fingers of god” effect). As a result, the flux power spectrum can go up (on large scales) or down (on small scales), with the zero crossing not too different from the pivot point of the current data. The actual dependence of the flux power amplitude on the linear amplitude is thus rather sensitive to the range of scales covered by the data and the model around which one studies the dependence. This effect probably explains some of the differences found in the scalings between the linear amplitude and the mean flux. It highlights the need to use a grid of simulations that covers the parameter space of interest.

In addition to the change in the mean value we also investigated the effects of increase in the error estimate. If we use $\bar{F} = 0.705 \pm 0.027$ the 68% contour again expands significantly (figure 1b). If we use $\bar{F} = 0.742 \pm 0.027$, our current best estimate, the error contours expand relative to $\bar{F} = 0.742 \pm 0.012$, although the effect is less dramatic. The change in \bar{F} moves the points and error contours along a degeneracy line diagonal in the Δ^2 - n_{eff} plane. Note that one cannot parameterize the error contours with a Gaussian form.

4 COMBINED CMB-LY- α FOREST ANALYSIS

To assess the effects of our improved treatment we present a joint WMAP+CBI+ACBAR (“WMAPext”) +Ly- α forest analysis (Verde et al. 2003; Mason et al. 2002; Kuo et al. 2002), which can be compared to the WMAP results (Spergel et al. 2003; Peiris et al. 2003). While the WMAP team also included the 2dFGRS galaxy clustering results we have chosen not to do so in the present analysis. The main reason is that the 2dFGRS results do not significantly strengthen the significance of the results on the primordial power spectrum. While a joint analysis with the Ly- α forest gives a 2- σ indication of $n < 1$ and $dn/d \ln k < 0$, the result with 2dF is actually less significant than the result from the CMB alone and both $n = 1$ and $dn/d \ln k = 0$ are statistically acceptable at the 1.5- σ level. In addition, the galaxy power spectrum is measured at low redshift, so one must include growth factor effects due to the possible

nonstandard dark energy component. Finally, there remain concerns about the accuracy of the linear bias and redshift distortion assumptions on scales used in the 2dF analysis, as well as the remaining effects of luminosity bias.

Our approach to parameter estimation is a standard one. We run CMBFAST (Seljak & Zaldarriaga 1996) to generate both the CMB power spectrum and the matter power spectrum at $z = 2.72$. We use the former to obtain the CMB χ^2 . We interpolate the latter onto the grid of simulations to obtain χ^2 for Ly- α forest, marginalizing over all the nuisance parameters (\bar{F} , T_0 , and $\gamma - 1$). We generate a Monte Carlo Markov chain using the combined χ^2 . We obtain a sequence of chain elements that sample the probability distribution in the space of cosmological parameters. We ran several chains both with and without tensors. We verified that we obtain results similar to those reported by Spergel et al. (2003) using the CMB data only.

It is instructive to look at the constraints in the Δ^2 - n_{eff} plane at the $k = 0.03$ s/km pivot discussed before. These are shown in figure 2 together with our Ly- α contours. The CMB data define a degeneracy line in Δ^2 and n_{eff} , which is essentially determined by the curvature in the primordial power spectrum: low Δ^2 and n_{eff} indicates a very negative curvature, while for high values of Δ^2 and n_{eff} one has $dn/d \ln k > 0$. Note that this degeneracy is very similar to the \bar{F} degeneracy in the Ly- α forest. The filled triangles in figure 2 show the models produced by the CMB only chain with $dn/d \ln k > 0$. One can see that while these models are between the 68% and 95% contours for the CMB, they are inside 68% for the Ly- α forest. In contrast, these models are 2- σ away from the Ly- α constraint as used by the WMAP team (with or without tensors). The Ly- α constraint used by the WMAP team reinforces the slight preference for negative $dn/d \ln k$ that appears in the CMB data alone. Our reanalysis of the Ly- α forest, by contrast, pulls away from the direction of the best-fit CMB-only model towards $dn/d \ln k \approx 0$.

We ran Markov Chains with a combined WMAPext+Ly- α analysis with and without tensors (we fix the tensor slope to $T/S = -8n_T$). We only focus on the parameters that may be affected by our reanalysis (the primordial power spectrum and tensors) and we emphasize that the other constraints as presented by WMAP remain unchanged. We find that the running of the slope is strongly correlated with the tensors, in such a way that a larger tensor contribution requires more negative running. This is not surprising, since negative running reduces power on large scales (compared to no running), which can then be filled in by tensors. It however suggests that any one-dimensional distribution of $dn/d \ln k$ will depend on the assumed prior for T/S and that positive $dn/d \ln k$ may appear unlikely only because of the bound $T/S > 0$. In the limit of a perfect degeneracy between the two parameters and assuming a uniform prior on $T/S > 0$ the probability for $dn/d \ln k > 0$ simply decreases with the assumed upper cutoff on T/S . This means that adding two or more additional parameters when they are not required by the data can lead to misleading results, especially if only one-dimensional projections are used in interpretation. If we fix $T/S = 0$ we find that the no running model is within the 68% contours, with about 1/3 of all chain elements having $dn/d \ln k > 0$. Even more impressive is the success of the simplest scale invariant model with no tensors

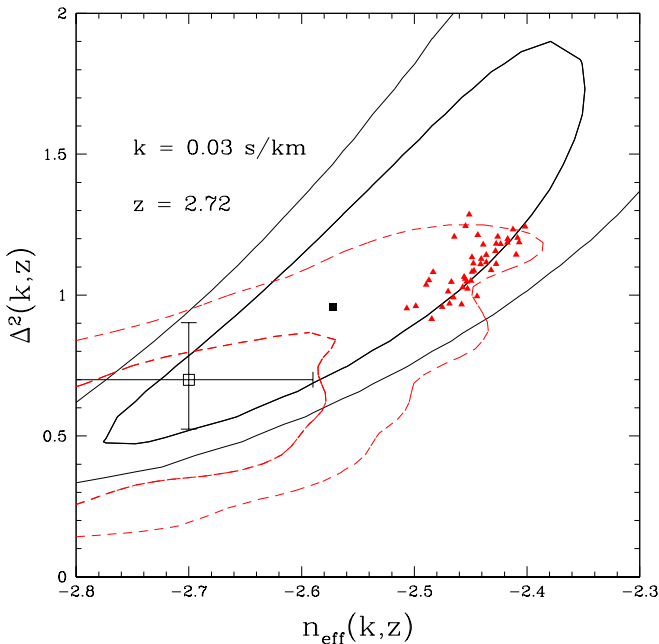


Figure 2. Our Ly- α constraints (solid square and solid contours), CMB (WMAP+ACBAR+CBI) constraints (dashed contours) and the Ly- α constraint used by WMAP (empty square with errors). The CMB contours follow $dn/d \ln k$, with more negative values leading to low amplitude and low slope: very roughly, $n_{\text{eff}} \sim -2.45$ corresponds to $dn/d \ln k = 0$, $n_{\text{eff}} \sim -2.7$ to $dn/d \ln k = -0.03$ (final WMAP value) and $n_{\text{eff}} \sim -2.85$ to $dn/d \ln k = -0.05$ (WMAPext value). We warn that due to a small number of Markov Chain elements (around 25000) the contours shown here may be an underestimate of the true 68% and 95% contours for the CMB. While the Ly- α constraint as used by WMAP pulls towards negative $dn/d \ln k$, our Ly- α constraints have a strong degeneracy, with the best fit preferring less negative $dn/d \ln k$. Filled triangles show the models obtained from the CMB-only chain with $dn/d \ln k > 0$.

($T/S = 0$, $n = 1$ and $dn/d \ln k = 0$). The best fitted model has $\chi^2 = 1449$ for the combined data, while the best fitted model without restrictions on tensors, slope or running in our chains has $\chi^2 = 1445$ (for the Ly- α forest the χ^2 is around 6-7 for 9 effective degrees of freedom, indicating a good fit to the data). The difference is thus only $\Delta\chi^2 = 4$ for effectively 3 degrees of freedom, corresponding to close to 68% confidence level. Adding these 3 degrees of freedom thus hardly improves the fit at all! This should be compared to the WMAP results where both $n = 1$ and $dn/d \ln k = 0$ were excluded at the (roughly) 2-sigma level (Spergel et al. 2003) (see table 8 of that paper; note that $n = 1$ was allowed if tensors were included, and that the correlation between the errors on n and $dn/d \ln k$, which affects the precise level of confidence for the combined result, was not given). Our re-analysis thus reduces the statistical significance of the WMAP result on these two parameters. While we did not perform the combined analysis with 2dF it seems likely that with the expanded errors the addition of Ly- α does not improve the constraints, except for eliminating very strong runnings with $dn/d \ln k < -0.05$.

To summarize, we use the simulations described in McDonald et al. (2003) to reanalyze the current Ly- α forest

observations. Our results are based on hundreds of simulations that cover the range of parameter space needed with sufficient dynamic range. We compare simulations directly to the data rather than performing an inversion from the flux power spectrum to the linear power spectrum. We also emphasize the importance of external constraints such as the mean flux decrement, which is still poorly determined and can lead to a significant expansion of the errors on the cosmological constraints. The degeneracy direction due to the uncertainty in the mean flux is similar to that from the CMB data. Above all, we argue that previous analyses used an error bar on mean flux decrement that is too small, and probably a value that is too low, and that better estimates of it and its uncertainty, combined with a grid of simulations that fully covers parameter space, bring the Ly- α forest data into better agreement with a scale-invariant model. We find that at the moment the combined CMB+Ly- α forest data do not rule out the simplest $n = 1$, $T/S = 0$ and $dn/d \ln k = 0$ model.

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