

Leading nucleon and inelasticity in hadron-nucleus interactions

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Abstract

We present in this paper a calculation of the average proton-nucleus inelasticity. Using an Iterative Leading Particle Model and the Glauber model we relate the leading particle distribution in nucleon-nucleus interactions with the respective one in nucleon-proton collisions. To describe the leading particle distribution in nucleon-proton collisions we use the Regge-Mueller formalism.

1 Introduction

It is well known that the inelasticity is one of the most important variables to analyse cosmic ray data on hadronic cascade and on extensive air showers. A wide and diversified range of hadron interaction calculations in cosmic ray and accelerator physics is strongly dependent on the inelasticity parameter, whose energy dependence is presently in question.

Inelasticity is understood as the fraction of the available energy released for multiple particle production in an inelastic hadron-nucleus collision. Its value was estimated a long time ago from cosmic ray experiments [1] as being around 0.5, as was later confirmed at the CERN ISR [2],

The question of the energy dependence of the inelasticity was dealt by a number of authors through different approaches, most of them in a model dependent way. The obtained results are not consistent [3],[4] and the question remains unsolved. Considering the average inelasticities in hadron-proton collisions, a careful analysis of inclusive reaction data was done by Bellandi *et al.* [5]. The average partial inelasticities were extracted in a model-independent way from experimental data of inclusive reactions initiated by pp collisions ($pp \rightarrow c$, $c = \pi^\pm, K^\pm, p, \bar{p}$). From the results obtained for π, K, p, \bar{p} partial inelasticities it was also estimated the behavior of the total proton inelasticity, which turned out to be rapidly increasing with energy in the high energies region.

Bellandi *et al.* [6],[7],[8] have already discussed this question in connection with the behavior of the hadronic cascade and extensive air showers in the atmosphere, showing that the average proton-air inelasticity is also an increasing function of the energy. A model dependent analysis of the average inelasticity in [8] was done by means of the so called Interacting Gluon model (IGM) [9], [10], which includes, besides soft gluonic interactions, semi-hard QCD interactions responsible for minijet production. This model in the original version [9] had predicted inelasticity decreasing with energy. With the addition of the semi-hard component [10] the total inelasticity is an increasing function of the energy [8].

For proton-nucleus scattering, at low energy, several models for describing the leading particle spectrum have been proposed (Interacting Gluon model and Regge-Mueller formalism) [11], [12]. Here, we shall work in the Iterative Leading Particle Model [13], [14] and use the notation of Fricther, Gaisser and Stanev [15]. In this model the leading particle spectrum in $p + A \rightarrow N(\text{nucleon}) + X$ collisions is built from successive interactions with ν interacting protons of the nucleus A and the behaviour is controlled by a straightforward convolution equation. It should be

mentioned that, strictly speaking, the convolution should be 3-dimensional. Here we only considered the 1-dimension approximation.

2 Inelasticities

In this model [15] an iterative scheme was used to write the longitudinal distributions for multiple scattering of nucleons with wounded nucleons. After ν collisions the longitudinal distributions are related by means of the following 1-dimensional Mellin convolution integral

$$M_\nu^p(x) = \int_x^1 \frac{dy}{y} [S_{\nu-1}^+(y)\beta_{\nu-1}M_{\nu-1}^p(x/y) + S_{\nu-1}^-(y)(1 - \beta_{\nu-1})M_{\nu-1}^n(x/y)] \quad (1)$$

for protons and

$$M_\nu^n(x) = \int_x^1 \frac{dy}{y} [S_{\nu-1}^+(y)\beta_{\nu-1}M_{\nu-1}^n(x/y) + S_{\nu-1}^-(y)(1 - \beta_{\nu-1})M_{\nu-1}^p(x/y)] \quad (2)$$

for neutrons. $M_\nu^{p,n}(x)$ are the proton and neutron distributions normalized as

$$\int_0^1 dx M_\nu^{p,n}(x) = n_\nu^{p,n} \quad (3)$$

with $n_\nu^p + n_\nu^n = 1$. The numbers n_ν^N express the outgoing nucleon (p and n) multiplicities for ν wounded target nucleons. The superscripts (\pm) describe interactions which preserve and change the projectile isospin, respectively, and the parameters β_ν specifying the fraction of isospin preserved reactions. The $S_{\nu-1}^\pm(y)$ define the probability of transition of a nucleon with longitudinal momentum fraction x/y to a state with longitudinal momentum x , after $\nu - 1$ collisions. For the probability functions $S_\nu^\pm(y)$, we have for the first collision

$$S_o^\pm(y) = \frac{M_1^{p,n}(y)}{\int_o^1 dy M_1^{p,n}(y)} \quad (4)$$

with appropriate definitions of M_o^p and M_o^n [15].

In this model it is assumed that we have different inelasticities upon subsequent collisions. Adopting a power law form with adjustable factor for $\nu > 1$, we write

$$S_\nu^\pm(y) = \frac{y^{\alpha_\nu} M_1^{p,n}(y)}{\int_o^1 dy y^{\alpha_\nu} M_1^{p,n}(y)} \quad (5)$$

In order to compute the average nucleon-nucleus elasticity, $\langle x \rangle_{N-A}$ we use the Glauber model [16]. The $N - A$ leading particle can be obtained by means of the relation

$$M_{N-A} = \sum_{\nu=1}^{\infty} P_{\nu} M_{\nu}, \quad (6)$$

where P_{ν} is the probability of ν -fold collisions of the nucleon inside the nucleus, given by

$$P_{\nu} = \frac{\int d^2b P_{\nu}(b)}{\sigma_{in}^{N-A}} \quad (7)$$

and

$$P_{\nu}(b) = \frac{1}{\nu!} [\sigma_{tot}^{pp} AT(b)]^{\nu} \exp[-\sigma_{tot}^{pp} AT(b)], \quad (8)$$

where $T(b)$ is the nuclear thickness, given in terms of the nuclear density by

$$T(b) = \int_{-\infty}^{+\infty} dz \rho(b, z)$$

and normalized in the following way

$$\int d^2b T(b) = 1$$

The inelastic cross section σ_{in}^{N-air} is given by the following Glauber model [16] relation

$$\sigma_{in}^{N-A} = \int d^2b [1 - \exp[-\sigma_{tot}^{pp} AT(b)]] \quad (9)$$

As in [15] we shall assume that the $S_{\nu}^{\pm}(y)$ are the same for all interactions with more than one collision, $\nu > 1$. Using Eqs. (1) and (2), it is straightforward to show that the serie in Eq. (6) is an absolutely convergent serie and we can write

$$\begin{aligned} (1 - \langle x \rangle_{N-A}) &\equiv \langle K \rangle_{N-A} \\ &= \frac{1}{\sigma_{in}^{N-A}} \int d^2b [1 - \{\eta \exp[-(1-k)\sigma_{tot}^{pp} AT(b)] + \\ &\quad + (1-\eta) \exp[-\sigma_{tot}^{pp} AT(b)]\}] \end{aligned} \quad (10)$$

where

$$k = \beta_{\nu-1} \int_0^1 dy y S_{\nu}^{+}(y) + (1 - \beta_{\nu-1}) \int_0^1 dy y S_{\nu}^{-}(y) \quad (11)$$

and

$$\eta = \frac{n_1^p \langle x \rangle_1^p + n_1^n \langle x \rangle_1^n}{k} = \frac{\langle x \rangle_N}{k} \quad (12)$$

This parameter η defines the relationship between the nucleon elasticity in the first interaction and the one for the successive interactions of protons and neutrons with nucleus. The Eq. (10) gives a relation between the inelasticity in a nucleon-nucleus collision with inelasticities in nucleon-nucleon scattering. In a recent paper [17] we have used this model to analyse cosmic ray data on the hadronic flux and we have shown that the preserved momentum fraction of the successive interactions is the same as the one for first interactions, that is $k \approx \langle x \rangle_N$, $\eta \approx 1$. We then simply have

$$\begin{aligned} (1 - \langle x \rangle_{N-A}) &= \langle K \rangle_{N-A} \\ &= \frac{1}{\sigma_{in}^{N-A}} \int d^2b [1 - \exp[-(1-k)\sigma_{tot}^{pp}AT(b)]] \end{aligned} \quad (13)$$

In this situation $1 - k = \langle K \rangle_N$ and Eq. (13) gives a relationship between average inelasticities [15].

It is clear from this relationship that only in small σ_{tot}^{pp} limit is $\langle K \rangle_{N-A} \simeq \langle K \rangle_N$. In general, $\langle K \rangle_{N-A} \geq \langle K \rangle_N$, the effect increasing with the increase of σ_{tot}^{pp} . If $\langle K \rangle_N \rightarrow 0$, one also has $\langle K \rangle_{N-A} \rightarrow 0$. On the other hand, if $\langle K \rangle_N = 1$, then $\langle K \rangle_{N-A} = 1$, and Eq. (13) coincides with Eq. (9).

We use here the Woods-Saxon model [18] for the nuclear distribution which is given by

$$\rho(r) = \rho_o \left[1 + \exp\left[\frac{r - r_o}{a_o}\right] \right]^{-1} \left(1 + \omega \frac{r^2}{r_o^2} \right) \quad (14)$$

where the factor $(1 + \omega \frac{r^2}{r_o^2})$ corresponds to the Fermi parabolic distribution correction. The parameter ρ_o is a normalization factor,

$$\int d^3r \rho(r) = 1 \quad (15)$$

The parameters r_o , a_o and ω can be derived from experimental data and we have $r_o = 0.976A^{1/3}$ fm, $a_o = 0.546$ fm and the parameter ω is given by

$$\omega = -0.25839 \quad \text{if} \quad A \leq 40$$

$$\omega = 0 \quad \text{if} \quad A > 40$$

In order to calculate $\langle K \rangle_{N-A}$ we use for $\langle K \rangle_N$ the values calculated by means of the Regge-Mueller formalism [12] and as input for σ_{tot}^{pp} we have used the UA4/2 parametrization for the energy dependence [19]. In the Fig. (1) we

show the results of this calculations for the following nuclei: C, Al, Cu, Ag, Pb and air ($A=14.5$). In this figure we also show recent emulsion chamber data for p -Pb, $\langle K \rangle = 0.84 \pm 0.16$ [20] and for p -C, $\langle K \rangle = 0.65 \pm 0.08$ [21].

In the Fig. (2), we compare the calculated $\langle K \rangle_{p-air}$ with results from some models used in Monte Carlo simulation [22]; the Kopeliovich *et al.* [23] (KNP) QCD multiple Pomeron exchanges model; the Dual Parton model with sea-quark interaction of Capella *et al.* [24]; the statistical model of Fowler *et al.* [25] and with calculated values derived from cosmic ray data by Bellandi *et al.* [26]. We note that the calculated $\langle K \rangle_{p-air}$ in [26] was done assuming for the $T(b)$ nuclear thickness the Durand and Pi model [27], which gives small values for the average inelasticity. In the Fig. (2) we also show the average inelasticity values as calculated by means of this model.

3 Conclusions

We have here calculated the average proton-nucleus inelasticity in the Glauber framework, relating the leading particle distributions in nucleon-nucleus interactions with the respective one in nucleon-proton collisions. We have compared our results with recent emulsion chamber data for $p - Pb$ [20] and for $p - C$ [21] at $p_{lab} = 1.20 \times 10^7 \text{ GeV}/c$. At least in the experimental errors limit our calculation is in agreement with these experimental data. We have also calculated the average $p - air$ inelasticity in a wide range of energy. In order to describe the nuclear thickness we have used two models: the Woods-Saxon model [18] and the Durand and Pi model [27]. The average proton-air inelasticities calculated by means of the Woods-Saxon model are larger than the ones calculated by using of the Durand and Pi model.

One remark should be stressed. The calculated $\langle K \rangle_{p-air}$ values derived from cosmic ray data [26] were obtained assuming an approximation for the leading particle distribution in proton-air collisions. Therefore, it is model dependent. The discrepancies between the values of the $\langle K \rangle_{p-air}$ at low \sqrt{s} are consequence of the fact that two different sets of experimental data were used: nucleonic flux and hadronic flux at sea level (for discussions see [26]). Finally, we note that the calculated $\langle K \rangle_{p-air}$ with the Woods-Saxon model shows a behavior with \sqrt{s} wich goes between the values calculated by means of the QCD multiple Pomeron exchanges model (KNP) and that one calculated by means of the Dual Parton model.

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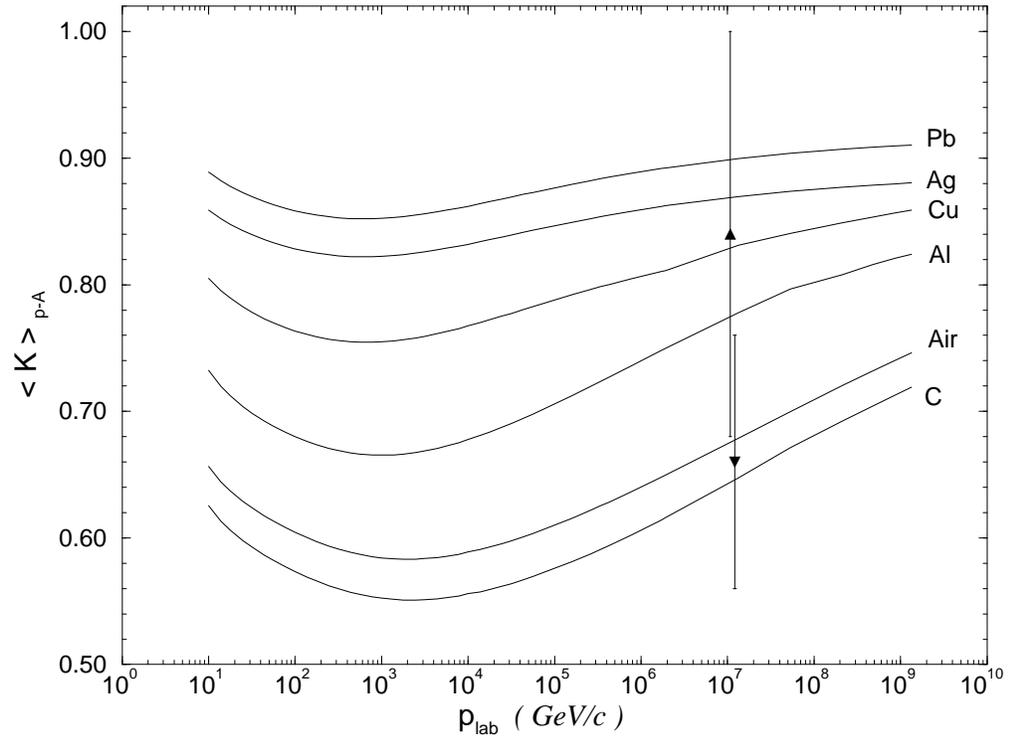


Figure 1: Proton-nucleus inelasticities calculated by means of Eq. (13). The Pb data (up triangle) from [20] and C data (down triangle) from [21].

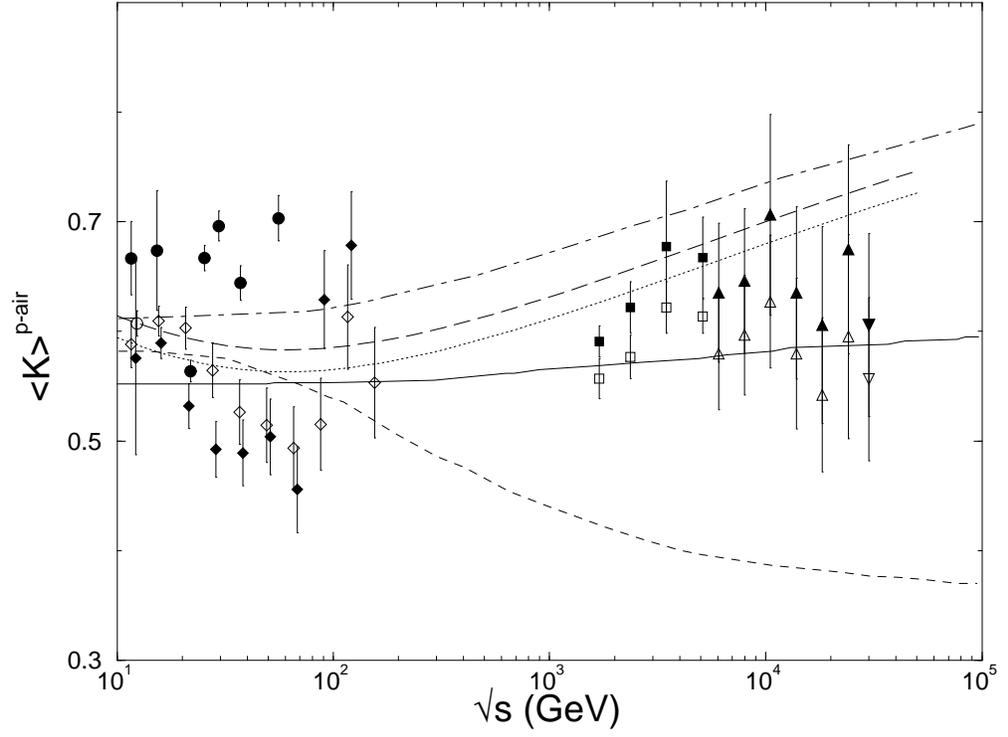


Figure 2: The $\langle K \rangle^{p-air}$ as a function of \sqrt{s} in GeV . The experimental data from [26]. Dashed line from [25]. Full line from [24]. Dot-dashed line from [23]. Dotted line from Eq.(13) with Woods -Saxon model [18]. Long dashed line from Eq. (13) with Durand-Pi model [27].