Decentralized Adaptive Coordinated Control of Multiple Robot Arms

without Using a Force Sensor

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Abstract

This paper presents a distributed adaptive coordinated control method for multiple robot arms grasping a common object. The cases of rigid contact and rolling contact are analyzed. In the proposed controller, the dynamic parameters of both object and robot arms are estimated adaptively. The desired motions of the robot arms are generated by an estimated object reference model. The control method requires only the measurements of the positions and velocities of the object and robot arms, but not the measurements of forces and moments at contact points. The asymptotic convergence of trajectory is proven by the Lyapunov-Like Lemma. Experiments involving two robot arms handling a common object are shown.

Keywords: coordinated robots, robotic manipulator, adaptive control, force/position control, Lyapunov stability.

1. INTRODUCTION

A robotics system consisting of multiple robots has greater capacity than a single robot system for tasks such as the handling of heavy materials and assembly. Many researchers have studied the coordinated control of multiple robot arms. When multiple robots grasp a single object, the robotics system forms a closed-chain mechanism that is extremely nonlinear and coupled. The control of such a system is highly complicated because a set of holonomic equality constraints is imposed, and the number of actuators exceeds the mobility of the system. The control objectives are specific motion of the grasped object and specific internal force exerted on the object by the robot arms, the latter of which does not affect the motion of the object.

Most of the coordinated controls are based on impedance control (Schneider et al., 1992; Kosuge et al., 1995) and hybrid position/force control (Hayati et al., 1988; Yoshikawa et al., 1993), for which the dynamic parameters of the object and the robot arms are known. It is well known that it is very difficult and time-consuming to precisely identify the dynamic parameters of the object and the robot arms. Moreover, the dynamic parameters of the object often vary according to the task, which is variable. To control multiple robot arms whose dynamic parameters are unknown, control methods using neural networks (Hwang et al., 1994), a learning control (Nakayama et al., 1995), and sliding mode controls (Yao et al., 1992; Su et al., 1995) have been proposed. Additionally, Pagilla et al. (1995) and Liu et al. (1995) have proposed adaptive controls. These methods require measurements of forces and moments at contact points and adopt a centralized controller. The force sensor is relatively expensive and is easily broken. The centralized controller suffers from its complicated architecture because the state space of the dynamics of multiple robots has a high dimension. Methods of decentralized control (Hsu, 1989; Liu et al., 1996), in which each robot is controlled separately by its own controller, are thus desirable approaches because of their simple architecture. However, these decentralized controllers are not adaptive and require a force sensor to measure the interactions between the robot and the target object.

In this paper, a decentralized adaptive coordinated control method without a force sensor for multiple robot arms grasping a common object is proposed. The cases of rigid contact and rolling contact are analyzed. In this method, the dynamic parameters of both the object and the robot arms are estimated and the desired motions of the robot arms are generated by use of an estimated object reference model. The motion of each robot arm is controlled independently and adaptively. The asymptotic convergence of trajectory and the
boundedness of force error applied to the object are proven by the Lyapunov-Like Lemma. If the persistently exciting condition of motion is satisfied, the force error applied to the object converges to zero. Experiments involving two robot arms handling a common object are shown.

2. MODELING OF SYSTEM DYNAMICS

Consider the dynamic equations of \( k \geq 1 \) arms holding a common rigid object as shown in Fig. 1, in which all robot end effectors hold the same object moving in three-dimensional space. The coordinate systems are defined as follows: \( \Sigma_p \) is the task coordinate system, \( \Sigma_o \) is the object coordinate system fixed on the object, and \( \Sigma_i \) is the \( i \)-th arm coordinate system fixed on the \( i \)-th end effector.

We also use notations defined as follows: \( p_i \in \mathbb{R}^3 \) is the position vector of the origin of \( \Sigma_i \), \( \omega_i, \eta_i \in \mathbb{R}^3 \) is the orientation vector of \( \Sigma_i \) with respect to \( \Sigma_o \), \( \omega_o, \eta_o \in \mathbb{R}^3 \) is the angular velocity of \( \Sigma_i \) with respect to \( \Sigma_o \), \( p_{ic} \in \mathbb{R}^3 \) is the position vector of the origin of \( \Sigma_i \) with respect to \( \Sigma_o \), \( p_{ac} \in \mathbb{R}^3 \) is the position vector from the origin of \( \Sigma_i \) to the contact point \( C_i \) with respect to \( \Sigma_o \), and \( p_{ao}, p_{ao} \in \mathbb{R}^3 \) is the position vector from the origin of \( \Sigma_i \) to the contact point \( C_i \) with respect to \( \Sigma_o \). To facilitate dynamic formulation, the following assumptions are made:

A1): The \( i \)-th robot arm has \( \alpha_i \geq 6 \) joints.
A2): All end effectors of robot arms are rigidly attached to the common object so that no relative motion occurs between the object and any end effector, and it is possible to generate arbitrary force and moment at any contact.
A3): \( \mathbf{r}_e = (\mathbf{p}^T_0, \eta_0^T)^T, \mathbf{r}_e = (\mathbf{p}^T_i, \eta_i^T)^T, \mathbf{r}_{ac}, \mathbf{r}_{ao} \) are measurable.
A4): The desired position \( \mathbf{r}_e^d \), desired velocity \( \dot{\mathbf{r}}_e^d \), and desired acceleration \( \ddot{\mathbf{r}}_e^d \) of the object are time-continuous and bounded.

2.1 Force and moment

Each robot arm applies a force \( f_i \in \mathbb{R}^3 \) and a moment \( \mathbf{n}_i \in \mathbb{R}^3 \) through the contact point \( C_i \) to the object. The resultant force and moment, \( \mathbf{f}_o, \mathbf{n}_o \in \mathbb{R}^3 \), applied to the object by multiple robot arms are then given by \( \mathbf{f}_o = \sum f_i \) and \( \mathbf{n}_o = \sum (\mathbf{n}_i + \mathbf{p}_{ac} \times f_i) \), respectively. These equations can be rewritten as

\[
\mathbf{f}_o = \sum_{i=1}^{k} W_i f_i, \quad (1)
\]

where \( W_i = (f_i^T, n_i^T)^T \) is the external force affecting the object, \( F_i = (f_i^T, n_i^T)^T \) is the force and moment applied to the object by the \( i \)-th robot arm, and \( W_i \) is a grasp form matrix at the contact point \( C_i \) given by

\[
W_i = \begin{pmatrix} I & 0 \\ \hat{\mathbf{p}}_{oi} & I \end{pmatrix} \in \mathbb{R}^{6 \times 6} 
\]

where \( I \) is an identity matrix and \( \hat{\mathbf{p}} \) is defined such that the relation \( \hat{\mathbf{p}} a = \mathbf{p} \times a \) is satisfied for any vectors \( a, p \in \mathbb{R}^3 \).

Equation (1) can be represented in a more compact form as follows:

\[
\mathbf{F} = \mathbf{W} \mathbf{F}_v 
\]

where \( \mathbf{W} = \begin{pmatrix} W_1 & \cdots & W_k \end{pmatrix} \) and \( \mathbf{F}_v = (F_1^T, \ldots, F_k^T)^T \).

Similarly, a force \( \mathbf{f}_v \in \mathbb{R}^3 \) and a moment \( \mathbf{n}_v \in \mathbb{R}^3 \) at the \( i \)-th robot end effector is given by

\[
\mathbf{F}_i = \mathbf{G}_i \mathbf{F}_v 
\]

where \( \mathbf{F}_i = (f_i^T, n_i^T)^T \) is the force and moment at the \( i \)-th robot end effector, and \( \mathbf{G}_i \) is given by

\[
\mathbf{G}_i = \begin{pmatrix} I & 0 \\ \hat{\mathbf{p}}_{oi} & I \end{pmatrix} \in \mathbb{R}^{6 \times 6}. 
\]

Equation (4) can be represented in a more compact form as follows:

\[
\mathbf{F} = \mathbf{G} \mathbf{F}_v 
\]

where \( \mathbf{G} = \text{block diag}(\mathbf{G}_1, \ldots, \mathbf{G}_k) \) and \( \mathbf{F} = (F_1^T, \ldots, F_k^T)^T \).

2.2 Object dynamics

In general, the dynamic equation of the object is represented by:

\[
\mathbf{M}_o(\mathbf{r}_o) \ddot{\mathbf{r}}_o + \mathbf{C}_o(\mathbf{r}_o, \dot{\mathbf{r}}_o) \dot{\mathbf{r}}_o + \mathbf{g}_o(\mathbf{r}_o) = \mathbf{F}_o 
\]

where \( \mathbf{v}_o = (\mathbf{p}_o^T, \omega_o^T)^T \) is the velocity vector of the object with respect to the task coordinate system, \( \mathbf{M}_o \) is the symmetric positive definite inertia matrix, \( \mathbf{C}_o \) is the damping coefficient matrix, and \( \mathbf{g}_o \) is the gravitational force term. When the orientation vector of the object is represented by the ZYZ-Euler angles as \( \mathbf{\eta}_o = (\phi_o, \theta_o, \phi_o)^T \), the relationship between \( \mathbf{v}_o \) and \( \mathbf{\dot{r}}_o \) is given by \( \mathbf{v}_o = \mathbf{T}_o(\mathbf{\eta}_o) \mathbf{\dot{r}}_o \), where \( \mathbf{T}_o(\mathbf{\eta}_o) \) is a transformation matrix between the two velocity vectors defined by

\[
\mathbf{T}_o(\mathbf{\eta}_o) = \begin{pmatrix} I & 0 \\ 0 & -\sin \phi_o & \cos \phi_o \sin \theta_o \\ 0 & \cos \phi_o & \sin \phi_o \sin \theta_o \\ 0 & 0 & \cos \theta_o \end{pmatrix} \in \mathbb{R}^{6 \times 6}. 
\]
It is well known that 1) a suitable definition of $C_o$ makes matrix $M_o - 2C_o$ skew-symmetric, and 2) the dynamic model of the object is linear in the dynamic parameter vector, as follows:

$$M_i(r_{\omega})\dot{v}_{\omega} + C_i(r_{\sigma}, \dot{r}_{\sigma})v_{\sigma} + g_i(r_{\sigma}) = Y_i(r_{\sigma}, \dot{r}_{\sigma}, v_{\sigma}, \dot{v}_{\sigma})\sigma_o$$  \hspace{1cm} (8)

where $v_{\sigma}$ is an arbitrarily defined velocity, $\sigma_o \in \mathbb{R}^p$ is a dynamic parameter vector of the object, and $Y_i(r_{\sigma}, \dot{r}_{\sigma}, v_{\sigma}, \dot{v}_{\sigma})$ is a regressor with respect to $\sigma_o$.

2.3. Robot arm dynamics

The dynamic equation of the $i$-th robot arm is represented by

$$M_i(r)\dot{v}_{\omega} + C_i(r, \dot{r})v_{\sigma} + g_i(r) = Y_i(r, \dot{r}, v_{\sigma}, \dot{v}_{\sigma})\sigma_o$$  \hspace{1cm} (9)

where $v_{\sigma}$ is a position vector of the $i$-th arm with respect to $\Sigma_o$, $r_{\sigma} = (r_{i1}' \cdots r_{in}')'$ is the position and orientation vector of the $i$-th arm with respect to $\Sigma_o$, $M_i$ is the symmetric positive definite inertia matrix, $C_i$ is the damping coefficient matrix, $g_i$ is the gravitational force term, $u_i$ is the control input given in $\Sigma_o$, and $v_{\sigma} = T_o(\eta_i)'$. It is well known that the robot dynamic model is linear in its dynamic parameters, as follows:

$$M_i(r)\dot{v}_{\omega} + C_i(r, \dot{r})v_{\sigma} + g_i(r) = Y_i(r, \dot{r}, v_{\sigma}, \dot{v}_{\sigma})\sigma_{o,i}$$  \hspace{1cm} (10)

where $v_{\sigma} \in \mathbb{R}^p$ is an arbitrarily defined velocity, $\sigma_{o,i} \in \mathbb{R}^n$ is a dynamic parameter vector of the $i$-th robot arm, and $Y_i(r, \dot{r}, v_{\sigma}, \dot{v}_{\sigma})$ is a regressor. The robot dynamics can be rewritten more concisely as follows:

$$M(r)\dot{v} + C(r, \dot{r})v + g(r) = u - GF_o$$  \hspace{1cm} (11)

where $\dot{r} = (r_{11}' \cdots r_{in}')'$, $v = (v_{i1}' \cdots v_{in}')'$, $u = (u_{i1} \cdots u_{in})$, $M = \text{block diag}(M_{1} \cdots M_{n})$, $C = \text{block diag}(C_{1} \cdots C_{n})$, and $g = (g_{i1}' \cdots g_{in}')'$. The robot model given in Eq. (11) is characterized by the following structural properties: 1) $M$ is a symmetric positive definite; 2) suitable definitions of $C_i$ for all $i$ make matrix $M - 2C$ skew-symmetric; and 3) the robot model is linear in its dynamic parameters as

$$M(r)\dot{v} + C(r, \dot{r})v + g(r) = Y(r, \dot{r})v_{\sigma} + \dot{\sigma}_{o,i}$$  \hspace{1cm} (12)

where $v_{\sigma} = (v_{i1}' \cdots v_{in}')'$, $Y = \text{block diag}(Y_1 \cdots Y_n)$, and $\sigma_{o,i} = (\sigma_{o,1} \cdots \sigma_{o,n})'$. These properties are utilized in our controller design.

2.4. Integrated dynamics

By eliminating $F_o$ and $F_i$ from the object dynamics and the robot dynamics by using Eq. (3), the integrated dynamics are obtained as follows:

$$M_i(r_{\omega})\dot{v}_{\omega} + C_i(r_{\sigma}, \dot{r}_{\sigma})v_{\sigma} + g_i(r_{\sigma}) + W G^{++}[M(r)\dot{v} + C(r, \dot{r})v + g(r)] = W G^{++}u$$  \hspace{1cm} (13)

These integrated dynamics do not include the force and moment at the contact points. This property leads to an adaptive coordinated control that does not require measurement of force and moment at the contact points.

3. ADAPTIVE COORDINATED CONTROL

The control objective is to provide a set of input joint torques such that the motion of the object converges to the desired trajectory asymptotically for case in which the dynamic parameters of the object and the robot arms are unknown. The proposed adaptive coordinated control does not require measurement of the force and moment at the contact point; it requires only measurements of the position and velocity of the object and all robot arms. Moreover, in the present system, it is not necessary to calculate the desired position of each arm.

3.1. Desired external force applied to the object

Let us define a reference velocity of the object by

$$v_{\omega} = T_o(\dot{r}_{\omega})$$  \hspace{1cm} (14)

where $\dot{r}_{\omega} = r_{\omega} - r_o$ is a position error vector and $\rho > 0$ is a scalar constant. The following desired external force is then generated by an estimated reference model of the object:

$$F_o^d = M_o(r_o)\dot{v}_{\omega} + \hat{C}_o(r_o, \dot{r}_o)\dot{v}_{\omega} + \hat{g}_o(r_o)$$  \hspace{1cm} (15)

where $\hat{\sigma}_o$ is a parameter estimate of $\sigma_o$, and $\hat{M}_o$, $\hat{C}_o$, and $\hat{g}_o$ are estimates of $M_o$, $C_o$, and $g_o$, respectively, which are computed using $\hat{\sigma}_o$. An adaptive law of the dynamic parameter of the object is given by

$$\dot{\hat{\sigma}}_o = - \Gamma_o Y_o^T(r_o, \dot{r}_o, v_{\omega}, \dot{v}_{\omega})s_o$$  \hspace{1cm} (16)

where $\Gamma_o > 0$ is an adaptive gain matrix and $s_o$ is a residual error given by

$$s_o = v_{\omega} - v_o = T_o(-\dot{r}_o - \rho e_o).$$  \hspace{1cm} (17)

The desired external force applied to the object is updated based on the parameter estimates of the dynamic parameters of the object.

3.2. Desired force and moment at contact points

The existence of a set of forces and moments equilibrating the external force at contact points is assumed. The desired force and moment at a contact point equilibrating the desired external force should satisfy the relation given in Eq. (3). Moreover, the forces and moments at contact points generate the internal force in the object; hence, the general solution of the desired force and moment at a contact point is given by

$$F^d = (F^d_1 \cdots F^d_n)^T = W^* F_o^d + (I - WW^*)f^d_{int}$$  \hspace{1cm} (18)

where $I \in \mathbb{R}^{n \times n}$ is an identity matrix, $f^d_{int}$ is a bounded vector to generate desired internal force, and $W^*$ is a pseudo inverse of $W$ given by $W^* = W^T (WW^T)^{-1}$.
3.3. Control law of the robot arm

The duality of force and velocity yields a relationship between the velocities of the contact point of the object and the i-th robot arm as follows:

\[ G_i^T v_i = W_i^T v_o \]  

(19)

This relation leads to the desired velocity of the i-th robot arm as follows:

\[ v_i^d = G_i^T W_i^T v_i^d. \]  

(20)

where \( v_i^d = T_i \dot{\theta}_i \). Let us define a reference velocity of the i-th robot arm by

\[ v_o = v_i^d + \rho G_i^T W_i^T T_i e_o. \]  

(21)

The control law of the i-th robot arm in the task space is then given by

\[ u_i = Y_i^T (r_i, \dot{r}_i, v_o, \dot{v}_o) \dot{\sigma} + G_i F_i^d - K_i s_i, \]  

(22)

where \( \dot{\sigma} \) is an estimate of \( \sigma \), \( K > 0 \) is a feedback gain matrix, and \( s_i = (v_i - v_o) \) is a residual error between the reference velocity and the actual velocity, which is rewritten by

\[ s_i = G_i^T W_i^T s_o. \]  

(23)

An adaptive law of the parameter estimate of the i-th robot arm is given by

\[ \dot{\sigma} = \Gamma_i Y_i^T (r_i, \dot{r}_i, v_o, \dot{v}_o) s_i, \]  

(24)

where \( \Gamma > 0 \) is an adaptive gain matrix. The joint input torque of the i-th robot arm \( \tau_i \) is given by \( \tau_i = J_i^T u \), where \( J_i \) is a Jacobian for the i-th robot arm. On the right-hand side of Eq. (22), the first term is a feedforward input based on an estimated reference model, the second term is a feedforward input for the desired force and moment at the contact point, and the third term is a feedback input for the trajectory errors of position and velocity of the object. Note that the adaptive coordinated controller does not require measurement of force and moment at each contact point and that each robot arm is controlled independently without communication between robot arms.

The integrated control law and adaptive law are represented by

\[ u = Y(r, \dot{r}, v, \dot{v}) \dot{\sigma} + GF^d - K s \]  

(25)

and

\[ \dot{\sigma} = \Gamma Y(r, \dot{r}, v, \dot{v}) s \]  

(26)

where

\[ Y(r, \dot{r}, v, \dot{v}) = \text{block diag} (Y_1, \ldots, Y_k), \]

\[ K = \text{block diag} (K_1, \ldots, K_k), \]

\[ v_o = (v_o^1, \ldots, v_o^k) \]

\[ \dot{\sigma} = (\dot{\sigma}_1, \ldots, \dot{\sigma}_k) \]

\[ F^d = (F_{i,1}^d, \ldots, F_{i,k}^d) \]

and

\[ \Gamma = \text{block diag} (\Gamma_1, \ldots, \Gamma_k). \]

3.4. Asymptotic convergence of trajectory

For the proposed controller, the following theorem is proved.

**Theorem 1**: Consider a rigid object grasped by k robot arms, each robot having \( \alpha_i (\geq 6) \) joints. For a system (13) using the integrated control law (25) with the integrated adaptation law (26), in which the desired external force of the object is given by (15), the closed-loop system satisfies

1. \( r_i \rightarrow r_i^d \) as \( t \rightarrow \infty \), and
2. the force error at contact point \( F_e - F_e^d \) is bounded.

**Proof**: It is easily shown that

\[ Y(r, \dot{r}, v, \dot{v}) \Delta \sigma = Y(r, \dot{r}, v, \dot{v}) \sigma = -M s - C s. \]  

(27)

Let an estimate error vector of the object dynamic parameters be defined as \( \Delta \sigma = \hat{\sigma} - \sigma \); the following is then obtained:

\[ Y_o(r_o, \dot{r}_o, v_o, \dot{v}_o) \hat{\sigma}_o = M \hat{s}_o + C \hat{s}_o - Y_o(r_o, \dot{r}_o, v_o, \dot{v}_o) \sigma_o \]  

(28)

Consider a candidate of the Lyapunov function as follows:

\[ V = \frac{1}{2} (s_o^T M s_o + \Delta \sigma_o^T \Gamma \Delta \sigma_o + s^T M s + \Delta \sigma^T \Gamma^{-1} \Delta \sigma) \]  

(29)

where \( \Delta \sigma = \hat{\sigma} - \sigma \) is an estimate error vector of the robot dynamic parameters. By using the fact that \( M - 2C \) and \( M - 2C \) are skew-symmetric, applying \( s = G^{-T} W^T s_o \), and using Eqs. (7), (8), (11), (13), (16), (25), (26), (27) and (28), the time derivative along the solution of the error equation becomes

\[ \dot{V} = s_o^T (F_o - F_o^d + Y_o(r_o, \dot{r}_o, v_o, \dot{v}_o) \Delta \sigma_o) \]  

\[ + (-\Gamma Y_o(r_o, \dot{r}_o, v_o, \dot{v}_o) s_o)^T \Gamma^{-1} \Delta \sigma_o \]  

\[ + s^T (G F^d - K s - GF^d + Y(r, \dot{r}, v, \dot{v}) \sigma) \]  

\[ + (-\Gamma Y(r, \dot{r}, v, \dot{v}) s)^T \Gamma^{-1} \Delta \sigma \]  

\[ = -s^T K s \leq 0. \]  

(30)

This shows that \( V \) is the Lyapunov function; hence, \( s, s_o, \Delta \sigma \) and \( \Delta \sigma_o \) are bounded. Because \( \sigma \) and \( \sigma_o \) are constant, \( \dot{\sigma} \) and \( \dot{\sigma}_o \) are bounded. From the boundedness of \( T_o \), using Eq. (17) leads to the boundedness of \( e_o \) and \( \dot{e}_o \). These results then yield the boundedness of \( e_o, \dot{r}_o, v_o, \dot{v}_o, T_o \), and \( \dot{e}_o \) because of the boundedness of \( \dot{e}_o^d \) and \( \dot{e}_o^d \). Hence, \( Y_o(r_o, \dot{r}_o, v_o, \dot{v}_o) \) and \( F_o^d \) are bounded. From the boundedness of the contact point on the object, \( W, G, r \) and \( F^d \) are bounded. Hence, \( F_o^d \) is bounded. From Eqs. (19)-(21), we obtain \( v = G^{-T} W^T v_o \), \( v^d = G^{-T} W^T v_o^d \), \( v_o = G^{-T} W^T s_o \) and \( s = G^{-T} W^T s_o \), which show that \( v, v^d, v_o, v_o^d, \dot{r} \) and \( s \) are bounded. The boundedness of \( r_o, \dot{r}_o, r \) and \( \dot{r} \) give the boundedness of \( W \) and \( G \); hence, \( \dot{v}_o = G^{-T} (W^T \dot{v}_o + W^T v_o) + G^{-T} W^T v_o \) is bounded. The boundedness of these yields the fact that \( Y(r, \dot{r}, v, \dot{v}) \) and \( u \) are bounded. By substituting (25) and (27) into (13), the following equation is obtained:

\[ M \dot{v}_o + C v_o + g_o = W G \dot{Y}(r, \dot{r}, v, \dot{v}) \Delta \sigma - M \dot{s} - C s + K s + GF^d \]  

(31)

The use of Eqs. (15), (29) and (31) yields

\[ M \dot{s} + C s - Y(r, \dot{r}, v, \dot{v}) \Delta \sigma = -W G \dot{Y}(r, \dot{r}, v, \dot{v}) \]  

(32)
Substituting $\dot{s} = G^TW^T \dot{s}_0 + W^T \dot{s}_0 + 2 \dot{G}^TW^T s_0$ into Eq. (32) yields

$$
\begin{align*}
(M_i + W G^T G^{-1} W^T) \dot{s}_i &= (C_i - (I + W G^T G^{-1} W^T)) \dot{s}_0 \\
+ W G^T Y (r_i, \dot{r}_i, v_i, \dot{v}_i) \Delta \sigma - \gamma (\dot{e}_o - \dot{e}_s) \\
+ \gamma^T (K + C G^T G^{-1}) s_o.
\end{align*}
$$

(33)

It is evident that $(M_i + W G^T G^{-1} W^T)$ is a symmetric positive definite because $M_i > 0$, $G > 0$, and $W G^T G^{-1} W^T \succeq 0$; hence, $s_o$ is bounded, and therefore $s$ is bounded. Differentiating Eq. (30) with respect to time gives $\dot{V} = -2\kappa \dot{s} \dot{s}^T$.

$V$ is bounded because of the boundedness of $s$ and $\dot{s}$, which indicates that $V$ is uniformly continuous. From the Lyapunov-Like Lemma (Slotine et al., 1991), it is shown that $\dot{V} \rightarrow 0$ as $t \rightarrow \infty$. This implies that $s \rightarrow 0$, $s_o \rightarrow 0$, $\dot{e}_o \rightarrow 0$ and $\dot{e}_s \rightarrow 0$ as $t \rightarrow \infty$. Therefore, $r_o \rightarrow r^*_o$ and $\dot{r}_o \rightarrow \dot{r}^*_o$ as $t \rightarrow \infty$. Substituting $\Delta \sigma = \sigma - \sigma^*$, (11), (12) and (25) into (27) yields

$$F_{cc} - F_i = G^{-1}(M_s + C_s - \gamma Y (r_i, \dot{r}_i, v_i, \dot{v}_i)) \Delta \sigma + K s.$$  

(34)

Therefore, the force error at the contact point $F_{cc} - F_i$ is bounded. This result yields the boundedness of the internal force.

Remarks: $e_o$ and $\dot{e}_o$ asymptotically converge to zero, but $r_o$ and $\dot{r}_o$ are bounded. If the persistently exciting condition of motion is satisfied, the estimates of the dynamic parameters of the object and the robot arm converge at true values. In this case, Eqs. (33) and (34) show that $s \rightarrow 0$, $\dot{r}_o \rightarrow \dot{r}^*_o$ and $F_i \rightarrow F^*_i$ as $t \rightarrow \infty$. Finally, it is shown that internal force to $(I - W^TW) \mathbf{f}_m$ as $t \rightarrow \infty$.

4. ROLLING CONTACT

In the case in which the end effector of the robot arm does not grasp the object rigidly, but comes into contact with the object without slip, the proposed adaptive control can be modified to apply. First, assumptions A1), A2), and A3) in Section 2 are modified as follows:

A1): The $i$-th robot arm has $6$ (≥ 6) joints.

A2): All end effectors of robot arms contact the object at a single point, and rolling contact without slip is realized.

A3): $r_o$, $\dot{r}_o$, $r_i$, $\dot{r}_i$, $p_o$, $\dot{p}_o$, $p_w$, and $\dot{p}_w$ are measurable. Then, the resultant force and moment applied to the object by multiple robot arms are $F_s = \sum F_s$ and $n_s = \sum p_s \times \dot{f}_s$.

Hence, $F_o$ is given by $F_o = F_s$; and the grasp form matrix at the contact point $C_i$ is modified to $W_i = (I - \hat{p}_w)\gamma \in R^{6x3}$. The external force affecting the object is thus described by Eq. (1). Furthermore, $G_i$ is modified to $G_i = (I - \hat{p}_w)\gamma \in R^{6x3}$, and the force at the end effector is then described by Eq. (4).

The velocities of the contact positions on the object and the $i$-th arm are given by $W_i^T v_o + p_o$ and $G_i^T v_i + p_i$, respectively, and these are equal. When the $i$-th arm manipulates the object under rolling contact conditions without slip, on the rolling velocities of the contact positions on both surfaces with respect to $\Sigma_p$, the following relation is obtained (Cole et al., 1989):

$$\hat{p}_w = \dot{p}_w.$$  

(35)

This relation gives $G_i^T v_i = W_i^T v_o$, which is the same form as that given in Eq. (19). The desired velocity of the $i$-th arm should satisfy $G_i^T v_i = W_i^T v_o$. Hence, it is given by

$$v_i = (G_i^T G_i)^{-1} (G_i^T v_o - (I - (G_i^T G_i)^{-1})G_i) v_i.$$  

(36)

where $v_i$ is the arbitrary velocity, which is twice differentiable in time, and $G_i^T$ is a pseudo inverse of $G_i$, which is given by $(G_i^T G_i)^{-1}G_i^T$. By using the above equations, Theorem 1 is proved through the same approach. Noted that the measurement of the contact points on the arm and the object are required, but measurement of the contact force is not required.

5. EXPERIMENT

Experiments using two robot arms were performed to show the effectiveness of the proposed control method in cases in which the dynamic parameters of the object and the robot arms are unknown. In the experiment, as shown in Fig. 2, two robot arms, the 1st of which (left side) has 6 degrees of freedom (DOF), and the 2nd of which (right side) has 5 DOF, are used as 3-DOF robot arms that grasp a single object rigidly and carry it in a vertical plane. The link lengths from the 1st to the 3rd link of the 1st robot arm are 103.9, 90 and 71 mm, respectively, and those of the 2nd robot arm are 80, 134.4 and 46.3 mm. The positions of the arms and the object are measured using rotary encoders that are fixed to DC servo motors, and the velocities of the arms and object are obtained using a differential filter. Each arm is equipped with 6-axis force sensors (NANO sensors by BL Autotec, LTD.) at the end link to monitor the contact force, which is not used in the control. The object is a rectangular solid made of aluminum, 30 x 30 x 20 mm in size, and with a mass of 50 g. The position vectors from the origin of $\Sigma_0$ to the origin of $\Sigma_p$ with respect to $\Sigma_p$ are $\hat{p}_o = (-22,0,0)^T$ and $\hat{p}_w = (22,0,0)^T$. The desired trajectory of the object is a circle 40 mm in diameter, and whose motion time is 3 sec. The magnitude of the vector to generate desired internal force is set at $f_m = 0$ N and the controller gains were selected to be: $K_1 = K_2 = \text{diag}(500,500,5)$, $\rho = 100 \,$, and $\Gamma_o = \text{diag}(0.01,0.001)$, and all the diagonal elements of $\Gamma_i$.
and \( \Gamma_2 \) are 0.001. The initial values of the unknown dynamic parameters are set at zero. Coulomb friction and the damping friction coefficient at each joint are also estimated adaptively, and feedforward inputs using their estimates are added. The sampling cycle was 0.5 ms.

The experimental results of the trajectory of the object with respect to the task coordinate system are shown in Fig. 3, and the position error vector of the object, \( \mathbf{e}_p = (\Delta x, \Delta y, \Delta \theta)^T \), is shown in Fig. 4. These figures show that position and orientation errors decrease with repetition of motion. The control input of arm 1, \( \mathbf{u}_1 = (u_{1x}, u_{1y}, u_{1\theta})^T \), is shown in Fig. 5. These figures show that the feedforward term increases and the feedback term decreases with repetition of motion. The desired external force, \( \mathbf{F}^d = (F^d_x, F^d_y, N^d_z)^T \), and the external force, \( \mathbf{F} = (F_x, F_y, N_z)^T \), acting on the object in this experiment are shown in Fig. 6. The external force also decreases with repetition of motion. Figure 7 shows examples of parameter estimates; (1) is the estimated mass of the object, and (2) is the estimated inertia moment of link 1 of the 1st arm. These estimates converge to nearly constant values which are
not true values. Moreover, the external force is larger than the desired external force. There are several reasons why the parameter estimates do not converge to true values and the position errors do not converge to zero: the persistently exciting condition is not satisfied because of redundancy of the dynamic parameters; the dynamics of the mechanism such as the flexibility of the joint are not modeled; and the controller is not a continuous-time control system but a discrete-time control system whose accuracy of trajectory depends on the sampling cycle (Kawasaki, et al., 2003). Figure 8 shows the internal force acting on the object, which decreases with repetition of motion and is bounded. These results show that the adaptation in control is working successfully and effectively.

6. CONCLUSION

An adaptive coordinated control method for multiple robot arms grasping a common object has been proposed. Using this method, the desired motion of the robot arms is generated by an estimated reference model of the object, which is evaluated using parameter estimates and the desired motion of the object. The motion of each robot arm is controlled independently and adaptively without communication among the arms. The proposed control method does not require measurement of forces and moments at contact points. Asymptotic convergence of trajectory has been proven by the Lyapunov-Like Lemma. The internal force exerted on the object and the contact forces and moments at the contact points converge to the desired values if the persistently exciting condition is satisfied. Adaptive control can be applied in the case of rolling contact without slip between the arm end effectors and the object when contact points are measured. The results of experiments using two robot arms demonstrated that the control objective was successfully achieved.

References

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