ROBUST CONSTRAINED MOTION CONTROL OF MULTI-ARM ROBOTS HOLDING A COMMON OBJECT

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ABSTRACT

In this paper, a variable structure control (VSC) method is developed for robust constrained motion control of multi-arm robots holding a common object in the presence of parametric uncertainties and external disturbances. By introducing a set of generalized coordinates, the closed-chain dynamic equation of robotic system can be decomposed into position and force controlled subspace. In the controller design, position and force control are treated together and designed via VSC. For coordinating each robot, load distribution is carried out for minimizing weighted energy consumption. The proposed VSC controller can guarantee the robotic system with prescribed qualities both in the sliding mode and in the reaching transient response, and can be implemented in different space. Simulation results illustrate the proposed method.

1. Introduction

Control of multi-arm robots receives increasing attention in recent years. Large objects are usually beyond the load carrying capacity of a single arm and have centers of gravity that prohibit a single arm from maintaining a desired object orientation while moving. Moreover, multiple coordinated arms can increase positioning accuracy of the object, decrease the time required to complete an assembly task, etc.

When robots grasp a common object, they form a closed kinematic chain mechanism. This will impose a set of homogeneous constraints on the positions of the robots and cause interaction among the robots. To control the robots cooperatively, several control strategies have been proposed [1-15]. With the master-slave control methods [1-4], the master is assigned to carry the major part of the task and is controlled to follow the desired trajectory. The motion of the slave is adjusted accordingly to satisfy the imposed constraints resulted from the closed kinematic chain. In the closed kinematic chain methods [5-8], only the position of the whole system is controlled and the internal force control is not considered. As a result, the joint torque for a particular load of the object cannot be uniquely determined and load distribution is required. The hybrid position/force control methods [9-13] consider both the position and the internal force control of the whole system.

When the grasped object comes in contact with rigid surfaces, kinematic constraints are imposed on the motion of the object and contact forces are generated. It is necessary to control both the motion on the constraint surfaces and the generalized constrained force. The problem has been extensively studied in recent years for a single arm [16-20] which has been referred to as nonlinear singular system [17] or descriptor system [18], and has also been addressed for multi-arm robots in [14,15,21]. But less papers considered robust constrained motion control of robot manipulators. For dealing with parametric uncertainties, adaptive methods have been proposed in [19-21].

In this paper, we address the problem of controlling a cooperative task for multiple arms which is required to move an object constrained by the environment in the presence of both parametric uncertainties and external disturbances. By using a set of generalized coordinates [16], the closed kinematic chain dynamic equation of the robotic system can be decomposed into position and force controlled subspace. In the present study, position and force control are treated together, and designed via variable structure control (VSC) method. For coordinating each robot, load distribution is carried out for minimizing weighted energy consumption [5,10]. The proposed VSC controller can guarantee the robotic system with prescribed qualities both in the sliding mode and during the reaching transient, and can be implemented in different space. Simulation results of two three-DOF robots grasping a common object moving on a circular surface are given to illustrate the proposed method.

2. Dynamic Equation of Robotic System and Problem Formulation

In this Section, we establish the closed kinematic chain dynamic equation of m robot arms grasping a common object. The object is assumed to be rigid and the grasp is fixed with each robot being non-redundant such that the position of each robot is uniquely determined by the position of the robot, and vice versa. The dynamic equation of the ith robot arm with rigid link can be written as

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) + J_i^T(q_i)F_i + \tilde{f}_i(t) = \tau_i \tag{1}$$

where q_i is the nx1 joint displacement vector, τ_i is the applied joint torque, $M_i(q_i)$ is the inertia matrix, $C_i(q_i,\dot{q}_i)\dot{q}_i$ is the coriolis and centrifugal vector, $G_i(q_i)$ is the gravitational vector, F_i is the vector of forces/moments on the object exerted by the robot at the end-effector, $\tilde{f}_i(t)$ is the external disturbance, $J_i(q_i)$ is the manipulator Jacobin matrix which is assumed to be nonsigular in the work space Ω_i given by

$$J_i(q_i) = \frac{\partial x_i(q_i)}{\partial q_i} \tag{2}$$

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where x_i is the position and orientation vector of the end-effector frame in its base reference frame $O_iX_iY_iZ_i$.

The dynamic equation of the grasped object can be written as

$$H_0(p)\ddot{p} + C_0(p,\dot{p})\dot{p} + G_0(p) + \tilde{f}_0(t) + F_c = \sum_{i=1}^{m} L_i^{-T} F_i \tag{3}$$

where p is the position and orientation vector of the object frame in the world space OXYZ, H_0, C_0, G_0, $\tilde{f}_0(t)$ have the same meaning as in (1), F_c is the contact force vector on the environment exerted by the object, L_i is the nonsingular transformation Jacobin matrix between the object position and the ith robot end-effector position given by

$$L_i(p) = \frac{\partial p(x_i)}{\partial x_i} \tag{4}$$

Equation (1) and (3) have the following properties which we will use

Property 1 [20]. For any finite workspace Ω_i, $M_i(q_i)$ or $H_0(p)$ is a symmetric positive definite matrix. Moreover, there exist $k'_i>0$ and $k''_i>0$ such that

$$k_i' I \le M_i(q_i) \le k_i'' I \qquad \forall q_i \in \Omega_i \tag{5}$$

Property 2 [20]. The matrix $N_i(q_i,\dot{q}_i)=\dot{M}_i(q_i)-2C_i(q_i,\dot{q}_i)$ is a skew-symmetric matrix.

From (2) and (4), we have

$$\begin{aligned} &p = p(x_i(q_i)) = p(q_i) \\ &\dot{p} = A_i\dot{q}_i \qquad A_i = L_iJ_i \\ &\ddot{p} = A_i\ddot{q}_i + \dot{A}_i\dot{q}_i \end{aligned} \tag{6}$$

Substituting the above into (1), multiplying both sides by A_i^{-T}, and combining with (3), the closed-chain dynamic equation of the robotic system can be written as

$$H(p)\ddot{p} + C(p,\dot{p})\dot{p} + G(p) + \tilde{F}(t) = T - F_c \tag{7}$$

where

$$\begin{aligned} &H(p)=H_0(p)+\sum_{i=1}^{m} H_i(p) \qquad H_i(p)=A_i^{-T}M_i(q_i)A_i^{-1} \\ &C(p,\dot{p})=C_0+\sum_{i=1}^{m} C_i(p,\dot{p}) \qquad C_i=A_i^{-T}C_i(q_i,\dot{q}_i)A_i^{-1}-A_i^{-T}M_iA_i^{-1}\dot{A}_iA_i^{-1} \\ &G(p)=G_0(p)+\sum_{i=1}^{m} G_i(p) \qquad G_i(p)=A_i^{-T}G_i(q_i) \\ &\tilde{F}=\tilde{f}_0+\sum_{i=1}^{m} A_i^{-T}\tilde{f}_i \\ &T=A\tau \qquad A=[A_1^{-T},\dots,A_m^{-T}] \qquad \tau=[\tau_1^T,\dots,\tau_m^T]^T \end{aligned} \tag{8}$$

From (8) and noticing Properties 1 and 2, we have

Property 3. For any finite workspace Ω which A_i, $i=1,\dots,m$, are nonsingular, H(p) is a symmetric positive definite matrix. Moreover, there exist $k'>0$ and $k''>0$ such that

$$k' I \le H(p) \le k'' I \qquad \forall p \in \Omega \tag{9}$$

Property 4. The matrix $N(p,\dot{p})=\dot{H}(p)-2C(p,\dot{p})$ is a skew-symmetric matrix.

When the object comes in to contact with environment which is completely rigid, the object motion is constrained on the contact surfaces. Supposing that the constraints can be expressed by a set of k time-varying hypersurfaces:

$$\Phi(p,t)=0 \qquad \Phi(p,t)=[\phi_1(p,t),\dots,\phi_k(p,t)]^T \tag{10}$$

where $\phi_i(p,t)$ are twice continuously differentiable and mutually independent in Ω. The generalized constrained force F_c are given by [16]

$$F_c = J_{cl}^T(p,t)f_n \qquad J_{cl} = \frac{\partial \Phi(p,t)}{\partial p} \tag{11}$$

where f_n is a kx1 vector of Lagrange multipliers associated with the constraints, $J_{cl}(p,t)$ is a nonsingular matrix.

Supposing that there exists a set of (n-k) scalar functions $\{\psi_1(p,t),\dots,\psi_{n-k}(p,t)\}$ which is differentiable twice with respect to p and t such that $\{\phi_i(p,t), i=1,\dots,n;\ \psi_j(p,t),\ j=1,\dots,n-k\}$ are mutually independent in Ω for any t. We define a generalized coordinate frame as [16]:

$$r = [r_f^T, r_p^T]^T \qquad \begin{aligned} &r_f = [\phi_1(p,t),\dots,\phi_k(p,t)]^T \\ &r_p = [\psi_1(p,t),\dots,\psi_{n-k}(p,t)]^T \end{aligned} \tag{12}$$

Then, motion on the constraint surfaces (10) is completely determined by $r_p(t)$.

Remark 1. The existence of the supplemental coordinates r_p in local is guaranteed by the independent assumption of r_f, i.e., nonsingularity of J_{cl}. If J_{cl} is nonsingular, without loss of generality, we can suppose its first k column are independent. In this case, we can set $r_p=[p_{k+1},\dots,p_n]^T$.

From (12), we have

$$\dot{r} = J_c(p,t)\,\dot{p} + v_t(p,t) \tag{13}$$

where

$$J_c = \frac{\partial r(p,t)}{\partial p} \qquad J_c=\begin{bmatrix} J_{cl}^T & J_{r_p}^T \end{bmatrix}^T \qquad v_t = \frac{\partial r(p,t)}{\partial t} \tag{14}$$

In the new generalized coordinate frame, closed-chain equation (7) of the robotic system with the constrains (10) and the constrained force (11) can be expressed by

$$\begin{aligned} &H(r,t)\ddot{r} + C(r,\dot{r},t)\dot{r} + G(r,t) + F_t(r,t) + \tilde{F}(r,t) = T_r - J_c^{-T}F_c \\ &r = [\ 0\ ,\ r_p^T\ ]^T \end{aligned} \tag{15}$$

or

$$H_{12}(r_p,t)\ddot{r}_p + C_{12}(r_p,\dot{r}_p,t)\dot{r}_p + G_1(r_p,t) + F_{t1} + \tilde{F}_1(r_p,t) = T_{r_1} - f_n \tag{16}$$

$$H_{22}(r_p,t)\ddot{r}_p + C_{22}(r_p,\dot{r}_p,t)\dot{r}_p + G_2(r_p,t) + F_{t2} + \tilde{F}_2(r_p,t) = T_{r_2} \tag{17}$$

where

$$\begin{aligned} &H(r,t) = J_c^{-T}H(p)J_c^{-1} \qquad H(r,t) = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \\ &C(r,\dot{r},t) = J_c^{-T}C(p,\dot{p})J_c^{-1} - J_c^{-T}H(p)J_c^{-1}\dot{J}_cJ_c^{-1} \qquad C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \\ &G(r,t) = J_c^{-T}G(p) \qquad G = \begin{bmatrix} G_1^T & G_2^T \end{bmatrix}^T \\ &F_t = -H(r,t)\dot{v}_t - C(r,\dot{r},t)v_t \\ &\tilde{F}(r,t) = J_c^{-T}\tilde{F}(t) \qquad \tilde{F} = \begin{bmatrix} \tilde{F}_1^T & \tilde{F}_2^T \end{bmatrix}^T \\ &T_r = J_c^{-T}T \qquad T_r = \begin{bmatrix} T_{r_1}^T & T_{r_2}^T \end{bmatrix}^T \end{aligned} \tag{18}$$

which is decomposed into force and position controlled subspaces (16) and (17). In here, force and position controllers are treated together and designed with (15).

Equation (15) has the following properties.

Property 3. For any finite workspace Ω which $J_c(r,t)$ is nonsingular, H(r,t) is a symmetric positive definite matrix with

$$k_r' I \le H(r,t) \le k_r'' I \qquad \forall p \in \Omega \tag{19}$$

where

$$0<k_r' \le \frac{k'}{c_1^2} \qquad k_r'' \ge \frac{k''}{c_2^2} \tag{20}$$
$$c_1 = \max_{p\in\Omega,\, t\in R} [\sigma_{max}(J_c(p,t))] \quad c_2 = \min_{p\in\Omega,\, t\in R} [\sigma_{min}(J_c(p,t))]$$

and $\sigma(J_c)$ means singular value of matrix J_c.

Proof. see Appendix A.1.

Property 4. The matrix $N(r,\dot{r},t)=\dot{H}(r,t)-2C(r,\dot{r},t)$ is a skew-symmetric matrix.

Proof. see Appendix A.2.

Assume that the available values of $H(r,t)$, $C(r,\dot{r},t)$, $G(r,t)$, $F_t(r,t)$ are $\hat{M}(r,t)$, $\hat{C}(r,\dot{r},t)$, $\hat{G}(r,t)$, $\hat{F}_t(r,t)$ and the modelling errors in (15) due to parametric uncertainties, external disturbances are assumed to be bounded by

$$\begin{array}{ll} |\Delta H(r,t)| \le \delta H(r,t) & \Delta H(r,t) = H(r,t) - \hat{H}(r,t) \\ |\Delta C(r,\dot{r},t)| \le \delta C(r,\dot{r},t) & \Delta C(r,\dot{r},t) = C(r,\dot{r},t) - \hat{C}(r,\dot{r},t) \\ |\Delta G(r,t)| \le \delta G(r,t) & \Delta G(r,t) = G(r,t) - \hat{G}(r,t) \\ |\Delta F_t(r,t)| \le \delta F_t(r,t) & \Delta F_t = F_t - \hat{F}_t \\ |\tilde{F}(r,t)| \le \delta \tilde{F}(r,t) & \end{array} \tag{21}$$

where $|A| \le B$ is true in element, i.e., $|A_{ij}| \le B_{ij}$ (In the following, the operation of matrix is understood in the same meaning).

The robust control problem under consideration is stated as that of designing a controller so that the robotic system with the constraints (10) and the constrained force (11) follows the desired motion trajectory $r_{pd}(t)$ while exerting the desired generalized constrained force $f_{nd}(t)$ on the environment under the modelling errors (21).

3. VSC Constrained Motion Control of Robotic System

For VSC, the switching function is selected as

$$s = [s_f^T,\ s_p^T]^T \qquad s_f = K_I \int_0^t e_f(\mu)d\mu \qquad e_f = f_n(t) - f_{nd}(t) \\ s_p = \dot{e}_p + K_p e_p \qquad e_p = r_p(t) - r_{pd}(t) \tag{22}$$

where K_p is any positive definite matrix, K_I is a weighted matrix. The resulted sliding mode equation { $s=0$, $\dot{s}=0$ } is described by

$$f_n(t) - f_{nd}(t) = 0 \\ \dot{e}_p + K_p e_p = 0 \tag{23}$$

which position and force responses are decoupled, and the robotic system asymptotically follows the desired motion trajectory $r_{pd}(t)$ while exerting the desired constrained force $f_{nd}(t)$ on the environment. Moreover, by choosing a suitable matrix K_p [23], prescribed quality can be guaranteed in the sliding mode.

The control torque can be determined so that the robotic system reaches the sliding mode in finite time and has prescribed reaching transient response against the modelling errors.

Theorem 1. For the robotic system with the constraints (10) and the constrained force (11), the robotic system will asymptotically follow the desired motion trajectory $r_{pd}(t)$ and exert the desired constrained force $f_{nd}(t)$ under the modelling errors (21), if the following control torque is applied

$$T_r = \hat{H}(r,t)\ddot{r}_{eq} + \hat{C}(r,\dot{r},t)\dot{r}_{eq} + \hat{G}(r,t) + \hat{F}_t + J_c^{-T}F_c - Rs - \varepsilon sgn(s) - T_d \tag{24}$$

where

$$\dot{r}_{eq} = \begin{bmatrix} -K_I\int_0^t e_f(\mu)d\mu \\ \dot{r}_{p_d}(t) - K_p e_p \end{bmatrix} \qquad \ddot{r}_{eq} = \begin{bmatrix} -K_I e_f \\ \ddot{r}_{p_d}(t) - K_p \dot{e}_p \end{bmatrix}$$
$$(T_d)_i = (\delta T)_i sgn(s_i) \qquad i=1,\ldots,n \tag{25}$$
$$\delta T = \delta H(r,t)|\ddot{r}_{eq}| + \delta C(r,\dot{r},t)|\dot{r}_{eq}| + \delta G(r,t) + \delta F_t(r,t) + \delta \tilde{F}(r,t)|$$
$$\varepsilon > 0 \qquad sgn(s) = [sgn(s_1),\ldots,sgn(s_n)]^T$$

and R is any positive definite matrix. Moreover, the reaching time t_r which the system reaches the sliding mode is

$$t_r \le t_{max} \qquad t_{max} = \frac{2}{c_3}\ln(1+\frac{c_3}{c_4}\sqrt{V_0}) \tag{26}$$

where

$$c_3 = \frac{2\lambda_{min}(R)}{k_r''} \qquad c_4 = \varepsilon\sqrt{\frac{2}{k_r''}} \qquad V_0 = \frac{1}{2}s(0)^T H(r_0,0)\, s(0) \tag{27}$$

and the reaching transient response is shaped by

$$\|s\| \le \sqrt{\frac{2}{k_r'}}[(\sqrt{V_0}+\frac{c_4}{c_3})\exp^{-\frac{c_3}{2}t} - \frac{c_4}{c_3}] \tag{28}$$

where $\lambda_{min}(R)$ means minimum eigenvalue of the matrix R. ■

Proof. For the constrained robotic system (15), we choose a Lyapunov function as

$$V = \frac{1}{2}s^T H(r,t)s \tag{29}$$

From property 3, we have

$$\frac{1}{2}k_r'\|s\|^2 \le V \le \frac{1}{2}k_r''\|s\|^2 \tag{30}$$

Differentiating V with respect to time yields

$$\begin{aligned} \dot{V} &= s^T H\dot{s} + \frac{1}{2}s^T\dot{H}s \\ &= s^T H(\begin{bmatrix} 0 \\ \ddot{r}_p \end{bmatrix} - \ddot{r}_{eq}) + s^T C(r,\dot{r},t)(\begin{bmatrix} 0 \\ \dot{r}_p \end{bmatrix} - \dot{r}_{eq}) \\ &= s^T[T_r - G(r,t) - F_t - J_c^{-T}F_c - \tilde{F}(r,t) - H\ddot{r}_{eq} - C(r,\dot{r},t)\dot{r}_{eq}] \end{aligned} \tag{31}$$

where property 4 has been used to eliminate the term $1/2s^T\dot{H}s$ due to the time nature of inertia matrix. Substituting control torque (24) into it and noticing (30), we have

$$\begin{aligned} \dot{V} &= -s^T Rs - \varepsilon s^T sgn(s) - s^T T_d \\ &\quad - s^T[\Delta H\ddot{r}_{eq} + \Delta C\dot{r}_{eq} + \Delta G + \Delta F_t + \tilde{F}(r,t)] \\ &\le -s^T Rs - \varepsilon s^T sgn(s) - s^T T_d + |s^T|\delta T \\ &\le -s^T Rs - \varepsilon\sum_{i=1}^n |s_i| \\ &\le -\lambda_{min}(R)\|s\|^2 - \varepsilon\|s\| \\ &\le -c_3 V - c_4\sqrt{V} \end{aligned} \tag{32}$$

So

$$\sqrt{V} \le (\sqrt{V_0}+\frac{c_4}{c_3})\exp^{-\frac{c_3}{2}t} - \frac{c_4}{c_3} \tag{33}$$

which means that in finite time $V=0$, i.e, $s=0$. Moreover, from (30), the reaching transient response is shaped by (28). The upper limit t_{max} of the reaching time t_r is solved by setting right hand of (33) equal zero which is given by (26). Hence, Theorem is proved. ■

Remark 2. In the above Theorem, the role of the discontinuous control torque T_d is to overcome the modelling errors (21) so that the system reaches the sliding mode in finite time. Introducing the discontinuous term $\varepsilon \mathrm{sgn}(s)$ enhances this effect and enables us to explicitly control the reaching transient. As can be seen from (26) that the larger ε and $\lambda_{min}(R)$ are, the smaller t_{max} will be, i.e., the reaching transient will be shorter. However, if ε is large, there would probably appears a strong chattering in practice due to its discontinuity. Therefore the better choice is to take small ε and large $\lambda_{min}(R)$, such that the reaching transient is rapid enough and at the same time the chattering is relative small [24,26]. Moreover, since r_p can be chosen as either some independent joint vector q_{ij}, the object position coordinates p_i, or orthogonal curvilinear coordinates on the constraint surfaces (10), the controller can be implemented in different space.

For implementation, the control torque (24) is converted into world space.

Theorem 2. For robotic system (7) with constraints (10), the robotic system will asymptotically follow the desired path $r_{pd}(t)$ and exert the desired constrained force $f_{nd}(t)$ if the following control torque is applied:

$$T = \hat{H}(p)\ddot{p}_{eq} + \hat{C}(p,\dot{p})\dot{p}_{eq} + \hat{G}(p) + F_c - J_c^T[\,Rs+\varepsilon sgn(s)+T_d\,] \tag{34}$$

where

$$\begin{aligned} \dot{p}_{eq} &= J_c^{-1}(\dot{r}_{eq}-v_t) \\ \ddot{p}_{eq} &= J_c^{-1}(\ddot{r}_{eq}-\dot{J}_c\dot{p}_{eq}-\dot{v}_t) \end{aligned} \tag{35}$$

To implement the required control torque $T=A\tau$ in (34), the joint torque τ_i for each robot is not unique. Load distribution is needed to coordinate each robot. Here, minimizing weighted energy consumption [5,10] is used, i.e., minimizing

$$Q = \tau^T W \tau^T \tag{36}$$

where W is a positive definite weighted matrix. If there is no constraint imposed on the joint torque τ, the solution is straightforward [27] and is given by

$$\tau = W^{-1}A^T(AW^{-1}A^T)^{-1}T \tag{37}$$

4. Simulation

Fig.1 shows two planer robots grasping a common object moving on a circular surface. Configuration of each robot is shown in Fig.2. The dynamic equation of each robot is given by

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) + J_i^T(q_i)F_i = \tau_i \tag{38}$$

where

$$M_i=\begin{bmatrix} m_{i1}l_{ic1}^2+m_{i2}L_{i1}^2+m_{i3}L_{i1}^2+I_1 & (m_{i2}L_{i1}l_{ic2}+m_{i3}L_{i1}L_{i2})C_{i21} & m_{i3}L_{i1}l_{ic3}C_{i32} \\ (m_{i2}L_{i1}l_{ic2}+m_{i3}L_{i1}L_{i2})C_{i21} & m_{i2}l_{ic2}^2+m_{i3}L_{i2}^2+I_{i2} & m_{i3}L_{i2}l_{ic3}C_{i32} \\ m_{i3}L_1l_{ic3}C_{i31} & m_{i3}L_{i2}l_{ic3}C_{i32} & m_{i3}l_{ic3}^2+I_{i3} \end{bmatrix}$$

$$C_i=\begin{bmatrix} 0 & -(m_{i2}L_{i1}l_{ic2}+m_{i3}L_{i1}L_{i2})S_{i21}\dot{q}_{i2} & -m_{i3}L_{i1}l_{ic3}S_{i31}\dot{q}_{i3} \\ (m_{i2}L_{i1}l_{ic2}+m_{i3}L_{i1}l_{i2})S_{i21}\dot{q}_{i1} & 0 & -m_{i3}L_{i2}l_{ic3}S_{i32}\dot{q}_{i3} \\ m_{i3}L_{i1}l_{ic3}S_{i31}\dot{q}_{i1} & m_{i3}L_{i2}l_{ic3}S_{i32}\dot{q}_{i2} & 0 \end{bmatrix}$$

$$G_i=\begin{bmatrix} (m_{i1}l_{ic1}+m_{i2}L_{i1}+m_{i3}L_{i1})g\cos q_{i1} \\ (m_{i2}l_{ic2}+m_{i3}L_{i2})g\cos q_{i2} \\ m_{i3}gl_{ic3}\cos q_{i3} \end{bmatrix}$$

$$J_i=\begin{bmatrix} -L_{i1}\sin q_{i1} & -L_{i2}\sin q_{i2} & -L_{i3}\sin q_{i3} \\ L_{i1}\cos q_{i1} & L_{i2}\cos q_{i2} & L_{i3}\cos q_{i3} \\ 0 & 0 & 1 \end{bmatrix}$$

and $C_{i21}=\cos(q_{i2}-q_{i1})$, $C_{i31}=\cos(q_{i3}-q_{i1})$, $C_{i32}=\cos(q_{i3}-q_{i2})$, $S_{i21}=\sin(q_{i2}-q_{i1})$, $S_{i31}=\sin(q_{i3}-q_{i1})$, $S_{i32}=\sin(q_{i3}-q_{i2})$. Parameter values of the ith robot are

$$m_{i1}=10kg \quad m_{i2}=10kg \quad m_{i3}=5kg \quad I_{i1}=1kgm^2$$
$$I_{i2}=1kgm^2 \quad I_{i3}=0.2kgm^2 \quad L_{i1}=1.25m \quad L_{i2}=1.25m$$
$$L_{i3}=0.2m \quad l_{ic1}=0.65m \quad l_{ic2}=0.65m \quad l_{ic3}=0.1m \quad d_i=1.25m$$

Dynamic equation of the object is given by

$$\begin{bmatrix} m_0 & & \\ & m_0 & \\ & & I_0 \end{bmatrix}\ddot{p} + \begin{bmatrix} 0 \\ m_0g \\ 0 \end{bmatrix} + F_c = \sum_{i=1}^{2} L_i^{-T}F$$

where

$$p = [x, y, \theta]$$

$$L_1=\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \qquad L_2=\begin{bmatrix} -1 & & \\ & 1 & \\ & & -1 \end{bmatrix}$$

The exact values of m_0, I_0 are assumed to be unknown with actual values $m_0=40kg$, $I_0=4kgm^2$ and estimated values $\hat{m}_0=20kg$ $\hat{I}_0=2kgm^2$. The closed chain dynamic equation of robotic system can now be calculated by (8).

The constraint (10) and constrained force (11) imposed on the object are

$$\sqrt{x^2+y^2} - a = 0 \qquad a=1m$$

$$F_c = \begin{bmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{bmatrix} f_n \qquad \varphi=\tan^{-1}\frac{y}{x}$$

In the first simulation, we use orthogonal curvilinear coordinates of r_f as r_p, that is

$$\begin{aligned} r_f &= \sqrt{x^2+y^2}-a \\ r_{p1} &= (r_f+a)\varphi \\ r_{p2} &= \theta \end{aligned} \qquad J_c=\begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The switching function (22) is chosen as

$$\begin{aligned} s_1 &= 0.01\int_0^t e_f(\mu)d\mu & e_f &= f_n(t)-f_{nd}(t) \\ s_2 &= \dot{e}_{r_{p1}}+20e_{r_{p1}} & e_{r_{p1}} &= r_{p_1}-r_{p_{1d}}(t) \\ s_3 &= \dot{e}_\theta+20e_\theta & e_\theta &= \theta(t)-\theta_d(t) \end{aligned}$$

The control torque can be calculated by (34) and (37). For eliminating chattering, saturation function $sat(s_i/\Delta_i)$ instead of $sgn(s_i)$ is used [22]. The controller parameters are chosen as $R=diag\{3000, 1000, 1000\}$, $\varepsilon=diag\{1, 1, 1\}$, $\Delta=diag\{0.005, 0.005, 0.005\}$. The simulation results are shown in Fig.3-10. Fig.3, Fig.4, and Fig.6 show the time responses of motion and constrained force of the object which verify the motion and force tracking control of the proposed VSC controller. The tracking errors of the motion and force are shown in Fig.5 and Fig.7. The time response of the switching function is shown in Fig.8. Fig.9 and 10 shows joint torques of each robot.

In the second simulation, we simply use position coordinates x,θ as r_p, that is,

$$r_f=\sqrt{x^2+y^2}-a \qquad J_c=\begin{bmatrix}\cos\varphi & \sin\varphi & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$
$$r_{p1}=x$$
$$r_{p2}=\theta$$

Then, the controller is implemented in the world space. Similar simulation results are obtained.

5. Conclusion

In this paper, we have considered the problem of robust motion and force control of coordinated multi-arm robots grasping a common object constrained by the environment in the presence of parametric uncertainties and external disturbances. We first establish the closed chain dynamic equation of robotic system in the world space. Based on a transformed closed-chain dynamic equation, position and force control are designed together via VSC. For coordinating each robot, load distribution is carried out for minimizing weighted energy consumption. The VSC controller can guarantee the robotic system with prescribed qualities both in the sliding mode and in the reaching transient. Simulation results illustrate the proposed method.

APPENDIX

Appendix A.1: From (18), we have

$$y^T H(r,t) y = (J_c^{-1}y)^T H(p) (J_c^{-1}y) \qquad \forall y \in R^n$$

Noticing property 3, then

$$k' y^T (J_c^{-T} J_c^{-1}) y \le y^T H(r,t) y \le k'' y^T (J_c^{-T} J_c^{-1}) y$$
$$k' \sigma_{\min}^2 (J_c^{-1}) y^T y \le y^T H(r,t) \le k'' \sigma_{\max}^2 (J_c^{-1}) y^T y$$
$$\frac{k'}{\sigma_{\max}^2 (J_c)} y^T y \le y^T H(r,t) y \le \frac{k''}{\sigma_{\min}^2 (J_c)} y^T y$$

This leads to (19).

Appendix A.2: From (18), we have

$$\begin{aligned} y^T N(r,\dot{r},t) y = & \; y^T [J_c^{-T} \dot{H}(p) J_c^{-1} + J_c^{-T} H(p) \dot{J}_c^{-1}] y \\ & -2y^T [J_c^{-T} C(p,\dot{p}) J_c^{-1} - J_c^{-T} H(p) J_c^{-1} \dot{J}_c J_c^{-1}] y \\ = & (J_c^{-1} y)^T [\dot{H}(p) - 2C(p,\dot{p})] (J_c^{-1} y) \qquad \forall y \in R^n \end{aligned}$$

Noticing property 2, then

$$y^T N(r,\dot{r},t) y = 0$$

This leads to the property 2.

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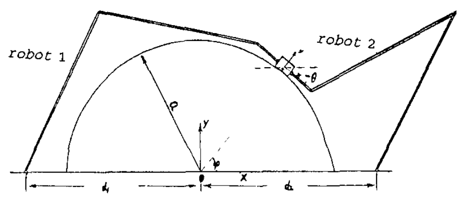

Fig.1 Configuration of robotic system

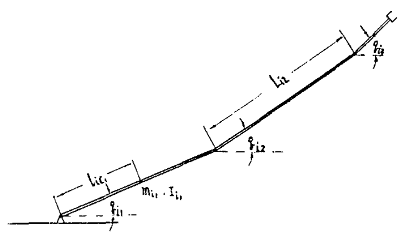

Fig.2 Configuration of the ith robot

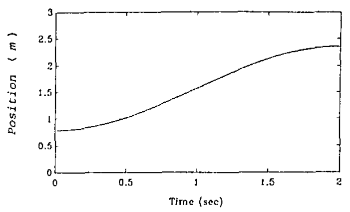

Fig.3 Time response of r_{p1}

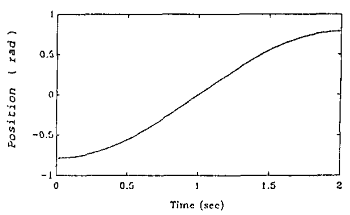

Fig.4 Time response of θ

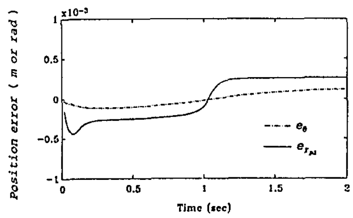

Fig.5 Tracking error of position r_p

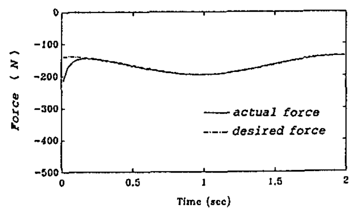

Fig.6 Time response of constrained force

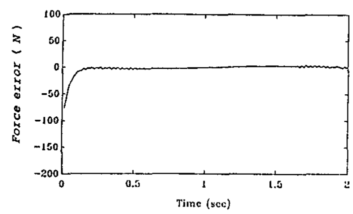

Fig.7 Tracking error of constrained force

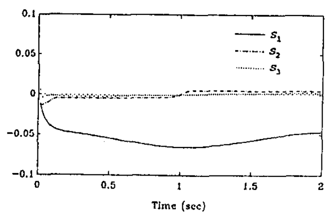

Fig.8 Time response of sliding surface s

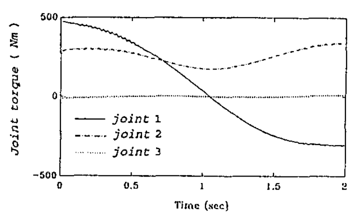

Fig.9 Joint torque of the first robot

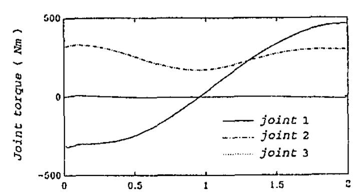

Fig.10 Joint torque of the second robot