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Spatiotemporal pattern formation in a diffusive predator-prey system: an analytical approach

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Abstract In this paper, we propose and analyse a mathematical model to study the mathematical aspect of reaction diffusion pattern formation mechanism in a predator-prey system. An attempt is made to provide an analytical explanation for understanding plankton patchiness in a minimal model of aquatic ecosystem consisting of phytoplankton, zooplankton, fish and nutrient. The reaction diffusion model system exhibits spatiotemporal chaos causing plankton patchiness in marine system. Our analytical findings, supported by the results of numerical experiments, suggest that an unstable diffusive system can be made stable by increasing diffusivity constant to a sufficiently large value. It is also observed that the solution of the system converges to its equilibrium faster in the case of two-dimensional diffusion in comparison to the one-dimensional diffusion. The ideas contained in the present paper may provide a better understanding of the pattern formation in marine ecosystem.

Keywords Prey-predator system · Reaction-diffusion equations · Marine ecosystem · Chaos · Spatiotemporal pattern

Mathematics Subject Classification (2000) 35B35 · 92C15 · 35K57

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1 Introduction

The fundamental importance of spatial and spatiotemporal pattern formation in biology is self-evident. Our understanding is such that the pattern we observe in the biological communities, it is almost certain that the process that produced it is still unknown. The field of pattern formation focuses on natural, social and technological sciences where the nonlinearities conspire to form spatial patterns that sometimes are stationary, traveling or disordered in space and time often referred as spatiotemporal chaos. The classical approach to the origin of spatial patterns was first suggested by Turing [30]. Later Wolpert [34] proposed a phenomenological concept of pattern formation. The general introductory paper by Wolpert [35] gives a very clear and non-technical description of development of pattern formation in animals which gives rise to an immense number of illuminating experimental studies, also associated with the marine ecosystem.

Plankton pattern formation is dependent on the interplay of various physical (temperature, light) and biological (nutrient supply, fish predation) factors [5, 10, 25]. Much of the research in developmental biology is devoted to determine the underlying mechanism which generates plankton pattern formation. Abraham [1] found that change in the environment due to turbulent advection causes characteristic spatial patterns of phytoplankton and zooplankton populations. Turing's idea [30] that differential-diffusion can act on a reacting system to produce time-independent spatial patterns in a homogeneous environment was explored by Segel and Jackson [27] and by Levin and Segel [18] in an ecological context in the first part of seventies. Levin and Segel [18] showed that this could serve as a mechanism for origin of planktonic patchiness in marine ecosystems. Turing spatial patterns have been observed in computer simulations of interaction-diffusion system by many authors [24, 36]. These structures of instability are generated under conditions of differential-diffusion for biological species like phytoplankton and zooplankton. This is not the case when dispersal of species is due to turbulent mixing. The more important observation in this context is that spatial patterns generated due to Turing instability are regular and stationary. Medvinsky et al. [22] examined plankton spatial patterns caused by diffusive interaction between nearby habitats some of which were fish free in a reaction-diffusion model with passive movement of individuals of two interacting species caused by turbulent mixing. Upadhyay et al. [31, 32] investigated wave phenomena and non-linear non-equilibrium pattern formation in the phytoplankton-zooplankton-fish system with Holling type II and Holling type IV functional responses. They observed that the wave of chaos still remains an effective mechanism for the propagation of chaotic dynamics. Periodic models in impulsive ecological systems were recently studied by Stamov [28] with variable diffusivity constant.

Recently many authors have performed analytical investigations of diffusive predator prey model systems in different aspects. Dubois [8] proposed a nonlinear model (Lotka-Volterra type ecological interaction) to explain the horizontal structuration of prey-predator populations in a turbulent sea. Vilar et al. [33] showed that biotic fluctuations and turbulent diffusion in standard prey-predator models are able to explain plankton field observations which includes not only the spatial pattern but also its temporal evolution. Brentnall et al. [3] investigated the effect of small-scale