Control and Estimation of Automotive Powertrains with Backlash

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Department of Signals and Systems
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden, 2004
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Doktorsavhandlingar vid Chalmers tekniska högskola
Ny serie nr 2167
ISSN 0346–718X

Technical Report No. 485
School of Electrical Engineering
Chalmers University of Technology
ISSN 1651–498X

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Cover:
Powertrain of a passenger car, © Volvo Car Corporation.
Mating gears with backlash. Illustrator: Mikael Cederfeldt.

Typeset by the author with the \LaTeX\ Documentation System

Printed by Chalmers Reproservice
Göteborg, Sweden 2004
Abstract

In automotive powertrains, backlash causes problems with vehicle driveability, specifically at so called tip-in and tip-out maneuvers. These maneuvers may trigger phenomena known in the automotive industry as *shunt* and *shuffle*, which are partially caused by the backlash. These phenomena are considered difficult to cope with, and until recently, only relatively simple controllers have been used to reduce the discomfort. Also, the tuning of these controllers is made quite subjectively.

A number of approaches to the control of systems with backlash are reported in the literature. Most of these approaches assume another system structure around the backlash than what is the case in automotive powertrains. Very few approaches are therefore directly applicable to the control of automotive powertrains.

The first part of this thesis gives an overview of available control strategies for backlash control. The strategies can be divided into active and passive strategies, depending on the way the controller handles the backlash. An active controller compensates the backlash nonlinearity by a more active control signal, while the passive controller becomes more cautious when the backlash gap is entered. Some of the strategies, e.g. switched linear controllers and model predictive controllers, are evaluated in the powertrain application by means of simulation. The results show that active nonlinear controllers have a potential for improved backlash control. However, the robustness of these controllers needs further investigation. Open-loop optimal control is used in this thesis as a way to find theoretical limits on backlash compensation performance.

High-performance controllers for backlash compensation require high-quality measurements of the current state of the powertrain. Information about the size of the backlash is also needed. These problems are addressed in the second part of this thesis. Two nonlinear estimators based on Kalman filtering theory have been developed, one for state estimation and one for the estimation of backlash size. Simulation and experimental results show that the resulting estimates are of high quality.

**Keywords:** Backlash control, powertrain control, backlash estimation, driveability, model predictive control, optimal control.
Preface

When I now finish writing this thesis, I realize that its contents is completely different from what I was anticipating when I first began my PhD studies. As a newly graduated chemical engineer, I joined the Control Engineering Laboratory¹ at Chalmers as a PhD student in 1992, and wrote my licentiate thesis (Lagerberg, 1996) on nonlinear control of chemical processes. After receiving my licentiate degree I decided to leave academia, to work with process control in the "real world". After a few years, I joined the School of Engineering at Jönköping University as a teacher in 2000. There, I was given the opportunity to continue my PhD studies at Chalmers. Due to various circumstances, I had to find a new research project, and I decided to join the Integrated Powertrain Control, IPC, research group. This group was formed to coordinate several PhD projects within powertrain control, and had members from the Control and Automation Lab., the Vehicle Dynamics and Mechatronics groups at Chalmers and from different Volvo companies.

I started to study problems caused by backlash in the powertrain. With my background, it was a new and interesting field for me to get familiar with the dynamics of engines, gearboxes etc. One of the reasons why I find automatic control interesting is that the same theory can be used to analyze both chemical and mechanical (and many other) system dynamics.

Contrary to the initial plan, it became apparent that a combination of the results from this project and the results presented in my licentiate thesis was impracticable. I ended up writing this PhD thesis from scratch, based only on the results from my last period of PhD studies.

Many people have contributed to my work throughout the years, and I would like to express my gratitude to all of them. A few persons deserve special recognition:

First of all I want to thank Prof. Bo Egardt, for accepting me as a PhD student (twice), and for acting as supervisor during my second period at the department. Bo has the ability to pinpoint the essential questions, and problems often seem easier to solve after discussions with him. Thank you Bo!

I have enjoyed being a member of the Control and Automation Laboratory at Chalmers during these years, and I want to thank all past and present members for various influences on my work. A special thanks goes to Stefan Pettersson for many discussions on various topics, and for proofreading parts of my thesis. Prof. Claes Breitholtz is acknowledged for

¹Currently the Control and Automation Laboratory at the Department of Signals and Systems.
supervising my licentiate project and for many interesting discussions. Jonas Fredriksson
and Fredrik Bruzelius allowed me to use their LATEX templates, which has helped me a lot.

I am grateful to all members of the IPC group, for introducing me to the exciting area of
powertrain control, and for suggesting me to focus on the backlash problem.

Sören Eriksson, Frank Mohr, Klas Bergqvist and many others at Volvo Car Corporation
are acknowledged for providing all sorts of information, and for allowing me to perform
real world experiments. I would like to thank Lars Bråthe at Volvo Powertrain for contin-
uous encouragements and for valuable comments on my thesis.

My daily work environment has been in Jönköping, and I would like to thank all my
colleagues at the School of Engineering for making it a good workplace. I am grateful to
the school head, Prof. Roy Holmberg, and my past and present heads of department, Prof.
Bengt Magnhagen and Prof. Shashi Kumar, for employing me and allowing me to spend
enough time on research to make it worthwhile. A special thanks also to Alf Johansson.

The work presented in this thesis has mainly been funded by the School of Engineering
in Jönköping. Valuable contributions to the funding have also been received from the
Volvo Research and Educational Foundations, the Swedish Automotive Research Pro-
gram (PFF) and Volvo, which is gratefully acknowledged.

I would like to express my gratitude and appreciation to my parents, Ingrid and Fredrik
Lagerberg, and my mother-in-law, Carola Fridén, for support in many forms. Finally,
my deepest gratitude goes to my wife Charlotte and our children, Göran and Märta. Our
children have often wondered what I was doing, working by the computer in the evenings
when they went to bed. My attempt to explain to them has been that "I work to become a
doctor, a car-doctor who cures backlashes...".

Jönköping / Göteborg, August 2004

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List of publications

This thesis is based on the work contained in the publications listed below, which will be referred to by Roman numerals. Only printing formats are changed and minor errors corrected in comparison with the original, published versions.


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Part I
Chapter 1

Introduction

1.1 Backlash in automotive powertrains

The automotive industry is characterized by mature products, that have been under development for a long time and thus are very optimized. Typical for the business is also the hard competition between brands, with virtually all manufacturers producing excellent products. At the same time, prices are just enough to finance product development.

Developments during recent decades include safety e.g. in terms of crashworthiness; environmental concerns in terms of fuel efficiency and exhaust and noise emissions; quality concerns in terms of less service, higher reliability and longer life; a higher degree of automatization of functions.

Image is important, and it is built up by the properties and perceived performance of the vehicle. In the following, only one specific aspect of this will be discussed: The influence of backlash in the powertrain on the properties of the vehicle, affecting noise emissions, driveability and quality impression. Lack in meeting the demands in this respect will influence the competition situation.

An automotive powertrain consists of all parts that are needed for the forward or reverse propulsion of a vehicle. The powertrain consists of engine, clutch, transmission, propeller shaft, final drive, drive shafts and wheels. See Figure 1.1 for an example of a rear wheel driven car. The same parts exist in the powertrain for a front wheel driven vehicle except that the propeller shaft is omitted. Also, the engine and transmission are usually oriented transversely in the engine bay.

The engine delivers the power needed for the propulsion of the vehicle. The power, and hence the torque, which the engine is able to produce is depending on the engine speed, and the efficient operating range is limited. To match the engine’s operating range to the vehicle speed chosen by the driver, a transmission is needed. The transmission consists of a gearbox, where different gear ratios are chosen depending on the vehicle’s speed and desired acceleration. The gearbox can be manually or automatically operated. The clutch is needed to make the transmission free from load during engagement and disengagement.
Chapter 1  Introduction

of the gears. In powertrains with automatic transmission, the clutch may be replaced by a torque converter. This thesis only considers powertrains with a clutch however. The differential distributes the torque to the two driven wheels, and allows the wheels to rotate at different speeds, for example in curves. It has also a final gear ratio, which together with the transmission yields a total transmission ratio from engine to wheels.

The existence of backlash in the automotive powertrain causes problems with driveability of the vehicle. The source of backlash is mainly the play that exists between teeth in the different gear components, such as the gearbox and the differential. Without play, the gears are unable to rotate. Also the clutch and the engine flywheel have more or less of backlash, introduced in order to reduce vibrations. This also adds to the total powertrain backlash. For most of the purposes in this thesis, the different backlash contributions can be lumped together, and treated as one backlash. In a new car on the first gear, the total backlash can be equivalent to 20–40 crankshaft degrees of the engine, or more. For older vehicles with worn components, 30 degrees on the wheel is reported, (Stewart et al., 2004), which corresponds to 360 crankshaft degrees.

When the driver goes from engine braking to acceleration or vice versa, so called tip-in and tip-out maneuvers, the backlash is traversed. Initially, the engine and wheel sides of the backlash will be in mechanical contact, but while the backlash is traversed, there is no contact between the sides. The engine will accelerate without the load of the vehicle mass. When the backlash is traversed, contact will be recovered, and if the relative speed between the backlash sides is high, some of the momentum that has been built up in the engine flywheel, will be transferred to the vehicle structure and to the wheels. The vehicle will be given a momentary acceleration, a so called shunt, which the driver will feel as an uncomfortable jerkiness of the vehicle, see Figure 1.2. However, if the backlash traverse is made too cautiously, the response time from driver command to vehicle acceleration will be too long.

Another aspect of this is the so called Conformity Of Production, COP, demand of legislation, meaning that the shunt phenomenon occurring in one specific vehicle should be
1.2 Driveability improvements

Driveability can be defined as phenomena in the vehicle that relate to either the comfort of the driver or the performance of the vehicle. It is a subjective measure of the driver–vehicle interaction. Examples of concepts, criteria and maneuvers that are included in a driveability evaluation are: Engine start, idle running, acceleration, pedal response, gear shift, declutch, tip-in, tip-out, hesitation, jerk, kick (shunt), overshoot, delay, oscillation (shuffle), noise and vibration (Schoeggl and Ramschak, 2000; Dorey and Martin, 2000; Biermann et al., 2000).

Shunt is caused by the powertrain backlash, and it is considered difficult to cope with. There are two main ways to decrease the shunt phenomena: The first is to physically decrease the backlash size by manufacturing or by special designs. Such designs are common for industrial robots where load reversals are very common in operation, and}

---

Figure 1.2: Measured tip-in response of a passenger car without backlash compensation functionality (Karlsson, 2001). The first increase in acceleration, from zero to the first peak, is called shunt, and the slowly decaying oscillation is called shuffle. It is seen how the acceleration stays close to zero while the backlash is traversed around t=80 s.

representative for other vehicles with identical specification. This makes type approval possible, in contrast to individual approval. In the future COP might also apply to aging cars, where the backlash sizes increase, and hence potentially also the shunt phenomena. Although COP is not explicitly addressed in this thesis, the presented results may be used for this purpose.

Still another automotive powertrain problem area, not treated here, is the rattling of unloaded components. This is a significant source of undesired noise, and backlash is contributing to the phenomenon.

1.2 Driveability improvements
where high operation speed as well as high precision is needed. The methods commonly used are either to force the gear shaft distance together or to axially pretension bevel gears. These strategies are used by some car manufacturers, but smaller backlash size means higher manufacturing costs for the gear components.

The second strategy, which is the field of this thesis, is to include backlash compensation functionality in the engine control system. The cost for this solution is instead on software development, a cost that can be distributed over a large number of produced vehicles.

Existing engine control systems employ relatively simple strategies for backlash compensation. There is an interest in the automotive industry to develop better backlash control systems, in order to decrease response time, and to increase passenger comfort. With increasing computing power in the control systems and more sensors in the vehicles, advanced controllers are potentially implementable today.

As indicated above, improved control system functionality has a potential for yielding higher driveability. It may also be possible to preserve the performance although the backlash size is increased, something that may reduce the manufacturing costs for powertrain components. These observations serve as motivation for performing research in this area. The purpose of this thesis is to contribute research results in the area of advanced control of powertrains with backlash. Since advanced control requires good information about the status of the powertrain components, a substantial part of the thesis is also devoted to state and parameter estimation in automotive powertrains. Although powertrains are in focus here, the presented results are applicable also to other mechanical systems with significant backlash.

This research work is part of a larger research project, Integrated Powertrain Control. The common goal in this project is to view the powertrain as an integrated system, where the engine is used as an actuator to the drivetrain, in order to improve the vehicle’s performance. This is in contrast to the more traditional view, where the engine and transmission control systems are designed independently and compensations are typically performed in other systems, or even separate add-on systems. Some examples: The engine control system, ECS, includes functions for control of the delivered torque to meet the driver’s request. How this torque is affecting the powertrain, e.g. resulting in shunt and shuffle, is not handled. In a vehicle with automatic transmission, the transmission control system mainly manages the choice of gear. This choice is based on wheel speed, engine speed and requested acceleration. It may also request modifications of the engine torque during the gear shifts in order to improve the shifting quality. The automotive industry is going towards tighter integration of these functions, but there is still potential for improvements. Backlash control is a typical application that requires this integrated view of the powertrain.
1.3 Nonlinear control aspects

Phenomena such as saturation, relays, friction, dead-zones, hysteresis and backlash are examples of discontinuous, or hard, nonlinearities. For continuous nonlinearities, described by smooth nonlinear functions, a linear approximation is often used. Based on this approximation, there exist a large spectrum of mature linear control theory that can be applied for the controller design and analysis. In contrast, hard nonlinearities are not linearizable in a straightforward manner. In the controller design for these systems, it is still common to rely on ad hoc solutions. A controller may be designed from experience, and the system performance is tested through computer simulations.

Nonlinear control theory for hard nonlinearities exists, but most of it applies to special cases. For example, in the describing function theory, (Slotine and Li, 1991), the nonlinearity is replaced by a describing function, which captures the dominant frequency response of the nonlinearity. If the nonlinearity is in feedback connection with other dynamics of low-pass character, the system stability may be analyzed by linear frequency response methods. The local performance when the system moves over the discontinuities is not possible to analyze however.

The theory of hybrid, or switched, systems may be used on systems with hard nonlinearities. Depending on internal or external events, a hybrid system switches between different modes, and in each mode the system is described by continuous dynamics. Research on hybrid systems theory is currently very active (Liberzon, 2003; Pettersson, 1999), but the results are not yet as mature as the linear systems theory, and therefore not as available for applications in general.

As will be detailed in Chapter 2, due to the hard backlash nonlinearity, the powertrain models used in this thesis are switching between different discrete modes. This observation intuitively leads to the use of switched controllers and switched Kalman filters to accomplish the control task. Furthermore, since all model dynamics except the backlash are linear, the models have the property that they are piecewise affine (or piecewise linear). This means that the problems may be tackled by hybrid systems theory, such as the analysis and design of stability in observers for hybrid systems (Juloski et al., 2002), and design of model predictive controllers for piecewise affine systems (Mayne and Raković, 2003; Morari et al., 2003).

The main focus of this thesis is to apply a selection of the methods above to the control and estimation of powertrains with backlash. Further details on control and estimation for the powertrain application will be given in Chapters 3 and 4.
1.4 Previous results

The published research results within the field of this thesis are rather sparse. The different fields under study are briefly listed here. More references to previous work are given in Chapters 3 and 4.

In the field of control of systems with backlash, most of the reported results are on systems where position or speed control is the focus, while acceleration control is the main focus in automotive powertrains. Typical applications are robot arm positioning and trajectory following in CNC-machines (Brandenburg and Schäfer, 1989; Tarng et al., 1997). Speed control is the focus in drive systems in e.g. paper machines and rolling mills (Dhaouadi et al., 1994).

In the literature on control of automotive powertrains, the importance of taking backlash into consideration is often emphasized, see e.g. (De La Salle et al., 1999; Jansz et al., 1999), but no explicit solutions are presented.

State estimation in systems with backlash is not frequently described in the literature, even for other applications than automotive powertrains. One example of existing results is (Schröder, 2000). General results concerning estimators for hybrid systems exist as well, see e.g. (Alessandri and Coletta, 2001).

Estimation of the backlash size has not been presented for automotive powertrains. There are some results for robot arms and paper machines (Hovland et al., 2002; Nordin et al., 2001). The book (Schröder, 2000) presents general results for systems with isolated non-linearities, based on neural networks.

1.5 Main contributions

The results presented in this thesis are all developed with the powertrain application in mind. The results should be applicable also to other mechanical systems where backlash cannot be neglected. This section lists the main contributions of the thesis.

• The thesis gives an overview of the possible methods for control of powertrains with backlash, see Chapter 3. Some of the methods are evaluated in Paper I and Paper III. When comparing different methods, it may be useful to know the theoretical limit for what may be achieved. Paper II presents a method to find the optimum, based on optimization of an open-loop control sequence.

• The theory for calculation of model-predictive control laws off-line is currently evolving. To evaluate the value of this theory, applications of the theory to realistic, non-trivial systems is of interest, and Paper III provides an example.

• For the design of advanced controllers for systems with backlash, it is often assumed that the backlash size is known. This is not always true in a real application.
The backlash may vary over time, due to e.g. wear or temperature. In Paper V, a new method for estimation of backlash size is presented, with the backlash size treated as a slowly varying parameter. In Paper VI, the developed estimator is experimentally validated.

- Also the dynamic state of the system is assumed measurable. When this is not the case, state estimators are used. Paper IV and Paper VI present a state estimator for systems with backlash and its experimental validation.

- The most common type of sensors in rotating systems is the pulse sensor. From the pulses, speed, and sometimes position, is often calculated based on the number of pulses in a given sampling period. In Paper IV, a linear estimator for fast and accurate estimation of the angular position of a rotating wheel is presented. It utilizes standard pulse sensors and is based on event based sampling.

- In Paper II, a Matlab environment for (open-loop) optimization of dynamic systems is presented. It is in turn based on existing solvers for constrained nonlinear optimization problems, which only handle static optimization.

### 1.6 Thesis outline

The thesis is organized as follows. Chapter 2 is aimed at presenting the models used for the controller and estimator designs. The controller results are summarized in Chapter 3. Chapter 4 gives an overview of the parameter and state estimators that have been developed. In Chapter 5, some general conclusions are drawn, together with suggestions for future work. The thesis ends with the appended papers, Paper I to Paper VI.
Chapter 2

Backlash and powertrain models

The controllers and estimators that are designed and analyzed in this thesis are based on dynamic models. Most of the models used for this are collected in this chapter, together with a discussion of the model choices that are made. The chapter is divided into three sections: Section 2.1 emphasizes the importance of using a correct model structure when analyzing systems with backlash. Section 2.2 describes different models for the backlash. Section 2.3 presents the complete powertrain models. Appendix A lists most of the symbols that are used in this thesis, together with representative numerical parameter values.

2.1 Backlash structures

Depending on how the backlash is connected to the surrounding parts, two principally different model structures can be identified: Backlash feed-through and backlash feedback systems. It is important to distinguish between these structures in order to understand the applicability of backlash control results in the literature.

2.1.1 Backlash feed-through systems

In backlash feed-through systems, the backlash has an input side and an output side, and the dynamics on the output side does not influence the dynamics on the input side, see the first system in Figure 2.1. This is sometimes referred to as a sandwiched backlash. In the second and third systems in the figure, only one of the dynamics is present. These systems are referred to as input backlash and output backlash systems respectively. Examples of these are systems with actuator or sensor backlash.

An example of a feed-through system is a position servo application, where the pilot valve movements represent the input dynamics, the moved load represents the output dynamics, and a backlash exists in the servo valve. The movements of the load do not influence the input side of the system.
Figure 2.1: Different types of feed-through backlash systems. BL represents the backlash. \( G_i \) and \( G_o \) represent dynamic systems, possibly smooth nonlinear, on the input and output side respectively.

Feed-through backlash is often modeled by a hysteresis model (Slotine and Li, 1991), where the current output position is a function of current and earlier input positions to the backlash. This implies that the model must include some sort of memory.

2.1.2 Backlash feedback systems

In a backlash feedback system, both sides of the backlash can influence the state of the backlash, see Figure 2.2. A powertrain is a good example of a system having this structure. \( G_i \) represents the dynamics on the engine side, including the rotating engine mass. \( G_o \) represents the rotating mass on the driven side, i.e. shafts, transmission and wheels. Apparently the shaft torque at the backlash influences both the dynamics on the "input" and "output" sides. Hence the feedback character of the system. The feedback backlash is described by models of the types presented in Section 2.2.

2.1.3 Discussion

From the description above, it is obvious that the feed-through structure is inappropriate for modeling of powertrain applications. It is important to make this distinction, since many control strategies reported in the literature are based on the feed-through structure. For this structure, it is common to include a backlash inverse model in the controller, see Chapter 3.3.3. Throughout this thesis, the backlash feedback structure is assumed.
2.2 Backlash models

Backlash can be modeled in many ways. Depending on the needed level of accuracy of the model, different choices can be made. This section describes some of the most common models. For a thorough survey of backlash models, see (Nordin et al., 1997).

The physical sources of backlash in the powertrain are described in Chapter 1. For the models used here, it is assumed that the backlash is connected to a flexible shaft, see Figure 2.3. It is therefore common to model the shaft and the backlash together. Note that if the shaft is free from inertia, the torques in all cross-sections of the shaft are equal. Also, the physical position of the backlash along the shaft is irrelevant.

2.2.1 Dead-zone model

One of the simplest and most widespread backlash models is the dead-zone model (Slotine and Li, 1991):

\[
T = \begin{cases} 
  k(\theta_d - \alpha) & \text{if } \theta_d > \alpha \\
  0 & \text{if } |\theta_d| < \alpha \\
  k(\theta_d + \alpha) & \text{if } \theta_d < \alpha 
\end{cases}
\]  

(2.1)

where \( T \) is the shaft torque, \( \theta_d \equiv \theta_1 - \theta_3 \) is the total displacement, \( \alpha \) is half the backlash gap and \( k \) is the shaft stiffness. In this model, the shaft is considered to be a torsional spring. The torque characteristics of this model is seen in Figure 2.4. The model (2.1) is used in the MPC-controller design, described in Section 3.3.3.
If the internal shaft damping shall be considered, a commonly used model is the following, see e.g. (Brandenburg et al., 1986; Nakayama et al., 2000) for applications:

\[
T = \begin{cases} 
  k(\theta_d - \alpha) + c\dot{\theta}_d & \text{if } \theta_d > \alpha \\
  0 & \text{if } |\theta_d| < \alpha \\
  k(\theta_d + \alpha) + c\dot{\theta}_d & \text{if } \theta_d < \alpha
\end{cases}
\]  

(2.2)

where \( c \) is the shaft damping. This model is not physically correct. Due to the damping term, \( c\dot{\theta}_d \), a fast rotation of the shaft "into" the backlash may result in a non-physical sign change of the torque while having contact on one side.

### 2.2.2 Simplified dead-zone models

The dead-zone model without damping, (2.1), can be simplified if the stiffness is very big (no compliance). The system will then switch between two distinct modes: In contact mode only one total mass exists, in backlash mode two unconnected rotating masses exist (Tao, 1999). If the system structure is of feed-through type (see Section 2.1), the backlash nonlinearity may be seen as a hysteresis. Two special cases are then of interest (Nordin and Gutman, 2002): In the friction driven hysteresis model, the output side is assumed to retain its position when the backlash is open. In the inertia driven model, the output side moves with constant speed when the backlash is open. A mathematical analysis of each mode in such a system is simplified, but it should be noted that many simulation tools will have a difficulty in simulating systems where the number of states vary (Otter et al., 1997).

### 2.2.3 Physical model

(Nordin et al., 1997) presents a physical model of the backlash, valid also for shafts with damping. This model includes one extra state variable, which makes it possible to model both the backlash angle and the remaining twist of the shaft. See Figure 2.3 for notation.
2.3 Powertrain models

The shaft torque is given by:

\[ T_s = k(\theta_d - \theta_b) + c(\omega_d - \omega_b) \]  

(2.3)

where \( \theta_d \equiv \theta_1 - \theta_3 \) is the total shaft displacement, and \( \theta_b \equiv \theta_2 - \theta_3 \) is the position in the backlash. With \( \alpha \) denoting half the backlash size, the backlash position is governed by the following dynamics:

\[
\dot{\theta}_b = \begin{cases} 
\max(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)) & \text{if } \theta_b = -\alpha \\
\dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b) & \text{if } |\theta_b| < \alpha \\
\min(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)) & \text{if } \theta_b = \alpha 
\end{cases}
\]  

(2.4)

Note that this model eliminates the unphysical sign changes which may occur in the dead-zone model (2.2). Another property of this model is that it is piecewise linear, while the other models are piecewise affine.

This model is used in all powertrain models in this thesis, except for the MPC-control design model.

2.3 Powertrain models

2.3.1 Basic powertrain model

The basic powertrain model used in this thesis, see Figure 2.5, has two rotating inertias. One inertia represents the engine flywheel (motor). The second inertia (load) represents the wheels and vehicle mass, transformed to a rotating mass. The moment of inertia for the load is thus \( J_l = 4J_{\text{wheel}} + r_{\text{wheel}}^2m_{\text{vehicle}} \). The engine flywheel is connected to a gearbox, which in turn is connected to the load inertia via a shaft with backlash according
to (2.3-2.4). The equations for the inertias and the gearbox are defined by:

\[
\begin{align*}
J_m \dot{\theta}_m + b_m \dot{\theta}_m &= T_m - T_g \\
J_l \dot{\theta}_l + b_l \dot{\theta}_l &= T_s - T_l \\
T_g &= T_s / i, \theta_3 = \theta_l, \theta_1 = \theta_m / i
\end{align*}
\] (2.5)

The engine dynamics, except for the flywheel inertia, is modeled as a linear first order system, with time constant $\tau_{\text{eng}}$ and time delay $L_e$. The input $u$ represents the requested engine torque:

\[
\dot{T}_m(t) = (u(t - L_e) - T_m(t)) / \tau_{\text{eng}}
\] (2.8)

The external forces acting on the vehicle are rolling resistance, aerodynamic drag and climbing resistance. These forces are lumped together into a load force, $F_l$, and converted to the load torque, $T_l = r_{\text{wheel}} F_l$, acting on the load inertia, $J_l$ in (2.6). For the control design, this is treated as a disturbance input to the system. For the estimators in Chapter 4, this disturbance is modeled as a random walk process, driven by white noise.

### 2.3.2 Simplified design model

For the design of MPC controllers, see Section 3.3.3, which is very computationally demanding, a simplified powertrain model is used. Here, the simple dead-zone backlash model in Section 2.2.1 is used, together with the powertrain model in Section 2.3.1. Instead of modeling the road load as an external disturbance, $T_l$, the rotational friction, $b_l$ in (2.6) is increased, to model the road load. The engine delay, $L_e$, is also neglected. Using this model, the number of states is reduced from 6 to 4.

### 2.4 Discussion

The powertrain model described above is very simple. Some of the simplifications and potential improvements are discussed here.
2.4 Discussion

An engine, either spark-ignited (Otto) or compression-ignited (diesel), is a considerably nonlinear system. Much of the dynamics is dependent on the engine speed. The delay is also dependent on the current rotational angle in the combustion cycle in the cylinders. To catch all these phenomena, a much more complex model would be required, and such models exist, see e.g. (Karlsson and Fredriksson, 1999). The focus of this thesis is however backlash control in a general set-up. To study different ways to control the powertrain for this application, it is important to keep the engine model simple. For example, a simple model allows for the derivation of analytical solutions to some problems.

An engine can have a number of different actuators. Depending on the available actuators on a specific engine, the overall engine dynamics will vary a lot, which is also a reason for keeping the engine model very simple. A non exhaustive list of actuators for control of combustion engines includes:

- Spark-ignited engines: The throttle, which controls the air flow into the engine; Spark advance, controls the ignition time in each cylinder.
- Compression-ignited engines: Injection timing and injection amount for each cylinder.
- Turbo engines: The waste gate and Variable Geometry Turbine, VGT, which control the turbo pressure.
- In addition, an Integrated Starter Generator, ISG, is a potential means of adjusting the engine torque. The ISG is much faster in comparison with the other actuators in this list. This makes it very attractive for use in fast torque control applications, such as in backlash control.

In heavy vehicles, there are a number of actuators for braking the vehicle: Foundation (wheel) brakes; retarders, acting on different positions in the powertrain; and exhaust gas engine brakes, influencing the engine’s delivered torque. All these actuators influence the powertrain dynamics, and should be included in a more detailed powertrain model.

Wheel slip is ignored in the model used here. The jerk that the driver feels is damped through the slip, and thus the jerk levels that the model predicts are irrelevant for the purpose of quantifying the driveability of a control design. This is probably one of the biggest limitations of the model. However, for comparisons of different simulations, this jerk is still usable.

In the model used here, the engine is considered to be stiffly mounted to the engine bay. A real engine is flexibly suspended, which allows it to rotate a few degrees in response to changes in engine torque. A powertrain model including the engine suspension would be a three-mass model, with the third mass representing the engine block, and the engine suspension as an extra flexibility, possibly with additional backlash. The same reasoning applies to some extent also to the wheel and shaft suspensions.
Chapter 3

Control of systems with backlash

As described in Chapter 1, control system design for a powertrain with backlash is a trade off between obtaining a fast system response and avoiding impacts when the backlash is closed. How this trade off is made has influence on the vehicle’s driveability.

This chapter will first give an overview of current practice in the automotive industry. Then an overview of available methods for backlash control in general is given, with focus on the powertrain application.

The controllers presented in this chapter assume that all needed variables are measurable, and that the backlash size is known. Chapter 4 discusses how estimators can be used to alleviate these assumptions.

3.1 Driveability aspects of backlash control

The concept of driveability is introduced in Section 1.2. In this thesis, driveability problems related to backlash are in focus. In Figure 1.2, measurements of acceleration during tip-in on a car without backlash compensation control are seen. Using the model in Section 2.3.1, an open-loop step response is seen in Figure 3.1. By the use of a controller, the tip-in response can be modified. The performance goals are to minimize the jerk at the backlash contact instant, and simultaneously minimize the acceleration response time.

Existing engine control systems employ relatively simple strategies for backlash compensation. The control system may even lack any explicit function for backlash compensation. The engine torque controller is then tuned to be slow enough, so that the backlash is not excited too much. A somewhat more sophisticated solution is to detect situations when the sign of the requested torque is changing. At these instances, the engine torque is first controlled to a low torque of opposite sign. After a pre-specified time, long enough to ensure that the backlash has been traversed, full torque demand is allowed, see Figure 3.2. Even for a backlash compensation strategy such as this, no feedback of the backlash state is used. Neither is any information about the backlash size used, more than implicitly in the tuning process of the control system.
Figure 3.1: Step in requested engine torque for the powertrain model. The upper plot shows the vehicle acceleration, together with the driver’s desired acceleration. The middle plot shows the engine’s requested (solid) and delivered (dashed) torque, and the lower plot shows the vehicle jerk. Note that since the powertrain model does not include wheel slip, this jerk will not be equal to the driver’s jerk experience. It is however useful for comparison of different simulation results.

3.2 Performance limitations and open-loop control

The ultimate solution to the problem of performing a backlash traverse in minimum time, and with constraints on jerk etc. can be found by optimization methods (Bryson, 1999). The optimization problem can be formulated as finding either an optimal open-loop control sequence, or an optimal feedback control law.

The open-loop solution is easier to find, but not very useful as a controller for the real system, since it will have no robustness to model errors, disturbances and errors in initial conditions. The solution can however be used as an indicator for the best possible performance of a feedback controller. A preliminary investigation is presented in (Karlsson, 2001), where the backlash is traversed in minimum time, with constraints on allowed jerk at the contact instant.

Contributions in the thesis

In Paper II, optimization is applied to the powertrain model used in this thesis. The results, see Figure 3.3, show the following:

The backlash can be traversed in approximately 0.17 s without causing any jerk at the contact instant. The time to reach the desired acceleration and reduce all driveline oscillations...
3.3 Feedback strategies for backlash control

When a feedback controller for backlash shall be chosen, a large number of methods are available. This section reviews some of the methods described in literature for the control of systems with backlash. The results presented in the appended papers are briefly commented in this review.

Most of the literature on backlash control focuses on speed or position control. In powertrain applications of backlash control, it is more important with torque (or acceleration) control. For general systems, there is often stick-slip friction in the system in combination with the backlash. Since the powertrain is always rotating, this phenomenon does not need as much attention here. These facts, together with the difference between the

![Figure 3.2: A "state of the art" backlash traverse. The engine torque is kept at a small positive value until \( t_{\text{wait}} \)=0.25 s is reached. The backlash is expected to be traversed during this time, and the torque is then allowed to approach the desired value. In a practical implementation, oscillation control functionality is added to the result depicted here. See Figure 3.1 for notation.](image-url)
Figure 3.3: (Paper II, Fig. 4) Open-loop optimal control of the backlash traverse. The system starts at constant retardation, and ends at a desired acceleration. At the contact instant, indicated by the vertical line at 0.17 s, the relative speed between engine and vehicle is zero, to avoid shunt. In the engine torque plot, the solid line is the control signal (requested torque), and the dashed line is the engine’s delivered torque, $T_m$. In the speed plot, the solid line is the engine speed, and the dashed line is the wheel speed, both transformed to equivalent vehicle speeds, $\omega_m / i \cdot r_{\text{wheel}}$ and $\omega_l \cdot r_{\text{wheel}}$ respectively.
feedback and feed-through backlash structures, see Section 2.1, should be kept in mind when considering the controllers found in literature.

In many applications, e.g. automotive powertrains, the backlash adds complexity to an already difficult control problem. The powertrain consists of two or more rotating masses, connected by flexible shafts. This system may be a challenge to control also without the backlash, see e.g. (Mo et al., 1996; Chen, 1997; Fredriksson et al., 2002). The importance of taking backlash into consideration in powertrain control is demonstrated e.g. in (De La Salle et al., 1999). When backlash is included, the control problem becomes more complex. For a survey on control strategies for mechanical systems in general, both with and without backlash, see (Nordin and Gutman, 2002).

Most controllers for systems with backlash can be put into one of the following groups:

**Linear backlash control**, where the controller must be robust enough to handle the nonlinearity.

**Nonlinear passive control**, which takes the backlash into account, with the philosophy of being cautious in the backlash gap.

**Nonlinear active control**, which also takes the backlash into account, but with the philosophy of quickly getting out of the backlash, but in a controlled manner.

In each group, there may be both feedback controllers and controllers of more or less feed-forward or open-loop character. The three groups are discussed in the following subsections.

### 3.3.1 Linear backlash control

Linear controllers for systems with backlash must be robust enough to handle this nonlinearity. A few different approaches have been reported in literature.

The most straightforward control method for systems with backlash is to use a basic PID-controller that is tuned not to excite oscillating modes originating from the backlash or the shaft flexibility (Hori et al., 1994; Nakayama et al., 2000).

If all states are measurable, a state feedback controller can be used. If load or motor signals are measured, the state feedback can be complemented with a state observer. An application of the latter is presented in (Hori et al., 1994), where motor speed is measured.

A linear controller can be designed using some of the existing robust design methods. The backlash nonlinearity is modeled as a suitable uncertainty, fitting into the used design method’s framework. Examples of robust design methods are $H_\infty$ (Green and Limebeer, 1995) and Quantitative Feedback Theory, QFT, (Horowitz, 1991). Applications of QFT synthesis to backlash systems are presented in (Yang, 1992; Nordin and Gutman, 1995).

A frequently used approach in handling backlash is to treat it as a torque disturbance, which enters the system at the location of the backlash. The system is then treated as
linear, and the controller is designed to compensate for the torque disturbance instead of the backlash, see Figure 3.4.

With this system model, a torque observer can be used to estimate the shaft torque. The torque is then compensated for in an inner feedback loop. In an outer loop, a PID or other controller is used to control a "backlash-free" system, see Figure 3.4. See e.g. (Brandenburg, 1987; Brandenburg and Schäfer, 1987; Odai and Hori, 1998; Nakayama et al., 2000; Shahruz, 2000; Dhaouadi et al., 1994).

**Contributions in the thesis**

In Paper I, a simple PID controller is used to control the powertrain. The PID controller is also complemented with a torque compensator as described above. In Figure 3.5, the results for the controller with torque compensator are seen. The parameters of both controllers are optimized for minimum rise time, with a constraint on the allowed jerk at the contact instant. The PID controller is not performing well. Although the torque observer is too slow to follow the torque variations during the backlash traverse, the torque compensated controller has better performance than the PID controller. This is explained by the use of extra measured signals: engine torque and engine speed, and an extra tuning parameter. Two nonlinear controllers are also evaluated, (see Section 3.3.3), and in comparison, the two linear controllers are more robust to model uncertainties.

**3.3.2 Nonlinear passive control**

A nonlinear controller can be designed to become more careful when the backlash gap is open, or close to open, in order to avoid collisions and oscillations. Most controllers in this group consist of two separately designed controllers, often linear: One is designed for the system in backlash mode, the other for contact mode. The overall controller is then switching between the two controllers according to some switching scheme.
3.3 Feedback strategies for backlash control

Figure 3.5: (Paper I, Fig. 3) A PID controller with torque compensator. The same notation as in Figure 3.1 is used. The maximum jerk at contact is approximately equal to the result in Figure 3.2.

Switching between two state feedback control laws, based on the backlash angle, is described in (Friedland, 1997).

In (Nordin, 2000, Paper E), a continuous gain scheduling is used between two linear controllers designed by QFT. The controller’s output signal is used as scheduling variable. The linear controller used at low torques is tuned to be cautious so that no oscillations arise.

The "sophisticated" method used in practice, as described in Section 3.1, can be characterized as a nonlinear passive controller. During the backlash traverse, the engine torque is controlled to a low value, to avoid a large contact jerk. Note that this is an open loop controller with respect to the backlash traverse.

3.3.3 Nonlinear active control

When the backlash gap is entered, the nonlinear active controllers try to get the system back to contact mode, without generating a collision at the contact instant. The controllers can be divided into subgroups:

Inverting control, where the controllers include an explicit inverse model corresponding to the nonlinearity in the system. Many results exist for this group, e.g. (Tao and Kokotovic, 1996; Ma and Tao, 2000; Taware and Tao, 2000). However, inversion of the backlash is only meaningful for the feed-through backlash structure, see Section 2.1, and is not further discussed here.
Switching control, where the controllers detect when the backlash mode is entered, and try to quickly get back to contact mode. Note the difference between active and passive switching controllers: The passive ones get more cautious when the backlash mode is entered.

Optimal control, in which the backlash traverse is optimized e.g. to be made in minimum time. The structure of these controllers is however not of an explicit switching character.

The Switching and Optimal control groups are discussed in the following.

Switching control

The principal idea in active switching is that two different controllers are designed, possibly using different design methods. The contact mode controller has the task of controlling the acceleration of the system, and should, in a powertrain controller, take care of the driveline oscillations. The backlash mode controller has the task of traversing the backlash as fast as possible, with a "soft landing" on the appropriate side. Some examples of results belonging to this group are presented here.

In (Ezal et al., 1997), a linear contact controller is combined with an optimal controller for the backlash traverse. The system under consideration is assumed to have a shaft without flexibility, and no actuator dynamics. The results are further developed in e.g. (Tao et al., 2001). In (Tao, 1999), switching is made between two linear controllers, designed by pole-placement. The same geartrain assumptions as in (Ezal et al., 1997) are used, which yields a simple controller structure.

Two cascaded QFT-designed linear controllers are used in (Boneh and Yaniv, 1999). In contact mode, the inner loop controller is bypassed. In backlash mode, this controller is positioning the motor for smooth contact with the load, while the outer loop gives the desired direction of backlash closure.

Optimal control

In the switching control group above, it is possible to use an optimal control law in each of the modes. In contrast, the optimal control group considers the backlash traverse as a more "global" optimization problem. The overall goal of this controller is to traverse the backlash in minimum time, with constraints on e.g. jerk at the contact instant.

In this setting, the system modes (backlash and contact) do not directly translate into different modes of the controller, they merely represent different dynamics during the backlash traverse. The optimal control group is the natural feedback control solution corresponding to the optimal open-loop controllers, discussed in Section 3.2.

Model Predictive Control, MPC, naturally fits into this group. MPC (Maciejowski, 2002; Mayne et al., 2000) is a method for combining optimal control calculations with feedback. In each sampling interval of the controller, a new optimal open-loop solution is calculated,
3.3 Feedback strategies for backlash control

for a specified number of samples ahead. This implies a large computational burden on the controller, and traditional MPC is mostly used in processes with very slow dynamics, such as in the process and petrochemical industries. MPC is recently used also in automotive applications.

The MPC theory is recently made attractive for implementation also in systems with fast dynamics. For linear and piecewise affine systems, it is possible to perform an off-line calculation of an optimal feedback control law that satisfies given constraints. See (Morari et al., 2003) for an introduction. This calculation can be extremely intense. However, the on-line implementation is only a matter of table look-up for the actual control law.

Contributions in the thesis

As an example of active switching control, the controller proposed in (Tao, 1999) (discussed above), is in Paper I applied to the powertrain model, see Section 2.3.1. A modification of this switching controller is also used: For the backlash mode, the pole-placement controller is re-derived for the system including engine dynamics. For the contact mode, the PID controller with torque compensator, see Section 3.3.1, is used. The same parameter optimization criteria as for the linear controllers in the paper are used.

The results, see Figure 3.6, show that the active switching controllers have potential to improve the performance of the vehicle, as compared to the linear controllers. The requested acceleration is reached slightly slower, but the backlash is traversed faster. This means that the vehicle will start to accelerate faster, which is more important from a driveability point of view. The switching controllers are however more sensitive to errors in the backlash size. On-line size estimation, described in Section 4.2, is suggested to remedy this.

Paper III describes an application of MPC to the backlash compensation problem, see Figure 3.7. The powertrain is controlled to a desired acceleration, and the driveline oscillations are reduced. The system is under MPC control between 0.04 and 0.32 s, when the acceleration target is reached. Then a linear controller, dedicated to "normal" acceleration control in contact, is used. At 0.17 s, the backlash is traversed, and contact is achieved with no relative speed between engine and wheels.

The MPC controller behaves similarly to the open-loop results in Figure 3.3 and Paper II, which shows that a feedback controller of MPC type can come close to the theoretical open-loop limit.

If compared to the feedback control results in Figures 3.5 and 3.6, it should be noted that the engine delay is neglected in the MPC control model. Otherwise, the models are similar, see Section 2.3.2.
**Figure 3.6:** (Paper I, Fig. 5) An active switching controller. In contact mode, a PID controller with torque compensator is used for acceleration control of the vehicle. In backlash mode, a pole-placement controller is used for positioning of the backlash sides at each other. See Figure 3.1 for notation. The maximum jerk at contact is approximately equal to the result in Figure 3.2.

**Figure 3.7:** (Paper III, Fig. 7) Simulation of a backlash traverse using MPC control. Upper plot: Engine speed, $\omega_m$, (solid) and wheel speed, $\omega_l$, (dashed), both scaled to vehicle speed. Middle plot: Total shaft displacement. ($\theta_d = 0.1$ rad corresponds to a chassis acceleration of 1.5 m/s$^2$ in the previous plots in this section.) The backlash limits ($\pm \alpha$) are indicated by dashed horizontal lines. Lower plot: Control signal, requested engine torque, $u$ (solid), and engine torque, $T_m$, (dashed).
3.4 Discussion

Various feedback strategies for backlash control are presented in the previous section. Here, a few conclusions are drawn.

Since the active nonlinear controllers seem to be the least investigated for powertrain applications, and have most potential for achieving good system performance, this group is the focus among the controller strategies used in this thesis. Both the results in Paper I and Paper III indicate that there is a risk that these methods are too sensitive to model errors and noisy signals. It should therefore be further studied whether the active controllers are robust enough for the powertrain application. A passive or linear approach would be more robust, but possibly at the cost of lower performance.

For switching controllers, the switching itself is not trivial. In Paper I, the controller switches mode at the instants when the backlash is entered or exited. It is not certain that this is the optimal choice. It may be better to switch as soon as the driver’s requested torque changes sign, or at some other instant. The use of optimal controllers may give advice in this issue, as discussed in Section 3.2.

In Paper I, it is observed that the control signal changes abruptly at the switching instants, which is needed in order to get a fast response. This is contrary to general recommendations for the practical implementation of control systems. There, switching between manual and automatic mode, or between different controller parameters in a gain-scheduled controller, should be made without discontinuities in the control signal. For a controller with integral action, it is possible to choose the mode-switch characteristics. How this choice should be made requires further investigations.

The stability of each mode of the switched controllers is possible to analyze, and for some of the used controllers, stability is guaranteed by the synthesis method. This however does not automatically lead to a globally stable system. Analysis methods for hybrid systems could be used here, but the stability check is often made by means of computer simulations of the system.

The optimal controllers are attractive for further study. The experience from Paper III is that the computational load of the synthesis methods is still significant when applied to realistic systems. There are also limitations in the class of systems and constraints that can be described.

More realistic engine and powertrain models are needed for the controller designs. A trade-off has to be made here between using detailed models as discussed in Section 2.4, and models that are simple enough to use e.g. in MPC controller synthesis.

The results presented in this chapter assume that the system state is measurable and that the backlash size is known. Next chapter discusses how these variables can be estimated when they are not directly available. When the estimator dynamics is included in the loop, it may be necessary to retune the controller parameters and the system performance may degrade. Chapter 5 discusses these issues further, and gives an example of a closed-loop system with controller and estimators.
Chapter 4

Estimation of systems with backlash

In order to design a high performance control system for backlash compensation, information about both the backlash size and the current state of the system is important. Most of these quantities are not directly measurable. This chapter gives an overview of available methods for getting this information, including those presented in the appended papers. The size and state estimation problems are treated separately in the following sections.

In an estimator implementation, the size and state estimators will be running simultaneously, and should be interconnected as in Figure 4.1. There, also prefilters described in Section 4.1.3 are shown.

4.1 State estimation

4.1.1 Background

Not much is written on the problem of estimating the state of a rotating system with backlash. Results for systems with feed-through backlash structure (see Section 2.1), are treated e.g. in (Tao and Kokotovic, 1996). This structure is however not relevant for backlash in powertrains. Estimators for systems with isolated static nonlinearities are presented in (Schröder, 2000). The nonlinearities are approximated by neural networks. Also the feedback backlash structure fits in this framework.

The state estimator presented in this thesis is an extension of the linear estimator along the following lines:

The general method for state estimation is to use an observer for the dynamic system. The observer contains a model of the system, and the known inputs to the real system are also fed into the model. The measured outputs from the real system are compared to their corresponding model outputs. The difference between these is used to adjust the state variable values of the model.
For linear systems, the optimal observer is the Kalman filter (Anderson and Moore, 1990). It is optimal in the sense that for Gaussian noise disturbances in states and measurements, with given covariances, the mean square of the estimation error is minimized by this filter.

For nonlinear systems, a well established observer is the Extended Kalman filter, EKF. As the name indicates, this is an extension of the linear Kalman filter. The model inputs and outputs are used as in the linear case, and the state variable adjustments are calculated based on a linearization of the nonlinear model. It should however be pointed out that there is no general optimality guarantee for the EKF.

The EKF approach is applied to the powertrain application, using the physical backlash model (2.3-2.4), and with the remaining dynamics being linear, see Section 2.3.1. It then turns out that due to the structure of the nonlinearity, the system only switches between two linear modes. This fact is further exploited in Section 4.1.2 and in Paper IV.

A class of nonlinear systems, hybrid systems, is defined as systems that switch between different discrete modes. In each mode, the dynamics are continuously evolving. At discrete events, triggered by external signals or by the continuous dynamics, the mode is switched. A special class of hybrid systems is the piece-wise linear systems. State estimation for piece-wise linear systems is described e.g. in (Alessandri and Coletta, 2001) for the case when the discrete mode is known. For the case when the mode is unknown, results exist for special cases. In (Juloski et al., 2002), an observer for a model structure similar to the one under investigation here is presented.

The observation that the powertrain model in Section 2.3.1 is a piece-wise linear system makes it possible to apply hybrid system theory to the design and stability analysis of the suggested state estimator. This approach is not pursued further here. Stability is merely indicated by means of simulation.
4.1 State estimation

4.1.2 State estimator design

One of the contributions of this thesis is the state estimator described in Paper IV.\(^1\) This subsection summarizes the derivation of the estimator.

Available sensors

The sensors that are normally available for measurements on a powertrain are the speed sensors on the wheels, used by e.g. the ABS functions (Antilock Brake System). There is also a speed sensor on the engine flywheel. The control signal (requested engine torque) is available from the engine control system. Depending on type of engine control system, various estimates of the engine’s delivered torque may be available, but these are not used for the derivation of the estimator under study here.

Each speed sensor consists of a toothed wheel, which rotates with the same speed as the wheel. A magnetic or optic pick-up senses the passage of the teeth, and generates an oscillating signal with one period per tooth passage. By use of e.g. a Schmidt-trigger, this signal can be transformed into a pulse train. In the ABS control system, this signal is filtered and used for speed estimation. Here, it is assumed that the pulsating signal is available directly, and counted. The current angular position of a wheel or the engine flywheel can then be calculated as:

\[ \theta_i = \frac{2\pi}{N_i} \# \text{pulses}, i \in \{l, m\} \]

(4.1)

where \(N_i\) is the number of teeth per revolution and \(\# \text{pulses}, i\) is the value of the corresponding pulse counter. \(l\) and \(m\) represent the wheel (load) and flywheel (motor) respectively.

In order to increase the quality of these measurement signals, the pre-filter described in Section 4.1.3 may be used.

Estimation model

If the powertrain model in Section 2.3.1 is analyzed, it is seen that the nonlinearity only consists of a switch between two linear modes, called contact mode, (co), and backlash mode, (bl). The mode switches depend on the conditions in the backlash model (2.4). From this model, it is seen that the backlash mode is entered as soon as the relative speed, \(\dot{\theta}_d\), into the backlash is high enough, even if the total shaft displacement, \(\theta_d\), is larger than the backlash width. Due to the switching structure, the powertrain model can be written on state-space form:

\[
\begin{align*}
\dot{x} &= \begin{cases} 
A_{co}x + Bu + v, & \text{co-mode} \\
A_{bl}x + Bu + v, & \text{bl-mode}
\end{cases} \\
y &= Cx + w
\end{align*}
\]

(4.2) (4.3)

\(^1\)In Paper IV to Paper VI, this estimator is referred to as a backlash position estimator. This is somewhat restrictive, since all states of the system are actually estimated.
where the following state and measurement vectors are used:

\[
    x = \begin{bmatrix}
        \theta_m & \omega_m & \theta_l & \omega_l & T_l & T_m & \theta_b
    \end{bmatrix}^T \tag{4.4}
\]

\[
    y = \begin{bmatrix}
        \theta_m & \theta_l
    \end{bmatrix}^T \tag{4.5}
\]

and \(v, w\) are the state and measurement noise vectors. The \(A, B,\) and \(C\)-matrices are presented in Paper IV.

**State estimator**

For the model above, an Extended Kalman filter, EKF, is derived. Since the model is switching, also the EKF will be switched:

\[
    \dot{\hat{x}} = \begin{cases}
        A_{co}\hat{x} + Bu + K_{co}(y - C\hat{x}), & \text{co-mode} \\
        A_{bl}\hat{x} + Bu + K_{bl}(y - C\hat{x}), & \text{bl-mode}
    \end{cases} \tag{4.6}
\]

\[
    \hat{y} = C\hat{x} \tag{4.7}
\]

where \(K_{co}\) and \(K_{bl}\) are stationary Kalman filter gains, designed for their respective modes. Note that these gains do not guarantee any optimality of the switched estimator. However, since no general optimality of an EKF exists, it is a choice as good as any, and the simulations and experimental results below indicate that the estimator is stable and performs satisfactorily.

The control signal is delayed with the same time delay as in the engine model, \(u(t) = u_{\text{control}}(t - L_e)\). The mode switches use the same conditions as described above, but based on the estimated signals.

Note that in the estimator, the backlash size is assumed to be known. In a complete system, see Figure 4.1, it is given by the backlash size estimator in Section 4.2. Since the size is only entering the equations via the switching conditions, it does not influence the design of the Kalman gains.

**Results**

In Paper IV, it is shown that the described state estimator has good tracking performance. As an example, a simulation of the estimated shaft displacement, \(\hat{\theta}_d\), is seen in Figure 4.2. In Paper VI, more details are given on an experimental validation of the estimator in data from a real vehicle.

**4.1.3 State estimation for a wheel with pulse measurements**

To increase the performance of the state estimator, it is possible to introduce a prefilter on the signals from the pulse sensors on the engine flywheel and the wheel. The prefilter estimates the angular position, velocity and acceleration of the individual wheel. In addition to the use in backlash applications, the signals from these filters may also be used e.g. for control of driveline oscillations.
4.1 State estimation

The "standard" use of pulse sensors for speed measurement is to count the number of pulses during a fixed sampling interval, and then calculate the speed. This approach is too slow for the current application. Therefore, an event based Kalman filter is presented in Paper IV.

In short, the event based method works as follows: A continuous model is used for prediction of the estimated states. When a sensor pulse arrives, a discrete update of the states is made. The covariance matrix, $P$, is updated in the same fashion. This is in contrast to the discrete-time infinite horizon filter, where the covariance matrix, and hence Kalman gain, is constant. Figure 4.3 shows a filter startup.
Figure 4.3: (Paper IV, Fig. 5) Prediction and measurement updates for the continuous-discrete filter. The plot shows a startup of the filter at $t = 35$ s. Dashed: Continuous quantized signal. $\ast$: Pulse measurements, used in this filter. $\triangle$: Discrete measurement updates. Solid: Predicted-updated estimate.

4.2 Size estimation

4.2.1 Background

The problem of backlash size estimation has been reported in a few different settings, most of which are offline methods, where specially designed experiments are needed to gather information about the backlash. In (Hovland et al., 2002), backlash size is estimated for an industrial robot arm. An offline method is described, where parameter estimation yields shaft torque as function of shaft displacement, i.e. a plot of the shaft stiffness and backlash characteristics. In (Nordin et al., 2001), a method suitable for rotating systems is described, in which a sinusoid is added to the control signal. A gap size parameter is then adjusted until the gains of a model plant and the physical plant coincide. More examples of size estimation in servo applications include (O’Donovan et al., 2004; Stein and Wang, 1996; Gebler and Holtz, 1998). In (Schröder, 2000), identification of isolated nonlinearities is presented, based on neural network models of the nonlinearity.

For the feed-through backlash structure, (see Section 2.1), backlash estimation is included in adaptive control schemes for systems with hard nonlinearities in (Tao and Kokotovic, 1996).

In the automotive powertrain application, the backlash size varies between vehicle individuals and with wear, but also with temperature. Therefore, the size estimator should be running continuously, and hence no experiments as described above are allowed. The
4.2 Size estimation

only excitation needed for the estimators presented in the following subsection is normal driving.

If the flexible engine suspension is considered as part of the powertrain backlash, the temperature dependence becomes evident. This flexibility is not explicitly taken into account in the powertrain model used here, but it is possible to extend the model, and hence the estimator, to include also this.

4.2.2 Size estimator design

This subsection summarizes the backlash size estimator, presented in Paper V. In the automotive powertrain, backlash and shaft flexibility contribute to an angular position difference between engine and wheels. The principle of the size estimator is to use a linear Kalman filter to estimate the shaft twist from a powertrain model without backlash. In this model, offset parameters are introduced in the measurement of angular position difference between engine and wheels. By estimation of one parameter for positive contact, and one for negative contact, the backlash size can then be calculated.

Available sensors

The input signals used by the state estimator, Section 4.1.2, are available also for the size estimator. Due to the lower bandwidth requirements of the size estimator, the pre-filter, Section 4.1.3, is not required, however.

Estimation model

In the estimation model, the powertrain model from Section 2.3.1 is used. However, the shaft is modeled as free from backlash, so instead of the backlash model (2.3-2.4), the following shaft model is used:

\[ T_s = k(\theta_1 - \theta_3) + c(\omega_1 - \omega_3) \]  (4.8)

The measurement signal from the wheel sensor is assumed to have an offset. The offset value is unknown, but assumed to change value when the backlash is traversed:

\[ \theta_{l,\text{meas}} = \begin{cases} 
\theta_l - \theta_{o+}, & \text{positive contact} \\
\theta_l - \theta_{o-}, & \text{negative contact} 
\end{cases} \]  (4.9)

The offset parameters are modeled as random walk processes with \( v_{\theta_o} \) white noise:

\[ \dot{\theta}_{o+} = 0 + v_{\theta_o} \]  (4.10)
\[ \dot{\theta}_{o-} = 0 + v_{\theta_o} \]  (4.11)
In this way the parameters can be augmented to the system state vector, making it possible to estimate their values. On state-space form the model becomes:

$$
\dot{x} = A^*x + B^*u + v \tag{4.12}
$$

$$
y = \begin{cases} 
C_+ x + w, & \text{positive contact} \\
C_- x + w, & \text{negative contact}
\end{cases} \tag{4.13}
$$

where the state and measurement vectors are:

$$
x = \begin{bmatrix} \theta_m & \omega_m & \theta_l & \omega_l & T_l & T_m & \theta_{o+} & \theta_{o-} \end{bmatrix}^T \tag{4.14}
$$

$$
y = \begin{bmatrix} \theta_{m,\text{meas}} & \theta_{l,\text{meas}} \end{bmatrix}^T \tag{4.15}
$$

and $v, w$ are the state and measurement noise vectors. The $A, B,$ and $C$-matrices are found in Paper V.

**Size estimator**

For each of the modes, a linear Kalman filter gain is calculated, and the following estimator is constructed:

$$
\dot{\hat{x}} = A^*\hat{x} + B^*u + K^*(y - C^*\hat{x}) \tag{4.16}
$$

$$
\hat{\alpha} = (\hat{\theta}_{o+} - \hat{\theta}_{o-})/2 \tag{4.17}
$$

The matrices $A^*, B^*, C^*$ and $K^*$ are found in Paper V. $C^*$ and $K^*$ switch according to the estimated mode of the system. In the two contact modes, the system states and the corresponding offset parameter are estimated. During backlash traverse, no offset parameter is estimated.

**Results**

In Paper V, it is shown that the size estimate obtained with this estimator is of good quality. It is however sensitive to errors in the shaft stiffness. In Paper VI, more details are given on an experimental validation of the estimator on data from a real vehicle. See also Figure 4.4.

The combination of state and size estimators, shown in Figure 4.1, is also evaluated in Paper V, see Figure 4.5. The size estimator is initiated with a small backlash size. After a few backlash traverses, the estimate has converged to the true value. From this point, the state estimator behaves as in Section 4.1.2.
4.2 Size estimation

Figure 4.4: (Paper VI, Fig. 3) Backlash size estimate from measurements on a passenger car, $\hat{\alpha} \text{ [rad]}$. The car is making continuous tip-in and tip-out maneuvers in order to excite the estimator. A "settled" sequence of $\hat{\alpha}$ is shown in the top left corner.

Figure 4.5: (Paper V, Fig. 7) Backlash size and position estimates during a filter startup. Solid: Backlash position estimate, $\hat{\theta}_b \text{ [rad]}$. Dash-dotted: Backlash size estimate, $\pm \hat{\alpha} \text{ [rad]}$. Dashed: True backlash size, $\pm \alpha \text{ [rad]}$. 
4.3 Discussion

The state and size estimators presented here are shown to perform well in both simulations and experiments. Although not formally proved, stability does not seem to be a problem. Section 4.1.1 discusses how stability can be proved by use of hybrid system theory. Most results, like (Alessandri and Coletta, 2001), assume that the system mode (backlash or contact) for the true system is known, which is not the case here. Proof of stability when only an estimate of the mode is available is suggested for future research.

In Section 2.4, the addition of a model for the engine suspension is discussed. It would be quite straightforward to extend the estimators to also include the engine suspension. This will lead to better estimates, but will require information on the stiffness of the engine mountings. This stiffness is both nonlinear and highly temperature dependent, which makes it a challenge to capture these phenomena in the model.

The time resolution of the sensor measurements for the experimental validation, Paper VI, is very high. Disturbances are then added to evaluate the robustness. A thorough investigation of the required sampling rates should be made.
Chapter 5

Conclusions

This thesis covers several aspects of backlash in automotive powertrains. Motivated by the need for improved driveability, results on both control and estimation of systems with backlash are presented.

In Chapter 3, a survey of existing control strategies for systems with backlash is presented. The conclusion from this survey is that many results are not relevant to the powertrain application. There are three main differences to the general backlash problem: The powertrain has the backlash in a feedback structure; The controlled variable is the acceleration rather than the speed or position; Stick-slip friction does not need special attention.

To find the theoretical performance limits of a given powertrain configuration, open-loop optimization is used. Results of such studies can be used to get an indication of whether a more advanced control strategy is worthwhile, or if an existing strategy is already close to the limit. Paper II and Paper III give an example of this type of analysis. Open-loop optimization can also be used to give indications on how a feedback control strategy should be configured, e.g. with respect to switching conditions.

A number of different feedback control strategies are evaluated: Linear controllers, switched active controllers and optimal controllers. The results show that there is a potential for improved performance with more advanced controllers. These controllers will however require high quality state estimates. The lower robustness of these controllers is also a potential drawback.

In Chapter 4, a state estimator is presented, which gives information about the current state of the powertrain. For the suggested control strategies, this is a necessary prerequisite. In particular, the possibility to keep track of the relative position of the sides of the backlash gap during a backlash traverse is important. It is equally important to have accurate information about the size of the backlash. For this purpose, also a size estimator is presented here.

It should be emphasized that the backlash estimators may be used also for other applications than advanced backlash control. One example is for diagnosis of component wear in the driveline, leading to larger backlash. The size estimate may also be combined with
adaptation (or gain-scheduling) of a simple backlash compensation controller, to make it more robust to backlash size variations.

The simulations and experiments presented here all relate to passenger cars. Most of the results are however equally applicable to heavy trucks and to other vehicles where backlash in the powertrain gives driveability problems.

## 5.1 Suggestions for future work

The discussions in Chapters 2 to 4 present some suggestions for future work in their respective field. Only a few general suggestions are given here.

A successful implementation of the advanced controllers presented in Chapter 3 relies on full state information as presented in Chapter 4. The subsystem interconnection is outlined in Figure 5.1. Figure 5.2 shows a preliminary result from simulation of the interconnected system. The controller has an active switching strategy (see Section 3.3.3) with two PID controllers, where the contact mode controller has a torque compensator. This is similar to the controller used in Figure 3.6. Only the angular position of engine and wheels are measured. All other signals are estimated by the size and state estimators proposed in Chapter 4. Although not optimized, it is seen that the performance is not degraded to any larger extent, in comparison to Figure 3.6.

This interconnection of estimator and controller needs further studies. Most importantly, the controller must be tuned with the dynamics of the estimator taken into account.

A related issue is a robustness analysis of the advanced control strategies, which are sensitive to errors in the backlash size. It is of interest to see whether the size estimator gives a sufficiently good estimate.
5.1 Suggestions for future work

Figure 5.2: Simulation of the interconnected system from Figure 5.1.
Chapter 6

Summary of appended papers

6.1 Paper I

_Evaluation of Control Strategies for Automotive Powertrains with Backlash_

A number of approaches to control systems with backlash are reported in the literature, but very few are directed towards control of automotive powertrains. This paper evaluates four different controllers for powertrains with backlash, by means of simulation.

Two of the controllers are linear: One is a PID controller which is conservatively tuned, to avoid too large jerk levels from the backlash impact. In the second controller, a shaft torque compensator is added to the PID controller.

The two other controllers are switching between two modes. In contact mode, the controllers follow an acceleration setpoint (driver’s command). In backlash mode, the engine side of the backlash is controlled towards contact with the wheel side in the appropriate direction. This is called active switching, since the controller actively tries to get out of the backlash.

The results show that the linear controllers are robust to model errors, but slower than the switching controllers. The torque compensator improves performance, but is sensitive to noise. The active nonlinear controllers have a potential for improved backlash control. However, the robustness of these controllers needs further investigation.

6.2 Paper II

_Open-Loop Optimal Control of a Backlash Traverse_

When evaluating the performance of control systems for a process, it may be of interest to know the theoretical performance limit. Open-loop optimal control can be used to find this limit. The character of the solution is also of interest for the choice of feedback controllers.
In this paper, open-loop minimum time control solutions are found for the powertrain application. The control problem is to perform a traverse of the backlash gap without any jerk at the contact instant.

Most standard optimization software does not handle problems with dynamics. To solve the problem above, a MATLAB implementation of a solver for open-loop optimal control problems is presented. The solver reformulates the control problem into a constrained nonlinear program, NLP, which is solved by the TOMLAB (Holmström and Göran, 2002) suite of optimization routines.

### 6.3 Paper III

**Model Predictive Control of Automotive Powertrains with Backlash**

The driveability motivation for backlash control can be formulated as: "The backlash should be traversed as fast as possible, but without any jerk at the contact instant." This can be regarded as a constrained, minimum-time control problem.

Model predictive control, MPC, is a control technology suited for control of systems with constraints. This motivates an investigation of the usability of MPC in this application. Recent developments in MPC theory make an off-line calculation of the control law possible. The on-line implementation only involves a fast table look-up for the feedback control law. This makes MPC attractive for implementation in fast control loops such as the one under study here.

The results indicate that MPC has a potential in this application. For example, performance of the MPC controllers presented here is close to the theoretical limit, which is found by the open-loop optimization described above. The used powertrain model is significantly simplified as compared to the other results in this thesis. For example, the engine delay is neglected. Despite this, the off-line computation time is significant, and further robustness investigations are needed.

### 6.4 Paper IV

**Backlash Gap Position Estimation in Automotive Powertrains**

In order to increase the powertrain controller performance, it is important to have information about the state of the system. For this purpose, a nonlinear estimator is developed, based on extended Kalman filtering theory. By exploiting the structure of the backlash nonlinearity, a switching Kalman filter is found. The filter switches between two linear modes, for the system in backlash and contact respectively. The mode switching is made depending on the estimated states. The estimator assumes that the backlash size is known.
The state estimator assumes that the angular position of engine and wheels are measured with high precision. Standard sensors, especially the wheel sensor normally used in the ABS system, have a low resolution. Therefore, a linear estimator for fast and accurate estimation of the angular position of a wheel or the engine is also described. It utilizes standard ABS sensors and engine speed sensors, and is based on event based sampling, at each pulse from the sensor.

The results show that the backlash state estimate is of high quality, and robust to modeling errors. The performance is increased further when the event based position estimators are used as pre-filters.

6.5 Paper V

*Estimation of Backlash with Application to Automotive Powertrains*

Advanced control of systems with backlash requires knowledge of the backlash size. Also the state estimator described above needs this information.

In this paper, a nonlinear estimator for the backlash size is described, based on Kalman filtering theory. The backlash is modeled as a state-dependent offset in one of the angular sensors. The offset is assumed to have one value in positive contact and one in negative contact. A powertrain model without backlash is augmented with the offsets as slowly varying parameters. A switching Kalman filter is used to estimate these parameter values together with the system states. The backlash size is taken as the difference between the two offset parameter values.

The size estimator is combined with the state estimator described in the previous paper, resulting also in accurate state estimation for the system.

6.6 Paper VI

*Estimation of Backlash in Automotive Powertrains — an Experimental Validation*

In the two previous papers, nonlinear estimators for powertrain states and backlash size are described. In this paper, an experimental validation of the performance of these estimators is described. High accuracy pulse measurements of engine and wheel angular sensor signals are made in a passenger car. The requested engine torque is also measured, but with lower accuracy. The data is then processed off-line. Noise is added to the measurement data before it is used as input to the estimators. The high quality measurements together with a-causal data processing and manual size measurements are taken as "true" system information.

The results show that the estimates are of high quality, and hence useful for improving backlash compensation functions in the powertrain control system.
Appendix A

Notation

This appendix lists most of the symbols used in the thesis and the appended papers. Symbols that only appear in the appended papers are not necessarily listed here. Table A.1 lists constants and slowly time varying parameters and Table A.2 lists variables. Table A.3 lists commonly used abbreviations and other symbols.

All simulation results in this thesis are based on model parameter values which correspond to a passenger car at the first gear. Note however that most of the results are equally applicable also to heavy vehicles. The values that are used in most simulations are listed in Table A.1.

Table A.1: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>Backlash size (half gap width)</td>
<td>rad</td>
<td>0.029</td>
</tr>
<tr>
<td>(\theta_{o+}, \theta_{o-})</td>
<td>Wheel sensor offsets</td>
<td>rad</td>
<td></td>
</tr>
<tr>
<td>(\tau_{\text{eng}})</td>
<td>Engine time constant</td>
<td>s</td>
<td>0.1</td>
</tr>
<tr>
<td>(b_l)</td>
<td>Viscous friction of wheels</td>
<td>Nm/(rad/s)</td>
<td>0 or 5.6</td>
</tr>
<tr>
<td>(b_m)</td>
<td>Viscous friction of engine</td>
<td>Nm/(rad/s)</td>
<td>0</td>
</tr>
<tr>
<td>(c)</td>
<td>Internal shaft damping</td>
<td>Nm/(rad/s)</td>
<td>120</td>
</tr>
<tr>
<td>(i)</td>
<td>Transmission ratio, (\theta_m/\theta_1)</td>
<td>rad/rad</td>
<td>12</td>
</tr>
<tr>
<td>(k)</td>
<td>Shaft stiffness</td>
<td>Nm/rad</td>
<td>10000</td>
</tr>
<tr>
<td>(m_{\text{vehicle}})</td>
<td>Vehicle mass</td>
<td>kg</td>
<td></td>
</tr>
<tr>
<td>(r_{\text{wheel}})</td>
<td>Wheel radius</td>
<td>m</td>
<td>0.33</td>
</tr>
<tr>
<td>(A, B, C, D)</td>
<td>State space matrices</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(J_i)</td>
<td>Moment of inertia of wheels and vehicle mass</td>
<td>kgm²</td>
<td>140</td>
</tr>
<tr>
<td>(J_m)</td>
<td>Moment of inertia of engine flywheel</td>
<td>kgm²</td>
<td>0.3</td>
</tr>
<tr>
<td>(J_{\text{wheel}})</td>
<td>Moment of inertia of one wheel</td>
<td>kgm²</td>
<td></td>
</tr>
<tr>
<td>(K)</td>
<td>Estimator gain (except in Paper III)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(L_e)</td>
<td>Engine delay</td>
<td>s</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Figure A.1: Powertrain model.

Table A.2: Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$</td>
<td>Angular position of shaft at gearbox end</td>
<td>rad</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>Angular position of shaft-side of backlash</td>
<td>rad</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>Angular position of wheel-side of backlash</td>
<td>rad</td>
</tr>
<tr>
<td>$\theta_b$</td>
<td>Angular position in backlash, $\theta_2 - \theta_3$</td>
<td>rad</td>
</tr>
<tr>
<td>$\theta_d$</td>
<td>Total shaft displacement, including backlash, $\theta_1 - \theta_3$</td>
<td>rad</td>
</tr>
<tr>
<td>$\theta_l$</td>
<td>Angular position of wheels (load)</td>
<td>rad</td>
</tr>
<tr>
<td>$\theta_m$</td>
<td>Angular position of engine flywheel (motor)</td>
<td>rad</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>Angular or longitudinal velocity corresponding to $\theta_c$</td>
<td>rad/s or m/s</td>
</tr>
<tr>
<td>$a_c$</td>
<td>Acceleration corresponding to $\theta_c$</td>
<td>rad/s² or m/s²</td>
</tr>
<tr>
<td>$k$</td>
<td>Discrete-time sample number</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>$u$</td>
<td>Control signal, requested engine torque</td>
<td>Nm</td>
</tr>
<tr>
<td>$v$</td>
<td>State disturbance noise vector</td>
<td></td>
</tr>
<tr>
<td>$w$</td>
<td>Measurement disturbance noise vector</td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>State variable vector</td>
<td></td>
</tr>
<tr>
<td>$y$</td>
<td>Measured system output variable vector</td>
<td></td>
</tr>
<tr>
<td>$F_l$</td>
<td>Load force (external forces on vehicle)</td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>Torque</td>
<td>Nm</td>
</tr>
<tr>
<td>$T_g$</td>
<td>Gearbox input torque</td>
<td>Nm</td>
</tr>
<tr>
<td>$T_l$</td>
<td>Load torque</td>
<td>Nm</td>
</tr>
<tr>
<td>$T_m$</td>
<td>Engine torque</td>
<td>Nm</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Shaft torque (equal at positions 1,2,3)</td>
<td>Nm</td>
</tr>
</tbody>
</table>

Table A.3: Abbreviations, indices and other symbols

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$bl$</td>
<td>Backlash</td>
</tr>
<tr>
<td>$co$</td>
<td>Contact</td>
</tr>
<tr>
<td>$l$</td>
<td>Load, refers to the wheel side of the backlash</td>
</tr>
<tr>
<td>$m$</td>
<td>Motor, refers to the engine side of the backlash</td>
</tr>
<tr>
<td>$+, (-)$</td>
<td>Referring to contact at continuous positive (negative) engine torque</td>
</tr>
<tr>
<td>$\hat{}$</td>
<td>Estimated variable or parameter value</td>
</tr>
<tr>
<td>$\dot{}$</td>
<td>Time derivative, $\frac{d}{dt}$</td>
</tr>
</tbody>
</table>
Bibliography


*http://www.tomlab.biz*


Maciejowski, J. (2002), Predictive Control with Constraints, Prentice Hall.


Part II
Paper I

Evaluation of Control Strategies for Automotive Powertrains with Backlash

Adam Lagerberg and Bo Egardt


Evaluation of Control Strategies for Automotive Powertrains with Backlash

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Abstract

Backlash is the source of an uncomfortable acceleration phenomenon, which in the automotive industry is called "shunt and shuffle". Until recently, only simple controllers have been used to reduce the discomfort, and the tuning of these is made quite subjectively. A number of approaches to control of systems with backlash are reported in the literature, but very few are directed towards control of automotive powertrains. This paper evaluates four different controllers for powertrains with backlash, by means of simulation. The results show that active nonlinear controllers have a potential for improved backlash control. However, the robustness of these controllers needs further investigation.

Keywords: Backlash control, Automotive powertrains, Nonlinear control.

1 Introduction

Backlash is a problem in powertrain control. The sources of backlash are mainly play between gears in the final drive and in the gearbox, but also other plays throughout the powertrain add to the backlash.

Backlash introduces a hard nonlinearity in the control loop for torque generation and distribution. When the nonlinearity is excited, i.e. when the backlash is traversed, no torque is transmitted through the shaft. Then, when contact is achieved, the impact results in a large shaft torque. The fast change between zero and large torque makes the system difficult to control and oscillations may occur. In the automotive industry the impact is often referred to as shunt, while the oscillations are called shuffle. The driver feels the torque oscillations as an uncomfortable jerkiness of the vehicle. The impacts in the gearbox can also yield noise.

The common solution of the backlash problem in real powertrain applications today is to use some conservative controller. The control system is tuned to be slow enough, so
that the backlash is not excited too much. A somewhat more sophisticated solution is to
detect situations when the sign of the requested torque is changing. At these instances, the
torque is first controlled to a low torque of opposite sign. After a pre-specified time, long
enough to ensure that the backlash has been traversed, full torque demand is allowed.

There is an interest in the automotive industry to develop better backlash control systems,
in order to decrease response time, but with maintained comfort. With increasing comput-
ing power in the control systems and more sensors in the vehicles, more advanced
controllers may potentially improve vehicle performance.

A number of approaches to control of systems with backlash are reported in the literature,
see e.g. (Nordin and Gutman, 2002) for an overview, but very few are directed towards
control of automotive powertrains. The reported approaches can be divided into three
main categories: linear controllers, passive and active nonlinear controllers. They are
briefly described below.

**Linear controllers.** A linear controller must be robust enough to handle the nonlinearity.
A simple PID-controller or a state feedback controller is used, see e.g. (Hori et al., 1994).
If the nonlinearity is modeled as an uncertainty in a linear system, various robust con-
trol design methods can be applied, e.g. $H_\infty$ or QFT (Yang, 1992). One approach is to
view the torque variations due to the backlash as a disturbance, and then include a dis-
turbance observer and compensator in the control system (Brandenburg, 1987; Odai and
Hori, 1998; Nakayama et al., 2000).

**Passive nonlinear controllers.** A nonlinear controller can be designed to become more
careful when the backlash gap is open, or when the probability for it is high, in order to
avoid collisions and oscillations. The most common method is to switch between two
controllers (often linear). One is designed for the contact mode, with high performance
goals. The other controller is designed for backlash mode, and is typically a detuned
version of the contact mode controller (Nordin, 2000; Friedland, 1997).

**Active nonlinear controllers.** A nonlinear active controller also takes the backlash into
account, but with the philosophy of quickly but smoothly getting out of the backlash.
The controller for the backlash mode is designed to make a fast but soft “landing” at the
contact point. Much work is done on inverting controllers e.g. (Tao and Kokotovic, 1996).
However, most of these results assume a mechanical structure with one input side and one
output side of the backlash. This is inappropriate for automotive powertrain systems,
where both the engine and the wheels affect the backlash position. In this case switching
controllers are more appropriate (Ezal et al., 1997; Tao, 1999; Yang and Fu, 1996; Boneh
and Yaniv, 1999).

This paper describes a comparative simulation study. Two linear and two active switching
controllers are applied to a powertrain model for a passenger car. The results are evaluated
with respect to speed and robustness.
2 Powertrain model

The powertrain model, see Figure 1, is representative for a passenger car on the first gear, at low speed. A two-inertia model is used, where one inertia represents the engine flywheel (motor). The other inertia represents the wheels and chassis (load). A gear is located close to the engine inertia.

\[ J_m \ddot{\theta}_m + b_m \dot{\theta}_m = T_m - T_g \]  
\[ J_l \ddot{\theta}_l + b_l \dot{\theta}_l = T_s - T_l \]  
\[ T_s = i T_g, \theta_3 = \theta_l, \theta_1 = \theta_m/i \]  

where \( b_m, b_l \) are viscous friction constants and \( i \) is the gearbox ratio. Remaining variables are defined in Figure 1. A flexible shaft with backlash is connecting the gear and the second inertia.

The shaft torque is modeled as in (Nordin et al., 1997). It is shown there that this model is more physically correct than the traditional dead zone model for backlash:

\[ T_s = k(\theta_d - \theta_b) + c(\dot{\theta}_d - \dot{\theta}_b) \]  
\[ \dot{\theta}_b = \begin{cases} \max(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)) & \theta_b = -\alpha \\ \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b) & |\theta_b| < \alpha \\ \min(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)) & \theta_b = \alpha \end{cases} \]

where \( \theta_d = \theta_1 - \theta_3 \) is the shaft twist, and \( \theta_b = \theta_2 - \theta_3 \) is the position in the backlash gap.

The engine is modeled as an idealised torque generator. Its dynamics is of first order with a delay and the available torque is limited:

\[ T_m(s) = \frac{e^{-L_{\text{eng}} s}}{\tau_{\text{eng}} s + 1} u(s) \]  
\[ T_m \in [T_{m,\text{low}}, T_{m,\text{high}}] \]

It is assumed that the position \( \theta \), speed \( \omega \) and acceleration \( a \) of both engine and wheels are measurable, as well as the engine torque, \( T_m \). In a real application, some of these...
quantities are available (speed, position), while acceleration may be estimated from the speed signal. The engine torque is often estimated in the engine control system. For this investigation of control principles, the assumptions are relevant, but work is planned on studying which signals are needed, and how to estimate them.

3 Controllers

As described above, there are three main strategies for backlash control: Linear, passive and active nonlinear. Note the difference between passive and active switching! For example, a passive controller for a powertrain would still try to control the wheel torque while in backlash mode, but with low performance goals. An active controller would switch from wheel torque control to position control of the backlash angle when backlash mode is entered.

In this paper, four different linear and active nonlinear controllers are evaluated in the powertrain application. These controllers are described in the following subsections.

3.1 Simple PID-controller

A linear PID controller of the following form is used:

\[ u_{PID} = k_P (a_{ref} - a_l) + k_I \int (a_{ref} - a_l) \, dt + k_D \frac{d}{dt} (b_D a_{ref} - a_l) \] (8)

Here, the load acceleration, \( a_l \), is viewed as controlled variable, \( k_P, k_I, k_D \) are tuning constants and \( b_D \) is set to zero, to avoid derivative action on the reference signal.

3.2 PID-controller with torque compensator

A linear observer/compensator for the backlash, as discussed in the introduction is evaluated. The concept is taken from (Nakayama et al., 2000), where it was applied to a system without engine dynamics. It can be shown that in that case, the resulting controller can be rewritten as a standard PID-controller with a specific choice of \( b_D \). In this paper, the concept is adapted to include the engine dynamics. The shaft torque observer is:

\[ \hat{T}_g = \frac{T_m - J_m s \omega_m}{\tau_{filt} s + 1} \] (9)

where the motor friction, \( b_m \), is neglected. \( \omega_m = \dot{\theta}_m \) is the measured motor speed.

From the shaft torque estimate, an engine control signal is computed, which compensates for variations in shaft torque. This compensator cancels the engine time constant, but does
not take the delay into account:

\[ u_{\text{comp}} = K \frac{\tau_{\text{eng}}s + 1}{\tau_{\text{filt}}s + 1} \hat{T}_g \]  

(10)

where the compensator gain, \( K \), is a tuning constant.

The torque compensator is combined with the PID controller from (8), forming the control signal:

\[ u_{\text{PIDComp}} = u_{\text{PID}} + u_{\text{comp}} \]  

(11)

### 3.3 Simple active switching controller

A simple active switching controller from (Tao, 1999) is evaluated. This controller was derived for a system without shaft flexibility, and without engine dynamics. The important feature is the switching between the contact and backlash modes. In contact mode, the controller follows an acceleration setpoint, \( a_{\text{ref}} \). In backlash mode, it controls the motor side position towards contact with the wheel side of the backlash.

The switching controller has a contact mode controller:

\[ u_{\text{CO}} = b_0 \omega_m + J_0 (ia_{\text{ref}} - k_1 (\omega_m - i\dot{\omega}_{\text{ref}}) - k_2 (\theta_m - i\dot{\theta}_{\text{ref}} - i\dot{\theta}_i)) \]  

(12)

where \( m \) denotes the motor state, \( r e f \) the desired load state, \( \theta_i = \pm \alpha \), the backlash angle with appropriate sign, the gear ratio \( i \) is used to translate all variable values to motor side magnitudes, \( b_0 \) is the total motor and load friction force parameter, \( J_0 \) is the total inertia and \( k_1, k_2 \) are tuning constants. The structure has been chosen so that the dynamics from reference signal to load position becomes:

\[ \frac{\theta_l(s)}{\theta_{\text{ref}}(s)} = \frac{s^2 + k_2 s + k_1}{s^2 + k_2 s + k_1} = 1 \]  

(13)

t.e. the powertrain dynamics are completely cancelled (for a rigid shaft and without engine dynamics). The tuning constants \( k_1, k_2 \) will determine the dynamics of disturbance rejection.

In backlash mode, the controller is:

\[ u_{\text{BL}} = b_m \omega_m + J_m (ia_{\text{ref}} - k_3 (\omega_m - i\omega_l) - k_4 (\theta_m - i\theta_l - i\dot{\theta}_i)) \]  

(14)

where \( \theta_l \) is the load position, \( b_m \) is the motor friction force parameter, \( J_m \) is the motor inertia and \( k_3, k_4 \) are tuning constants. In this mode, the load position is seen as the reference, towards which the engine is controlled. Similar to the contact case, the structure has been chosen so that the dynamics from load position to engine position becomes:

\[ \frac{\theta_m(s)}{\theta_l(s)} = \frac{s^2 + k_4 s + k_3}{s^2 + k_4 s + k_3} = 1 \]  

(15)

t.e. the engine flywheel dynamics are completely cancelled. However, the engine dynamics is not taken into account in this controller structure.
The overall controller switches between $u_{CO}$ and $u_{BL}$, depending on the size of the shaft twist:

$$u = \begin{cases} 
  u_{CO} & |\theta_d| \geq \alpha \\
  u_{BL} & |\theta_d| < \alpha 
\end{cases}$$

(16)

### 3.4 Modified switching controller

Since the switching controller described above is derived for a system without flexible shaft and engine dynamics, a modification is also evaluated. In the backlash mode, this controller is re-derived for a system including engine dynamics. The control law then becomes:

$$u_{BL,Mod} = a_{11}\dot{\omega}_m + a_{10}\theta_m + a_{20}T_m + b_2i_1 + b_1i_\omega l + b_0i(\theta_l + \theta_i)$$

(17)

with

$$\begin{align*}
  a_{11} &= \tau_{eng}(b_mk_7 - \frac{b_2^2}{J_m} - J_mk_6), \\
  a_{10} &= -k_5J_m\tau_{eng}, \\
  a_{20} &= 1 + \frac{b_m\tau_{eng}}{J_m} - k_7\tau_{eng}, \\
  b_2 &= J_m\tau_{eng}k_7, \\
  b_1 &= J_m\tau_{eng}k_6, \\
  b_0 &= J_m\tau_{eng}k_5
\end{align*}$$

With this controller, the polynomials in (15) are replaced by $s^3 + k_7s^2 + k_6s + k_5$.

In contact mode, no effort is made to cancel dynamics as described above. Instead, the PID controller with torque compensator from (11) is used. The overall controller switches according to:

$$u = \begin{cases} 
  u_{PIDComp} & |\theta_d| \geq \alpha \\
  u_{BL,Mod} & |\theta_d| < \alpha 
\end{cases}$$

(18)

### 4 Simulation results

#### 4.1 Evaluation criteria

To evaluate the control strategies, and to make a fair comparison of them, all controllers are tuned in a standardized way: The control systems are simulated imitating a driver going from engine braking to acceleration. Each controller configuration is tuned for minimal response time, while constraints on maximal acceleration and jerk (acceleration derivative) should be satisfied. The engine control signal must be within limits as well.

Robustness to variations in backlash size is evaluated, as well as the sensitivity to measurement noise.

#### 4.2 Nominal case

First, simulations are presented for the “nominal” case, with known backlash size, and without measurement noise. See Figures 2-5.
The simulation results show that the switched controllers traverse the backlash gap faster than the linear PID-based controllers. It is clearly seen that when the backlash is entered, the switching controllers retard the engine so that the impact is minimized. In contrast, the linear controllers increase the control signal when the backlash is entered.

When comparing the linear controllers, the performance of the torque compensator based controller is superior to the standard PID-controller. If the shaft torque observer is studied closer, it will be seen that the observed torque has no chance of following the true torque when the backlash is traversed. Nevertheless, the use of feedback from two extra measured signals can explain the improved performance.
When comparing the switched controllers, the modified controller traverses the backlash slightly faster than the original one. The original controller however, reaches the desired acceleration faster. The original controller doesn’t utilise the allowed range of the control signal, which is due to the simple model it is based upon.

The spike in the control signal of the modified switching controller just before backlash mode is entered is explained by the use of $\theta_d$, the total shaft twist, as indicator of backlash mode, while the internal state of the backlash model, $\theta_b$, governs the shaft torque. This will delay the switching into backlash mode a short instance.
4.3 Robustness

Robustness to variations in backlash size is evaluated by simulating the systems with double and half backlash size compared to what the controllers were tuned for, see Figure 6. The results show that the linear controllers are quite robust to backlash variations. The switched controllers are just as robust when the backlash is underestimated. However, if the backlash is overestimated, the switched controllers become unusable. The explanation is that the control system remains in backlash mode although contact is reached.

4.4 Measurement noise

As a test of sensitivity to noise, a noise with an amplitude of approximately 5% of the signal level is added to all measured signals. First order filters are also added at the measurement signals. The filters and the controller parameters are retuned to cope with the noise and extra filter dynamics. The results show that the observer-based approaches are more sensitive to noise than the others. This is due to the differentiation of the motor speed signal, as described in the previous section. Results for the systems with and without noise are compared for the worst and best case, for the PID-controller in Figure 7, and for the modified switched controller in Figure 8. The remaining two controllers are not shown: The simple switching controller is quite insensitive to noise and extra filtering, and the PID-controller with torque compensator performs well, but with a high noise level in the control signal, as in Figure 8.

5 Conclusion

Three different approaches to control of backlash in automotive powertrains are identified. Controllers from two of these were evaluated in this paper by simulations of a simplified
Figure 7: PID-controller with and without measurement noise and extra measurement filters. The noise-free case is the same as in Figure 2.

Figure 8: Modified switching controller with and without measurement noise and extra measurement filters. The noise-free case is the same as in Figure 5.

The simulations show that an active switching controller has potential for improving the vehicle performance. However, robustness to variations in backlash size has to be taken care of. When the shaft torque observer is introduced, the system becomes more sensitive to noise.

As seen in the simulations, the structure of the respective controller for contact and backlash may be improved. In contact mode, an LQG/LTR controller could be used, see (Fredriksson et al., 2002). Time-optimal control for the backlash mode could also be an
extension (Ezal et al., 1997). More advanced powertrain models should also be tested.

An important measurement signal is the backlash position. An investigation on the possibility of estimating this from speed sensors on both sides of the backlash should be performed. The standard sensors in the vehicle are preferable to use, and the signal quality of the sensors will influence this estimate.

Acknowledgements

This research was supported by the Volvo Research and Educational Foundations, the Swedish Automotive Research Program and the Volvo Corporation.

References


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Technical report R007/2004, Department of Signals and Systems,
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Abstract

This report describes the MATLAB implementation of a solver for open-loop optimal control problems. The control problem is numerically reformulated into a nonlinear program, NLP, which is solved by the TOMLAB suite of optimization routines. The solver is then used to solve the minimum time control problem of performing a smooth traverse of the backlash gap in an automotive powertrain. This solution may be used to give information about the limit on how fast a feedback controller for the same problem can be.

1 Introduction

Backlash is a problem in powertrain control. The sources of backlash are mainly play between gears in the final drive and in the gearbox.

Backlash introduces a hard nonlinearity in the torque control loop. When the backlash is traversed, no torque is transmitted through the shaft. Then, when contact is achieved, the impact results in a large shaft torque. The driver feels the fast change between zero and large torque as an uncomfortable jerk. Engine control systems must therefore compensate for the backlash. Typically, quite simple controllers, such as PI or PID controllers, are used in practical applications, while more advanced controllers are suggested in research papers, see e.g. (Lagerberg, 2001; Lagerberg and Egardt, 2002) and references therein. When investigating how much improvement that can be made from using an advanced feedback controller, a theoretical performance limit is of interest. For this, optimal (open-loop) control can be used, which is the main focus of this report.

There exist many methods for the solution of optimization problems. There are also numerous software tools implementing these methods. For optimal control problems, the problem description includes dynamics of the system. Solution methods exist, but software implementations for the general problem are uncommon. The second focus of
this report is to implement a method for reformulation of an optimal control problem into an optimization problem in the MATLAB (MathWorks, 2004) environment.

The connection between optimal control and general optimization methods is explained in Section 2, together with a general formulation of the optimal control problem. Section 3 describes how the optimal control problem is reformulated into a nonlinear program, NLP, while the MATLAB implementation is described in Section 4. The backlash traverse problem and its solution is described in sections 5 to 8.

2 Optimal control and Nonlinear Programming

2.1 Optimization methods

There exist many methods and software tools for the solution of different types of optimization problems. Examples include the Optimization Toolbox for MATLAB (MathWorks, 2004), NAG-routines (NAG, 2004), NPSOL (Gill et al., 1998), SNOPT (Gill et al., 2002) and TOMLAB (Holmström and Göran, 2002). One class of methods is Nonlinear Programming, NLP, which is used here. The specific software used is TOMLAB, an optimization environment based on MATLAB, which contains a large number of different optimization routines. All user interaction is made in the MATLAB environment, using a GUI, scripts and functions or directly from the command line. TOMLAB handles interfacing to the different solvers, which makes it easy to test different solvers on a given problem.

2.2 Optimal control

Let $x \in \mathbb{R}^{n_x}$ be the vector of state variables and $u \in \mathbb{R}^{n_u}$ the vector of control signals in a dynamic system. An optimal control problem can be formulated as follows:

Find the control function $u(t), t \in [t_I, t_F]$, that minimizes the cost functional

$$J = \phi(x(t_I), x(t_F), t_I, t_F) + \int_{t_I}^{t_F} L(x(t), u(t), t) \, dt$$

subject to the following constraints:

State equations, which define the dynamics

$$\dot{x}(t) = f(x(t), u(t), t), \quad t \in [t_I, t_F]$$

Simple boundary conditions

$$x(t_I) = x_I$$
$$x(t_F) = x_F$$
Complex boundary conditions

\[ g_I(x(t_I), u(t_I)) = 0 \]  \hspace{1cm} (5)  
\[ g_F(x(t_F), u(t_F)) = 0 \]  \hspace{1cm} (6)  

Simple constraints on state and control variables

\[ x_{\text{min}} \leq x(t) \leq x_{\text{max}}, \quad t \in [t_I, t_F] \]  \hspace{1cm} (7)  
\[ u_{\text{min}} \leq u(t) \leq u_{\text{max}}, \quad t \in [t_I, t_F] \]  \hspace{1cm} (8)  

Complex path constraints

\[ g_L \leq g(x(t), u(t), t) \leq g_U, \quad t \in [t_I, t_F] \]  \hspace{1cm} (9)  

The final time, \( t_F \), can be fixed, or allowed to vary and be a part of the optimization problem. The cost functional \( J \) is made up of two contributions: \( \phi(\ldots) \) is used to define a cost related to the initial and/or final states, and is called the \textit{Mayer} part. The integral defines a cost with contributions from the entire evolution of the system, and is called the \textit{Lagrange} part. When both parts are included, the problem is on \textit{Bolza}-form.

Optimal control problems can be solved by an \textit{indirect} method based on the Hamiltonian equation, in which the dynamics and the constraints are adjoined to the cost functional (Bryson, 1999). This results in a two point boundary value problem, with dynamics of both states and co-states that have to be solved. It is also possible to directly search for the input sequence that minimizes the given cost functional, while satisfying the dynamics and the constraints. This method, which is used in this report, is called a \textit{direct} method.

General NLP solvers such as those implemented in TOMLAB (Holmström and Göran, 2002), cannot handle the dynamics equation (2). To include the dynamics, different methods exist:

- In each iteration of the NLP solution, the dynamic equations are solved by an ODE solver. An advantage of this method is that it is easy to implement. A disadvantage is that the numerical errors from the ODE solver will vary from iteration to iteration, and this will be seen as a "noise" by the NLP-solver. The method is sometimes called the \textit{shooting method}.
- In the \textit{transcription method}, the dynamics are reformulated into a number of nonlinear constraints on a "static" optimization problem.

The latter method is used in this report and is further described in next section.

### 3 Optimal control in TOMLAB

As mentioned above, TOMLAB and its solvers cannot handle equations with dynamics, and hence not optimal control problems. A first version of an optimal control solver for
TOMLAB is described here, called TOMOC. Most of the ideas are taken from (Betts, 2001), and are also implemented in SOCS, a Fortran solver for optimal control problems (Betts, 2003).

The central idea in TOMOC is to implement a transcription formulation of the dynamics equation (2). The solution trajectory may be split into several phases, possibly with different dynamics. The following subsections describe these two main features of TOMOC.

In addition, constraints of the types (3-9) are handled. Functions for initial value generation and result presentation are also provided. These features are described in next section.

3.1 Transcription

In the transcription method, the differential equation (2) is replaced by a discretization scheme, such as Euler, Runge-Kutta, Trapezoidal or Hermite-Simpson. The discretization is made over $n_s$ intervals, called segments. As an illustration, the Trapezoidal discretization is used:

$$
\dot{x}_k \approx \frac{x_{k+1} - x_k}{h_k} \approx \frac{f_k + f_{k+1}}{2}, \quad k = 1, \ldots, n_s
$$

(10)

where $x_k = x(t_k)$, $f_k = f(x(t_k), u(t_k))$, $h_k = t_{k+1} - t_k$ and $t_1 = t_I$, $t_{n_s+1} = t_F$. Define defects, $\zeta_k$ as

$$
\zeta_k = x_{k+1} - x_k - \frac{h_k}{2}(f_k + f_{k+1})
$$

(11)

By adding defect constraints,

$$
\zeta_k = 0, \quad k = 1, \ldots, n_s
$$

(12)

to the NLP problem, the differential equations will be approximately satisfied. By increasing the number of intervals, $n_s$, a desired accuracy will be achieved.

3.2 Multiphase optimization

As described in (Betts, 2001), the dynamic trajectory can be divided into several phases. One transcription formulation is made for each phase. Also boundary conditions can be given for each phase. The purpose of the phases is to facilitate the following features of a problem:

- Different physical phenomena can be modeled on different phases, meaning that the differential equations may differ.

---

1After the first version of this report, SOCS is made available in MATLAB as a TOMLAB toolbox. Also, DIDO (Ross, 2004) has appeared as a TOMLAB-based solver for the same problem class.
• Different constraints can be applied in different phases.
• An event can be exactly found as the end point of a phase.
• The transient behaviour of the dynamics may differ in time, which means that the trajectory can be divided into several phases, with different number of segments on each phase.

4 TOMOC implementation

This section serves as a short introduction to the use of TOMOC. (Lagerberg, 2004). The solver implementation is written with the purpose of solving the backlash traverse problem in sections 5-8, but some effort is put on making it reusable. The code is available from the author.

4.1 Variables and parameter structures

The vector of variables used by the NLP solver is called \( X \). It contains all variables included in the optimization: dynamic state variables, control signals and trajectory duration times. It is structured in the following way:

\[
X = [X^{(1)}, X^{(2)}, \ldots, X^{(n_{ph})}]
\]  
(13)

where \( X^{(p)} \) is the vector for phase \( p, p = 1, \ldots, n_{ph} \).

\[
X^{(p)} = [\Delta t, x^1, u^1, x^2, u^2, \ldots, x^{n_s+1}, u^{n_s+1}]
\]  
(14)

where \( \Delta t \) is the time on phase \( p \), \( x^k \) is the state variable vector and \( u^k \) is the control signal vector in each segment node \( k, k = 1, \ldots, n_s + 1 \):

\[
x^k = [x^k_1, x^k_2, \ldots, x^k_{n_y}]
\]  
(15)

\[
u^k = [u^k_1, u^k_2, \ldots, u^k_{n_u}]
\]  
(16)

As in TOMLAB, many parameters etc. are stored in structures, which makes it easy to group them together, and pass them between different functions. TOMLAB’s standard structure \( \text{Prob} \) has a reserved substructure \( \text{Prob.user} \) which is used by TOMOC. \( \text{Prob.user} \) contains mainly the following variables:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>par</td>
<td>Substructure with problem specific parameters.</td>
</tr>
<tr>
<td>ph</td>
<td>A field of substructures, with phase-specific variables.</td>
</tr>
<tr>
<td>nph</td>
<td>Number of phases.</td>
</tr>
<tr>
<td>nXtot</td>
<td>Total number of optimization variables.</td>
</tr>
<tr>
<td>tomoc_dynfun</td>
<td>Name of function with dynamics.</td>
</tr>
<tr>
<td>tomoc_constraints</td>
<td>Name of function with nonlinear constraints.</td>
</tr>
</tbody>
</table>
The substructure `Prob.user.ph{1}` contains mainly the following variables:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>ny</code></td>
<td>Number of state variables.(^2)</td>
</tr>
<tr>
<td><code>nu</code></td>
<td>Number of control variables.</td>
</tr>
<tr>
<td><code>ns</code></td>
<td>Number of segments (intervals) on phase.</td>
</tr>
<tr>
<td><code>nyu</code></td>
<td>Number of states and controls. ((ny+nu))</td>
</tr>
<tr>
<td><code>nX</code></td>
<td>Number of variables on phase, including (\Delta t). ((1+(ns+1)\times nyu))</td>
</tr>
<tr>
<td><code>xi</code></td>
<td>Initial simple boundary value on phase, (x_I).</td>
</tr>
<tr>
<td><code>xf</code></td>
<td>Final simple boundary value on phase, (x_F).</td>
</tr>
<tr>
<td><code>Dt_g</code></td>
<td>Guess of (\Delta t) on phase.</td>
</tr>
<tr>
<td><code>xi_g</code></td>
<td>Guess of initial state.</td>
</tr>
<tr>
<td><code>xf_g</code></td>
<td>Guess of final state.</td>
</tr>
<tr>
<td><code>ui_g</code></td>
<td>Guess of initial control signal.</td>
</tr>
<tr>
<td><code>uf_g</code></td>
<td>Guess of final control signal.</td>
</tr>
</tbody>
</table>

### 4.2 File structure

The MATLAB scripts and functions needed for an optimization problem are listed in Table 1. Names starting with `tom_bl_*` serve as templates for how to formulate another problem, while names starting with `tomoc_*` are problem independent functions.

### 4.3 Problem setup

To setup a problem in TOMOC, the following files / functions have to be edited:

- Problem dimensions etc., e.g. variables in `Prob.user.ph`, are given in `tom_bl_start`.
- Problem specific parameters, i.e. the variable structure `Prob.user.par`, are given in `tom_bl_initpar`.
- The objective function (1) is given in `tom_bl_fun`.
- The dynamic state equations (2) are given in `tom_bl_dynfun`.
- The simple boundary conditions (3-4) are given in `tom_bl_start`.
- The complex boundary conditions (5-6) are given in `tom_bl_constraints`.
- Simple bounds (7-8) are given in `tom_bl_start`.
- Complex path constraints (9) are given in `tom_bl_constraints`.
- Optional guesses for variable values at phase start and endpoints, and phase times \(\Delta t\) can be given in `tom_bl_start`.

\(^2\)In standard optimization literature, \(x\) denotes the vector of optimization variables. In a control context, this corresponds to both states and controls, \(\{x, u\}\). Therefore, \(y\) is often used to denote the state variables. The TOMOC code uses both \(y\) and \(x\) for states, \(u\) for controls and \(X\) for the total optimization vector.
Function / script | Description
--- | ---
**Problem specific functions:**

- `tom_bl_start`
  
  Prob = `tom_bl_initpar(Prob)`

  Main script: Defines simple bounds and constraints, calls initializations, runs TOMLAB opt. routines.

- `J = tom_bl_fun(X,Prob)`

  Initialization of problem specific parameter values.

- `fk = tom_bl_dynfun(xk,u,k,Prob)`

  Objective function to minimize (1).

- `[gL, gF, g, gL, gU] = tom_bl_constraints(X,... Prob, type)`

  Problem specific (non)linear constraints.

- `tom_bl_plot*(xr,Prob)`

  Plots results.

**Problem independent functions:**

- `tomoc_initguess`

  Construct initial guess, see Section 4.4.

- `tomoc_con_ph`

  Administration of constraints from multiple phases.

- `tomoc_con`

  (Non)linear constraints. This includes the 'defects', i.e. the approximated differential equations.

- `tomoc_postproc`

  Post processing of results.

**Table 1:** Matlab functions and scripts for TOMOC.

### 4.4 Comments on specific features

Some of the functionality of TOMOC deserves more comments:

**Initial values** for the state and control variables at all time points are generated by the function `tomoc_initguess`:

If there is a prior solution vector, it is used as a new starting guess. If the number of segments has changed, the new guess is interpolated from the prior grid points.

If no prior value exists, an initial guess is built in the following steps:

- The boundary values \((x_i, x_f)\) as given by (3-4) are used.
- Where these are missing, the guessed values provided in \(x_i_g, x_f_g, u_i_g, u_f_g\) are used.
- If nothing else is provided, zeros are assumed.
- All state and control values at internal points on the phases are interpolated linearly between phase boundaries.

This functionality makes it easy to first solve the problem with few segments. Then the accuracy of the solution can be improved by iteratively increasing \(n_s\). This is of importance since the convergence speed of the NLP solution is dependent on a good initial value.
A result matrix is formed after an optimization run, in the function `tomoc_postprocess`. Since the solution from TOMLAB is in vector form, the result is transformed into a more convenient matrix form \(( \mathbf{x}_r \)) with the structure indicated in Table 2.

<table>
<thead>
<tr>
<th>Time</th>
<th>(x_1)</th>
<th>(\ldots)</th>
<th>(x_{ny})</th>
<th>(u_1)</th>
<th>(\ldots)</th>
<th>(u_{nu})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1</td>
<td></td>
<td></td>
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<tr>
<td>(t^1)</td>
<td>(x_1^1)</td>
<td>(\ldots)</td>
<td>(x_{ny}^1)</td>
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<td>(\vdots)</td>
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<td>(\vdots)</td>
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<tr>
<td>(t_{n_s+1})</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
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<tr>
<td>Phase 2</td>
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<tr>
<td>(t^1)</td>
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<td>(t_{n_s+1})</td>
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<tr>
<td>Phase (n_{ph})</td>
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<tr>
<td>(t^1)</td>
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<td>(t_{n_s+1})</td>
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<td>(\ldots)</td>
<td>(\ldots)</td>
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<td></td>
</tr>
</tbody>
</table>

Table 2: Structure of the result matrix \(\mathbf{x}_r\).

### 4.5 Limitations and possible improvements

There is a large potential for improvement of TOMOC:

- More effort can be put on making TOMOC computationally efficient.
- All complex constraints are treated as nonlinear. However, if they are linear, it is possible to put them in other parts of the TOMLAB problem structure, yielding a more efficient solution. (This is true for the backlash implementation below.)
- In (Betts, 2001), issues like sparsity exploitation and automatic mesh refinement (number of segments on phase) are discussed. This could be incorporated in TOMOC.
- The NLP solvers in TOMLAB work more efficiently if gradients (and Hessians) are provided for the objective function and the constraints. It is quite straightforward to include this functionality also in TOMOC.
- The possibility to model delays is not included, but could be an addition to current functionality.
5 Backlash control

In this report, open-loop optimal control is used to solve the backlash traverse problem. The specific problem addressed is to go from engine braking to acceleration, thereby traversing the backlash gap. The time to reach a certain acceleration should be minimized, with the constraint that the relative speed between engine and wheel sides of the backlash is zero at the contact instant.

Sections 6 to 8 describe the problem and its solution.

6 Powertrain model

![Powertrain model diagram]

**Figure 1:** Powertrain model.

The powertrain model, see Figure 1, is representative for a passenger car on the first gear, at low speed. A two-inertia model is used, where one inertia represents the engine flywheel (motor). The other inertia represents the wheels and chassis (load). A gearbox is located close to the engine inertia. The model is defined by:

\[ J_m \ddot{\theta}_m + b_m \dot{\theta}_m = T_m - T_g \]  
\[ J_l \ddot{\theta}_l + b_l \dot{\theta}_l = T_s - T_l \]  
\[ T_g = \frac{T_s}{i}, \theta_3 = \theta_l, \theta_1 = \theta_m / i \]

where \( b_m \) and \( b_l \) are viscous friction constants and \( i \) is the gearbox ratio. Remaining variables are defined in Figure 1. The engine is modeled as an idealised torque generator. Its dynamics is of first order with a delay:

\[ T_m(s) = \frac{e^{-L_\text{eng}s}}{\tau_{\text{eng}}s + 1} u(s) \]

Note that the delay is not included in the open-loop optimization. It is not possible in the current TOMOC version, and it only represents a shift in the time scale for open-loop optimization.
A flexible shaft with backlash is connecting the gear and the chassis inertia. The shaft torque is modeled as in (Nordin et al., 1997). This model is more physically correct than the traditionally used dead-zone backlash model. The shaft torque is given by:

\[ T_s = k(\theta_d - \theta_b) + c(\omega_d - \omega_b) \]  

(21)

where \( \theta_d = \theta_1 - \theta_3 \) is the total displacement, and \( \theta_b = \theta_2 - \theta_3 \) is the position in the backlash. With \( \alpha \) denoting half the backlash size, the backlash position is governed by the following dynamics:

\[
\dot{\theta}_b = \begin{cases} 
\max(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)) & \theta_b = -\alpha \\
\dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b) & |\theta_b| < \alpha \\
\min(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)) & \theta_b = \alpha
\end{cases}
\]  

(22)

7 Optimization problem formulation

As briefly discussed in Section 5, the problem under consideration is to start from a negative and constant acceleration, and in minimal time go to contact on the positive side of the backlash, with zero relative speed, and then reach a specified vehicle acceleration. The optimization problem can be stated as follows:

Find \( u(t) \) for \( t_I \leq t \leq t_F \) and the final time \( t_F \) to minimize

\[ J = t_F \]  

(23)

subject to the constraints

\[ \dot{x} = f(x, u) \]  

(24)

\[
x(t_I) : \begin{cases} 
\theta_d(t_I) = \theta_{dI} < -\alpha \\
\omega_d(t_I) = \omega_{dI} = 0 \\
\theta_b(t_I) = \theta_{bI} = -\alpha
\end{cases}
\]  

(25)

\[ x(t_{F1}) : \text{Entering backlash} \]  

(26)

\[ x(t_{F2}) : \begin{cases} 
\theta_b(t_{F2}) = \alpha \\
\omega_b(t_{F2}) = 0
\end{cases} \]  

(27)

\[ x(t_F) : a(t_F) = a_F \]  

(28)

\[ x_{\min} \leq x \leq x_{\max} \]  

(29)

\[ u_{\min} \leq u \leq u_{\max} \]  

(30)

The dynamics (24) are defined by (17-22), with a state variable vector

\[ x = [\theta_m, \omega_m, \theta_l, \omega_l, T_m, \theta_b]^T \]  

(31)

The initial condition (25) is defined as a stationary retardation, where contact is established on the negative side of the backlash. The first phase change, entering the backlash,
is equivalent to a certain linear combination of states being zero, $ax = 0$, which is shown in (Lagerberg and Egardt, 2003). The second phase change condition (27), going from backlash to contact, requires the backlash gap to be closed, and that the relative speed of the backlash sides is zero. The final condition (28) is the desired final wheel acceleration.

The state bounds (29) include a maximum and minimum available engine torque (in the $T_m$-state). In (30), the control signal is upwards bounded by the constraint on maximum available engine torque. Downwards however, the bound is set to $10 \cdot T_{m,\text{min}}$. This is a way to model the fact that it is possible to immediately shut off the ignition on a spark ignited engine. This means that the engine dynamics in the negative direction is much faster than what is modeled by $\tau_{\text{eng}}$ in (20).

### 8 Solution of the backlash traverse problem

The problem is solved by programming the TOMOC functions described in Section 4.3 with the equations (23-30).

Multiple phases are used in the solution so that:

**Phase 1**, Negative contact mode:
- Starts with a constant retardation.
- Ends when the backlash is entered (i.e. the backlash model switches mode).

**Phase 2**, Backlash mode:
- Ends when positive contact is reached, with the constraint of zero relative speed.

**Phase 3**, Positive contact mode:
- Ends when desired wheel acceleration is reached.

With this phase partitioning, only one of the backlash dynamics in (22) is active on each phase. Also, the exact times when the dynamics switch are found.

The optimization result is shown in Figure 2. For implementation in a real vehicle, this solution is probably too soft and slow to be acceptable. A certain amount of jerk is accepted by the driver. Therefore another optimization is also performed, where the relative speed at contact is constrained to a positive value which results in a jerk at the impact, but also a shorter response time, see Figure 3. This jerk level is on the same magnitude as in the simulations of feedback controllers in (Lagerberg and Egardt, 2002). Note however that the time delay should be added to these results for a fair comparison. A comfortable jerk level is probably found somewhere between these two solutions.

Both of the solutions above end with a difference in engine and wheel speeds. This means that after the final time, there will remain driveline oscillations which a control system has to reduce. In Figure 4, additional constraints on the speed difference and engine torque at the end point are added. The result is that there will be no remaining driveline oscillations after this point.
Figure 2: A soft backlash traverse. The phase boundaries are indicated by vertical lines. The backlash is traversed between 0.06 and 0.17 s. All values of speed, acceleration and jerk are transformed into vehicle longitudinal quantities. The subplots show:

- The chassis acceleration shows the soft contact after backlash traverse.
- In the engine torque plot, both requested torque, $u$, (solid) and actual torque, $T_m$, (dashed) is shown.
- The jerk is defined as the acceleration derivative.
- The speed plot shows how engine (solid, scaled with the gear ratio) and wheel (dashed) speeds are equal at the contact instant.
- The total shaft displacement $\theta_d$ (solid) and the backlash position $\theta_b$ (dashed).
Figure 3: A backlash traverse with jerk at the impact. The difference from soft landing is best seen in the jerk and speed plots. Note also that the response time has been shortened from 0.27 to 0.15 s.
Figure 4: A backlash traverse without jerk and without remaining driveline oscillations. The backlash is traversed similarly to the solution in Figure 2. It then takes approximately 0.1 s. to remove the driveline oscillations.
9 Conclusions

This report describes the implementation of a solver for optimal control problems in MATLAB. The implementation works well for the problem under consideration here. A number of possible improvements are listed in Section 4.5.

For the backlash traverse problem, it is shown that an optimal open-loop control sequence is possible to compute. The resulting traverse time can be used as a benchmark when evaluating feedback control solutions. There is also a possibility to use the characteristics of the solution to give ideas as to how a feedback control strategy should be outlined.

References


*http://www.boeing.com/phantom/socs


*http://www.sbsi-sol-optimize.com

*http://www.sbsi-sol-optimize.com

*http://www.tomlab.biz


*http://www.hj.se/~laad/ or http://www.s2.chalmers.se/~al/


*http://www.mathworks.com
*http://www.nag.com


*http://www.tomlab.biz*
Paper III

Model Predictive Control of
Automotive Powertrains with Backlash

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Technical report R010/2004, Department of Signals and Systems,

Model Predictive Control of Automotive Powertrains with Backlash

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Abstract

In automotive powertrains, the existence of backlash causes driveability problems, which to some extent are remedied by the engine control system. The control problem has a constrained, minimum-time character, which motivates an investigation of the usability of model predictive control, MPC, in this application. Recent developments in MPC theory make an off-line calculation of the control law possible. This makes MPC more attractive for implementation in fast control loops such as the one under study here. The results indicate that MPC has a potential in this application. However, the off-line computation time is significant, and further robustness investigations are needed.

1 Introduction

Backlash is a problem in powertrain control. The sources of backlash are mainly play between gears in the final drive and in the gearbox. Backlash introduces a hard nonlinearity in the powertrain.

When the driver goes from engine braking to acceleration or vice versa, so called tip-in / tip-out maneuvers, the backlash is traversed, and potentially uncomfortable "shunt and shuffle" phenomena are experienced. During the backlash traverse, no torque is transmitted through the shaft. Then, when contact is achieved, the impact results in a large shaft torque and a sudden acceleration of the vehicle. Engine control systems must compensate for the backlash. The goal of the control system is to traverse the backlash as fast as possible, but without a large acceleration derivative (jerk) at the contact instant. One existing control strategy today is to first control the engine torque to a low positive value. After a pre-specified time, under which the backlash is supposed to have been traversed, the controller starts to follow the driver’s torque request. With respect to the backlash traverse, this is an open-loop controller. A short review of feedback strategies is found in (Lagerberg and Egardt, 2002), where some of the strategies are also evaluated.
In (Lagerberg, 2004) is described how open-loop optimization can be used to find the optimal control signal trajectory, and get a theoretical lower limit on the time needed to perform a backlash traverse. The results presented there resemble the results in this paper, but here, feedback control laws are also computed.

Model predictive control, MPC, (Maciejowski, 2002; Mayne et al., 2000), is a method for combining open-loop optimal control calculations with feedback. In each sampling interval of the controller, an optimal open-loop solution is calculated, for a specified prediction horizon, and the first control step in this solution is taken. For the next sampling instant, a new optimal solution is calculated. This implies a large computational burden on the controller, and MPC is mostly used in processes with very slow dynamics, such as in the process industry. MPC is recently used also in automotive applications, e.g. for optimization of driving conditions (gear, speed etc.) for heavy trucks (Terwen et al., 2004), control of hybrid electric vehicles (Back et al., 2004) and control of diesel engines (Rückert et al., 2004).

The MPC theory is recently made attractive for implementation also in systems with fast dynamics. For linear and piecewise affine systems, it is possible to perform an off-line calculation of an optimal feedback control law that satisfies given constraints. See e.g. (Morari et al., 2003; Kerrigan and Mayne, 2002). This paper describes an application of this theory to the automotive powertrain with backlash.

The paper is organized as follows: Next section gives a brief description of model predictive control design for piecewise affine systems. Section 3 presents the powertrain model used in this paper. In Section 4, the control problem is further defined. Section 5 describes the synthesis of the MPC-based controller, and simulations of the resulting controller are discussed in Section 6. The paper ends with conclusions in Section 7.

2 MPC for piecewise affine systems

A piecewise affine, PWA, system is defined as

\[ x(k+1) = A_i x(k) + B_i u(k) + f_i \]
\[ L_i x + E_i u \leq W_i \]
\[ \text{if } x(k) \in D_i, \quad i = 1, \ldots, I_f \]

where \( D_i \) is a polytope defined by

\[ D_i = \{ x \in \mathbb{R}^n | H_i x \leq K_i \}, \quad i = 1, \ldots, I_f \]

and \( I_f \) is the number of dynamics that represent the system. \((H_i \in \mathbb{R}^{s_i \times n}, K_i \in \mathbb{R}^{s_i} \) where \( s_i \) is the number of scalar inequalities required to define the polytope.)

For this class of systems it is possible to make an off-line control law synthesis. In this paper, a currently developed MATLAB toolbox, the Multi-Parametric Toolbox, MPT,
Model Predictive Control of Automotive Powertrains with Backlash

(Kvasnica et al., 2004) is used for this synthesis. The toolbox contains a collection of algorithms for solving constrained optimal control problems by multi-parametric methods. For PWA systems, the general optimization problem that the MPT toolbox solves is

\[ \min_{u(0), \ldots, u(N-1)} \sum_{k=0}^{N-1} \| Q_f x(N) \|_l + \sum_{k=0}^{N-1} \| Q x(k) \|_l + \| R u(k) \|_l \]

subject to

\begin{align*}
  x(k+1) &= A_i x(k) + B_i u(k) + f_i \\
  L_i x + E_i u &\leq W_i, \quad x(k) \in \mathcal{D}_i, \\
  i &= 1, \ldots, I_f, \quad k = 1, \ldots, N-1 \\
  x(N) &\in \mathcal{X}_{set}
\end{align*}

and where \( Q, Q_f, R \) are appropriate weighting matrices for the chosen norm \( l \in \{1, 2, \infty\} \). \( \mathcal{X}_{set} \) in (9) is a polytope that defines a terminal constraint. This is used as a "setpoint" for the controller. It is also possible to define the minimum time optimization problem

\[ \min_{u(0), \ldots, u(N-1)} N \]

subject to the same constraints as above.

Multi-parametric programming is used to find the optimal control sequence \( \{u(k)\}_{k=0}^{N-1} \) parameterized by the initial state \( x(0) \). This parameterization is piecewise affine, and the first control signal in the sequence can be regarded as a state feedback control law

\[ u(k) = F_r x(k) + G_r, \quad \text{if} \quad x(k) \in \mathcal{P}_r \]

where \( \mathcal{P}_r \) is a polytope defined by

\[ \mathcal{P}_r = \{ x \in \mathbb{R}^n | H_r^c x \leq K_r^c \}, \quad r = 1, \ldots, R \]

Note that in general, each system dynamics \( \mathcal{D}_i \) may contain more than one controller partition, \( \mathcal{P}_r \).

The MPT synthesis algorithm for PWA systems, described in (Grieder et al., 2004), starts in the target set, \( \mathcal{X}_{set} \) and iteratively searches the state space for polytopes from where the target set can be reached in 1, 2, 3, \ldots steps. This iteration proceeds until all of the allowed state space, \( \bigcup_{i=1}^{I_f} \mathcal{D}_i \), is explored, or until no more polytopes can be found. After this iteration, multi-parametric programming is used to find feedback controllers of the form (11) for all the found polytopes, and stored in a table together with the polytopes they are valid on. The union of all found polytopes is called the controllable set, \( \mathcal{K}_{PWA}^\infty \).

The synthesis algorithm is computationally intense, and already for the relatively small sized problem under study here, the computation time is substantial (hours to days on a standard desktop PC). Therefore, the iterations described above may be interrupted when a large enough subset of the state space is explored, resulting in the controllable set \( \mathcal{K}_N^{PWA} \), where \( N \) is the maximum number of steps needed to reach the target set.
Although the off-line synthesis is time-consuming, the on-line implementation is very fast. At each sampling instant, the current state is used to find the appropriate controller partition, \( \{ P_r \}^R_{r=1} \). The corresponding control law (11) is found in a look-up table, and the control signal is applied to the system.

### 3 Powertrain model

The powertrain model under consideration in this paper is seen in Figure 1, and the following notation is used: The indices \( m \) and \( l \) refer to motor and load respectively. \( J_m, J_l \) \([\text{kgm}^2]\) are moments of inertia and \( b_m \) and \( b_l \) \([\text{Nm/(rad/s)}]\) are viscous friction constants. \( k \) \([\text{Nm/rad}]\) is the shaft stiffness. \( T_m, T_g, T_s \) and \( T_l \) \([\text{Nm}]\) are torques at the engine output, at the gearbox input, at the gearbox output and the load input, and the road load respectively. \( u \) \([\text{Nm}]\) is the requested engine torque. \( i \) \([\text{rad/rad}]\) is the gearbox ratio. \( 2\alpha \) \([\text{rad}]\) is the backlash gap size. \( \theta_m \) and \( \theta_l \) \([\text{rad}]\) are the angular positions of motor and load. \( \theta_1, \theta_2 \) and \( \theta_3 \) \([\text{rad}]\) are the angular positions of the indicated positions on the shaft.

As seen in the figure, the powertrain consists of two rotating masses, one representing the engine flywheel (motor) and one representing the vehicle mass (load) respectively:

\[
J_m \ddot{\theta}_m + b_m \dot{\theta}_m = T_m - T_g \quad (13)
J_l \ddot{\theta}_l + b_l \dot{\theta}_l = T_s - T_l \quad (14)
\]

The masses are connected by a gearbox (with fixed gear ratio),

\[
T_g = T_s / i, \theta_3 = \theta_l, \theta_1 = \theta_m / i \quad (15)
\]

and a flexible shaft with a backlash of size \( 2\alpha \). With the backlash modeled as a dead-zone nonlinearity, the shaft torque is described by:

\[
T_s = k \begin{cases} 
\theta_1 - \theta_3 - \alpha & \text{if } \theta_1 - \theta_3 \geq \alpha \\
0 & \text{if } |\theta_1 - \theta_3| < \alpha \\
\theta_1 - \theta_3 + \alpha & \text{if } \theta_1 - \theta_3 \leq -\alpha
\end{cases} \quad (16)
\]
where the three modes are referred to as the positive contact ($co^+$), backlash ($bl$) and negative contact ($co^-$) modes respectively.

The engine dynamics is modeled as a first order system with time constant $\tau_{eng}$:

$$\dot{T}_m = \frac{u - T_m}{\tau_{eng}}$$  \hspace{1cm} (17)

For the MPC-solution, the computational complexity of the controller design is increasing rapidly with the size of the plant model. Therefore the model used here is a simplification as compared to the model in e.g. (Lagerberg and Egardt, 2002) or (Lagerberg, 2004). Specifically, shaft damping and engine delay are neglected here.

### 4 Problem formulation

The powertrain control system is suggested to be switching between two different controllers. One is a vehicle acceleration controller, designed for the powertrain strictly in one of the contact modes. The other controller is an MPC-based controller, which is designed to traverse the backlash gap in an optimal way. The MPC-based controller will typically be involved in tip-in and tip-out maneuvers, and it is the controller under study in this paper.

In terms of the powertrain model described above, the control problem related to a tip-in maneuver can be formulated as follows. See also Figure 2.

The initial condition for a tip-in is the powertrain in negative contact mode, with a retardation of the system. At the starting point, the driver steps on the accelerator pedal, and requests a positive vehicle acceleration. This initiates the MPC-controller, which has as its goal to control the powertrain into positive contact, and then achieve a pre-specified acceleration. The transition into positive contact mode should be made with the vehicle jerk (acceleration derivative) below a specified level. This means that at the contact instant, the relative speed between engine and vehicle sides should be low.

The MPC controller is designed to reach a pre-specified acceleration. However, the driver’s requested acceleration may vary. It is possible to reformulate the MPC control problem into a tracking problem, but this implies the inclusion of more states in the control design model, and hence increased computation time for the controller synthesis. Instead, it is suggested to use the MPC controller to reach a low and fixed acceleration. Then the acceleration controller is used to transfer the system to the requested acceleration and to reduce the driveline oscillations.

In this paper, three different MPC-based approaches to the solution of the backlash traverse problem are presented:

- The transfer from backlash mode to positive contact mode has to be made with a small relative speed between the engine and vehicle sides of the backlash. The MPT
Figure 2: A schematic tip-in sequence. The state-space is divided into three regions with different dynamics. The tip-in sequence starts and ends in acceleration control, while during the backlash traverse, the MPC-controller is active. The MPC-controller is designed to reach the target in minimum time, with constraints on the jerk at the contact instant.

formulation of the problem does not allow such explicit constraints at a switching instant between two affine dynamics. Instead, it is possible to reformulate the constraint as a jerk constraint in the dynamics of the positive contact mode. This is explained further in Section 5.3.

B However, as will be seen in Section 6, the discrete-time implementation will cause problems with the jerk constraint. Therefore a two-step synthesis is made. Here, the jerk constraint in A is removed, and the problem is divided into two consecutive subproblems: First, the system is controlled to almost contact between the sides, and with a small relative speed. Then, the system is controlled into the positive contact mode, and to the specified acceleration. Each subproblem is solved individually, and will be referred to as phases in the following.

C The third approach investigates what would be gained from using the second phase MPC controller to also reduce the driveline oscillations, assuming that the requested acceleration is known.

As the acceleration controller mentioned above, a linear state feedback controller (LQ) is used in this paper. It is roughly tuned to reduce the oscillations when the system is in contact mode. Integral action is used to reach the desired acceleration. Bumpless transfer
is used to initialize the control signal to the same value as for the MPC controller when
the LQ controller is activated.

5 MPC Controller design

In this section, the problem definition above is transformed into the form presented in
Section 2. To summarize, the problem set-up comprises the following: The system dy-
namics is formulated as a PWA system and the "setpoint", $x_{set}$, is defined. Constraints on
the allowed state and control variables are defined on the form (2).

5.1 PWA-formulation of dynamics

Define $\omega_m \hat{=} \dot{\theta}_m$, $\omega_l \hat{=} \dot{\theta}_l$ and $\theta_d \hat{=} \theta_1 - \theta_3 = \theta_m/i - \theta_l$, and the state vector $x = [\omega_m \; \omega_l \; \theta_d \; T_m]^T$. The model (13-17) can then be written as the PWA-system

$$\dot{x} = \begin{cases} 
A_{co+}x + Bu + f_{co+} & \text{if } \theta_d \geq \alpha \\
A_{co-}x + Bu + f_{co-} & \text{if } \theta_d \leq -\alpha \\
A_{bl}x + Bu + f_{bl} & \text{if } -\alpha < \theta_d < \alpha 
\end{cases} \quad (18)$$

where

$$A_{co+} = \begin{bmatrix} 
-b_m/J_m & 0 & -k/J_m & 1/J_m \\
0 & -b_l/J_l & 0 & 0 \\
1/i & -1 & 0 & 0 \\
0 & 0 & 0 & -1/\tau_{eng} 
\end{bmatrix} \quad (19)$$

$$A_{co-} = A_{co+} \quad (20)$$

$$A_{bl} = \begin{bmatrix} 
-b_m/J_m & 0 & 0 & 1/J_m \\
0 & -b_l/J_l & 0 & 0 \\
1/i & 1 & 0 & 0 \\
0 & 0 & 0 & -1/\tau_{eng} 
\end{bmatrix} \quad (21)$$

$$B = \begin{bmatrix} 
0 \\
0 \\
0 \frac{1}{\tau_{eng}} 
\end{bmatrix} \quad (22)$$

$$f_{co+} = \begin{bmatrix} 
-k\alpha/J_m \\
-j I \frac{1}{\tau_{eng}} \\
0 
\end{bmatrix} \quad (23)$$

$$f_{co-} = \begin{bmatrix} 
-k\alpha/J_m \\
-j I \frac{1}{\tau_{eng}} \\
0 
\end{bmatrix} \quad (24)$$
Here, the load disturbance, $T_l$, is taken as a constant, which furthermore is set to zero in the current setting. This is due to the restrictions on the allowable class of systems in the current MPT implementation, that the origin should be an equilibrium point for some of the dynamics. The road friction and air resistance parts of $T_l$ are instead included in the load friction coefficient $b_l$. The MPT problem formulation can also be extended with a description of disturbances in the states. This can be used to gain some robustness to e.g. road slope disturbances.

The regions where the respective affine dynamics are valid is defined by the inequalities in (18), which are written as polytopes $D_i$, $i \in \{co+, co-, bl\}$ from (4).

The MPC theory is based on a discrete-time system description, so a discretization of the PWA model above is used for the MPT solution.

## 5.2 Target sets

The setpoints or target sets, $X_{set}$, are polytopes, defined by inequalities of the form $Hx \leq K$.

### 5.2.1 Target set for approach A

The final target set for a tip-in maneuver is chosen as achieving a minimum load acceleration, $a_l \geq a_{l,\text{min},A}$. For positive $a_{l,\text{min},A}$ and positive vehicle speed, $\omega_l$, this can only be achieved in the $co+$-mode. Using the dynamics of this mode, the relation can be written as:

$$a_l = \dot{\omega}_l = \begin{bmatrix} 0 & -\frac{b_l}{J_l} & \frac{k}{J_l} & 0 \end{bmatrix} x - \frac{k\alpha}{J_l} \geq a_{l,\text{min},A}$$

(26)

or

$$\begin{bmatrix} 0 & \frac{b_l}{J_l} & -\frac{k}{J_l} & 0 \end{bmatrix} x \leq -a_{l,\text{min},A} - \frac{k\alpha}{J_l}$$

(27)

### 5.2.2 Target sets for approach B

The first phase should end when the engine and vehicle sides of the backlash is almost in contact, and with a small relative speed. The engine torque should be close to zero at the contact instant, in order to get a globally optimal solution for the total backlash traverse. This is seen in the open-loop optimal results in (Lagerberg, 2004). These conditions are
formulated as
\[
\theta_{d,\text{min},B1} \leq \theta_d \leq \theta_{d,\text{max},B1}
\]  
(28)
\[
\omega_{\text{diff, min},B1} \leq \frac{\omega_m}{i} - \omega_l \leq \omega_{\text{diff, max},B1}
\]  
(29)
\[
T_{m,\text{min},B1} \leq T_m \leq T_{m,\text{max},B1}
\]  
(30)
where \(\theta_{d,\text{min},B1}, \theta_{d,\text{max},B1}\) are close to, but smaller than \(\alpha\) and \(\omega_{\text{diff, min},B1}, \omega_{\text{diff, max},B1}, T_{m,\text{min},B1}, T_{m,\text{max},B1}\) are close to zero.

Written on polytope form this becomes:
\[
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & -1 & 0 \\
\frac{1}{i} & -1 & 0 & 0 \\
-\frac{1}{i} & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
x
\end{bmatrix}
\leq
\begin{bmatrix}
\theta_{d,\text{max},B1} \\
-\theta_{d,\text{min},B1} \\
\omega_{\text{diff, max},B1} \\
-\omega_{\text{diff, min},B1} \\
T_{m,\text{max},B1} \\
-T_{m,\text{min},B1}
\end{bmatrix}
\]  
(31)

The second phase target is identical to (27) for approach A.

5.2.3 Target sets for approach C

The first phase is identical in approach B and approach C. For the second phase, the target set from B is used, with the additional requirement to reduce the driveline oscillations when the second target set is reached. This is accomplished by a speed difference limit similar to (29), but now in contact mode. In addition, an acceleration difference limit is required:
\[
\dot{\omega}_{\text{diff, min},C2} \leq \dot{\omega}_m/i - \dot{\omega}_l \leq \dot{\omega}_{\text{diff, max},C2}
\]  
(32)

Since a positive acceleration is required as above, only the positive contact mode is active here. Using that dynamics, the inequality can be written:
\[
\dot{\omega}_{\text{diff, min},C2} \leq H_C x + K_C \leq \dot{\omega}_{\text{diff, max},C2}
\]  
(33)
with
\[
H_C = \begin{bmatrix}
-\frac{b_m}{J_{m1}} & \frac{b_l}{J_l} \\
-\frac{k}{J_m} - \frac{k}{J_l} & \frac{1}{J_{m1}}
\end{bmatrix}
\]  
(34)
\[
K_C = \left( \frac{k}{J_{m1}^2} + \frac{k}{J_l} \right) \alpha
\]  
(35)

The target set is defined as the combination of (27), (29) and (33):
\[
\begin{bmatrix}
0 & \frac{b_l}{J_l} & -\frac{k}{J_l} & 0 \\
\frac{1}{i} & -1 & 0 & 0 \\
-\frac{1}{i} & 1 & 0 & 0 \\
H_C & -H_C
\end{bmatrix}
\begin{bmatrix}
x
\end{bmatrix}
\leq
\begin{bmatrix}
-ar_{l,\text{min},C2} - \frac{ka_l}{J_l} \\
\omega_{\text{diff, max},C2} \\
-\omega_{\text{diff, min},C2} \\
-\dot{\omega}_{\text{diff, max},C2} - K_C \\
-\dot{\omega}_{\text{diff, min},C2} + K_C
\end{bmatrix}
\]  
(36)
5.3 Constraints

5.3.1 General constraints

The engine can deliver torque in a limited interval

\[ T_{m, \min} \leq T_m \leq T_{m, \max} \]  

(37)

In order to allow for faster decreases of the engine torque than dictated by the engine time constant \((\tau_{\text{eng}})\), the control signal is constrained to the interval

\[ u_{\min} \leq u \leq u_{\max} \]  

(38)

where \(u_{\max} = T_{m, \max}\) while \(u_{\min} \leq T_{m, \min}\) (here, a factor 10 lower). This is motivated by the fact that it is possible (at least theoretically) to completely skip the firing of a cylinder in a spark-ignited engine.

5.3.2 Constraints for approach A

An explicit constraint on the system at a certain event is impossible in the optimization algorithms currently implemented in MPT Toolbox. The jerk constraint on the contacting point is therefore reformulated as a constraint on the dynamics in the system. Suppose the allowed jerk is proportional to the vehicle acceleration, something that may be motivated from a driveability point of view. Define a jerk index as

\[ I_j = \frac{\text{\ddot{a}}_l}{|\text{\dot{a}}_l| + a_{\min}} \]  

(39)

Then it is possible to formulate a jerk limitation as

\[ |\text{\ddot{a}}_l| \leq I_{j, \max} \left(|\text{\dot{a}}_l| + a_{\min}\right) \]  

(40)

It is seen that at the contacting instant, the acceleration \(a_l\) is small, and hence the jerk is limited. For the positive contacting instant, only the positive contact mode \(\text{co}^+\) is active. For a positive contact, only positive jerk is of interest, so \(|\text{\ddot{a}}_l|\) can be replaced by \(\text{\ddot{a}}_l\). The acceleration may however be negative in the beginning of the contact mode for a tip-in. Using the system dynamics in Section 3 and for positive acceleration, (40) can be written

\[ \text{\ddot{a}}_l = \dot{\omega}_l = \frac{1}{J_l} \left(-b_l \omega_l + \dot{T}_s - \dot{T}_l\right) \leq I_{j, \max} \left(\frac{1}{J_l} (-b_l \omega_l + T_s - T_l) + a_{\min}\right) \]  

(41)

Substituting \(T_s = k(\theta_d - \alpha)\), \(\dot{T}_s = k \dot{\theta}_d = k(\omega_m/i - \omega_l)\), setting \(T_l = 0\) and assuming \(|b_l \dot{\omega}_l| \ll |\dot{T}_s|\) this can be rewritten as the first row in the polytope inequality:

\[
\begin{bmatrix}
\frac{k}{J_l} & \frac{b_l I_{j, \max} - k}{J_l} & -k J_{\max} & 0 \\
0 & \frac{b_l}{J_l} & -\frac{k}{J_l} & 0 \\
\end{bmatrix}
\begin{bmatrix}
x \\
\end{bmatrix}
\leq
\begin{bmatrix}
I_{j, \max} \left(a_{\min} - \frac{ka}{J_l}\right) \\
0 \\
\end{bmatrix}
\]  

(42)
The second row guarantees that the acceleration is positive, and is taken from (27), with a zero reference acceleration.

For negative acceleration, the term corresponding to $a_l$ changes sign in (41). This is equivalent to a change of signs on $I_{j,\text{max}}$ and $a_{\text{min}}$ in (42). The second inequality guarantees negative acceleration:

$$\begin{bmatrix} \frac{k}{J_l} & -\frac{h}{J_l}I_{j,\text{max}}k & kI_{j,\text{max}} & 0 \\ 0 & -\frac{b}{J_l} & \frac{J_l}{k} & 0 \end{bmatrix} x \leq \begin{bmatrix} -I_{j,\text{max}} \left( -a_{\text{min}} - \frac{k\alpha}{J_l} \right) \\ \frac{k\alpha}{J_l} \end{bmatrix}$$

(43)

The constraints (42), (43) are both valid in $D_{co+}$. This region is therefore split into two disjoint parts, each constrained by one of (42), (43). See also Figure 4.

### 5.3.3 Constraints for approach B

In the first phase, the total shaft displacement, $\theta_d$, is restricted to be lower than the backlash width $\alpha$ in order to avoid overshoot into the positive contact phase before the target set is reached:

$$\theta_d = \theta_1 - \theta_3 \leq \alpha$$

(44)

In the second phase, only the general constraints above apply.

### 5.3.4 Constraints for approach C

Phase 1 is identical to phase 1 of approach B. Phase 2 also has the same constraints as in approach B.

### 5.4 Controller synthesis

For each of the approaches described above, MPC controllers are synthesized using the MPT Toolbox. Dynamics, target sets and constraints are given on the form presented here.

In the two-phase approaches, care must be taken that the target set of the first controller is enclosed in the set of controllable states for the second controller, $\mathcal{X}_{\text{set},1} \subset \mathcal{X}^{PWA}_{N,2}$, so that a switch between the controllers is possible.

Due to the extensive off-line computation times, the $\mathcal{K}^{PWA}_N$ sets may not be large enough to incorporate all possible initial values for the transients the controllers are designed for. In these cases, the acceleration mode (LQ) controller is used until $\mathcal{K}^{PWA}_N$ is entered. This will be seen in the simulation results below.
6 Simulation results

All simulations start with the system in constant retardation and in negative contact. At t=0 s, the acceleration setpoint is changed to $a_l = 1.5 \text{ m/s}^2$, which approximately corresponds to $\theta_d = 0.1 \text{ rad}$. All controllers have a sampling time of 10 ms.

6.1 Approach A

Figure 3 shows the response using approach A. This result shows the potential risk of unwanted inter-sampling behaviour in a discrete-time controller design for a continuous-time system. Since the controller is working in discrete time, the jerk constraint is only satisfied at the sampling instants. The first step into the contact mode is chosen by the controller so that the system is already accelerating. At that point, a large jerk value is allowed, and hence a large relative velocity. This is a way for the controller to reach the target set in minimum time. However, for the real, continuous-time system, this is not the desired behaviour. The continuous-time system will have a large relative velocity when contact is achieved at t=0.12 s. Figure 4 shows a 3-D plot of a cut through the 4-dimensional state-space, with the continuous system trajectory taking a "short cut" outside the allowed regions.

A potential remedy to this problem is to decrease the jerk limit $I_{j,\text{max}}$. Simulations show that to have an effect, $I_{j,\text{max}}$ has to decrease so much that the constraint will be restrictive throughout the entire acceleration trajectory. Another solution to the problem is to use a shorter sampling time, but this will inevitably lead to an even more time-consuming controller synthesis.

The observations here motivate the alternative approaches described in this paper.

Another observation is that no control law can be found (within reasonable time) for the speed range of interest. Therefore, a very low speed is simulated here as compared to the other approaches.

6.2 Approach B

In Figure 5 is seen that at t=0.17 s, the system has reached the first target set (almost contact), and switches to the second phase. The final target set ($a_l \geq 1.5 \text{ m/s}^2 \iff \theta_d \geq 0.1 \text{ rad}$) is reached at t=0.28 s. After this point, the acceleration (LQ) controller takes over, and reduces the driveline oscillations.

As described above, the LQ controller is used as a "back-up" controller until the controllable region for phase 1 is reached. In the figure, phase 1 is reached at t=0.04 s. During the LQ period, the control signal is taking on its maximum value. This is due to the tuning of the LQ controller and that it has the driver’s requested acceleration as setpoint. A maximum control signal during the initial time steps is intuitively correct, and according
Figure 3: Simulation of a backlash traverse according to approach A. Upper plot: Engine speed, $\omega_m$, (solid) and wheel speed, $\omega_l$, (dashed), both scaled to vehicle speed. Middle plot: Total shaft displacement. The backlash limits ($\pm \alpha$) are indicated by dashed horizontal lines. Lower plot: Control signal, requested engine torque, $u$, (solid), and engine torque, $T_m$, (dashed). Dashed vertical lines represent switches between different controller phases.

Figure 4: Cut through the state-space along $x_4 = T_m = 0$, for controller approach A. Shown dimensions are $\omega_m$, $\omega_l$, $\theta_d$. The lower region represents the backlash dynamics, while the two regions with slanting faces represent the two sub-divisions of the positive contact mode, for positive and negative acceleration respectively. The continuous trajectory is seen to pass outside the allowed region, while all sample instants fall inside the allowed region.
to the results in (Lagerberg, 2004), this is in fact optimal. Therefore, the back-up strategy does not seem to reduce the performance significantly.

The control signal trajectory may vary significantly depending on the initial conditions when $K_{N,1}^{PWA}$ is reached. The general appearance of the engine torque is however similar: First it is increased, then controlled to its minimum value and finally increased to zero.

The oscillations are not damped until at about $t=0.7$ s, partly due to the control signal becoming saturated, and that the MPC controller leaves the system with the oscillating mode maximally excited. Figure 6 shows the result if the LQ controller is used directly from the contact instant instead. It will then be able to reduce the oscillations while transferring the system to the desired acceleration, and the oscillations are damped after approximately $t=0.4$ s.

### 6.3 Approach C

Section 4 describes how the second phase controller can be modified to also reduce the oscillations. As seen in Figure 7, this will be the optimal solution. The drawback of this solution is that only a number of pre-computed acceleration levels can be achieved. The LQ solution is more flexible in this respect.

Similarly to phase 1, the LQ controller is used as back up in the beginning of phase 2, between $t=0.17$ and $t=0.20$ s, after which the second phase controller takes over. Since the second phase MPC controller starts with maximum control signal for a number of samples, it would probably be better to use the maximum control signal during the back up period.
Figure 6: Simulation of a backlash traverse according to approach B, but with the acceleration controller activated at the contact instant. See Figure 3 for notation.

Figure 7: Simulation of a backlash traverse according to approach C. See Figure 3 for notation.
7 Conclusions

The presented controller designs are examples of the use of recently developed MPC control algorithms to a realistic application. The experience from this study is that off-line computed MPC gives promising results. In comparison with the open-loop optimal results in (Lagerberg, 2004), the performance of the presented controllers are similar. For the powertrain model at hand, it is possible to come close to the theoretically optimal open-loop performance with a feedback controller.

However, more investigations are needed. For example, the model complexity has to be very low in order to achieve a controller within reasonable time. In the powertrain application, the major simplification made is that delays are ignored. The effects of these simplifications need to be investigated.

Robustness of the controller to disturbances and model uncertainties is not considered here, something that is of great importance for a successful implementation of this control strategy in a real application. The MPT toolbox provides options to include disturbance models in the problem formulation. This may be used to improve robustness also to model uncertainties such as the neglected delay.

It is also possible to include the delay in the model, but at the cost of more states, and hence longer computation times.

The jerk limitation at contact is not possible to achieve for the continuous-time system although the discrete-time controller respects the limitation at the sampling instants. This motivates the use of a two-phase solution for the MPC controller.

The controllers in this paper assume that all state variables are measured. A state observer for the powertrain system is presented e.g. in (Lagerberg and Egardt, 2003). A combination of observer and controller is a suggested future research direction.

Acknowledgements

The authors wish to thank Michal Kvasnica and Pascal Grieder at ETH in Zürich, for all support, bug-fixes and feedback on the use of the MPT Toolbox.

References


*http://control.ee.ethz.ch/~mpt/


Paper IV

Backlash Gap Position Estimation in Automotive Powertrains

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Backlash Gap Position Estimation in Automotive Powertrains

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Abstract

Backlash in automotive powertrains is a major source of driveability limitations. In order to increase the powertrain controller performance, knowledge of the backlash properties (size and current position) is needed. In this paper, a nonlinear estimator for the current angular position in the backlash is developed, based on extended Kalman filtering theory. A linear estimator for fast and accurate estimation of the angular position of a wheel and the engine is also described. It utilizes standard ABS sensors and engine speed sensors, and is based on event based sampling, at each pulse from the sensors. The results show that the backlash position estimate is of high quality, and robust to modeling errors. The performance is increased further when the event based position estimators are used as pre-filters.

Keywords: Backlash, Estimation, Event based sampling, Extended Kalman filter, Automotive applications.

1 Introduction

Backlash is always present in an automotive powertrain, and it causes problems with driveability of the vehicle. Therefore, engine control systems must compensate for the backlash (De La Salle et al., 1999), (Lagerberg, 2001). In order to design a powertrain control system with high performance, the current position in the backlash is an important input signal (Lagerberg and Egardt, 2002).

In this paper, estimators for the position in the backlash and the total shaft displacement is described. The position difference between wheels and engine comes from the powertrain backlash and from the shaft twist due to flexibility. The principle is to estimate these contributions based on position information from wheel and engine sensors. The engine control signal is also used. The estimators are based on the Kalman filter theory.

Speed and acceleration estimation by Kalman filters in backlash-free rotating systems were treated in (Hebbale and Ghoneim, 1991) and (Yoo et al., 1997). In (Hovland
et al., 2002) the size of the backlash was estimated together with the shaft stiffness for an industrial robot arm. In (Nordin and Bodin, 1995; Nordin et al., 2001) the backlash size in an industrial drive system was estimated. In these papers, the current position was however not estimated. The focus in this paper is on position estimation. In (Lagerberg and Egardt, 2003) the backlash size was estimated in the same application and framework as described here.

One of the filters described here is a continuous Kalman filter with discrete measurement updates at non-equally spaced sampling times. Purely event based sampling of wheel sensors was used in (Persson, 2002).

The data used to evaluate the estimators described in this paper are obtained from simulation of a backlash compensation controller, described in (Lagerberg and Egardt, 2002). There, it was assumed that the signals needed for feedback are available directly, from ideal sensors. Also, estimation from measured data is included as evaluation of the final estimator combination. The intention of this paper is to describe the estimators that have to be used when a controller is to be implemented in a vehicle.

The paper is organized as follows: The next section introduces the powertrain system under consideration. In Section 3, two estimators for the angular position of a rotating system are described. Estimation of the entire powertrain, including the backlash position is treated in Section 4.

2 System models

The following powertrain and backlash models represent the "true" system, i.e. measurement data is generated from a simulation of this model (Lagerberg and Egardt, 2002). The engine and wheel sensor models are then used to generate noisy (quantized) measurement data. The models are also used for the Kalman filters in this paper.
2.1 Powertrain model

The powertrain model is representative for a passenger car on the first gear, at low speed. A two-inertia model is used, where one inertia represents the engine flywheel (motor). The other inertia represents the wheels and vehicle mass (load). A gearbox is located close to the engine inertia.

The following notation is used, see also Figure 1: The indices m and l refer to motor and load respectively. \( J_m, J_l \) [kgm^2] are moments of inertia and \( b_m, b_l \) [Nm/(rad/s)] are viscous friction constants. \( k \) [Nm/rad] is the shaft flexibility and \( c \) [Nm/(rad/s)] is the shaft damping. \( T_m, T_g, T_s \) and \( T_l \) [Nm] are torques at the engine output, at the gearbox input, at the gearbox output and the load input, and the road load respectively. \( u \) [Nm] is the requested engine torque. \( i \) [rad/rad] is the gearbox ratio. \( 2\alpha \) [rad] is the backlash gap size. \( \theta_m \) and \( \theta_l \) [rad] are the angular positions of motor and load. \( \theta_1, \theta_2 \) and \( \theta_3 \) [rad] are the angular positions of the indicated positions on the shaft. In the sequel, \( \hat{\cdot} \) will denote an estimated variable.

The equations for the inertias and the gearbox are defined by:

\[
\begin{align*}
J_m \ddot{\theta}_m + b_m \dot{\theta}_m &= T_m - T_g \\
J_l \ddot{\theta}_l + b_l \dot{\theta}_l &= T_s - T_l \\
T_g &= T_s / i, \theta_3 = \theta_l, \theta_1 = \theta_m / i
\end{align*}
\]

A flexible shaft with backlash is connecting the gearbox and the load inertia. The shaft torque is modeled as in (Nordin et al., 1997). This model is more physically correct than the traditionally used dead-zone backlash model. The shaft torque is given by:

\[
T_s = k(\theta_d - \theta_b) + c(\omega_d - \omega_b)
\]

where \( \theta_d = \theta_1 - \theta_3 = \theta_m / i - \theta_l \) is the total shaft displacement, and \( \theta_b = \theta_2 - \theta_3 \) is the position in the backlash. With \( \alpha \) denoting half the backlash size, the backlash position is governed by the following dynamics:

\[
\dot{\theta}_b = \begin{cases} 
\max(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)) & \theta_b = -\alpha \\
\dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b) & |\theta_b| < \alpha \\
\min(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)) & \theta_b = \alpha
\end{cases}
\]

The engine is modeled as an idealised torque generator. Its dynamics is of first order with time constant \( \tau_{eng} \) and time delay \( L_e \):

\[
\dot{T}_m = (u(t - L_e) - T_m(t)) / \tau_{eng}
\]

The road load is modeled as a random walk process, with \( v_{T_l} \) white noise:

\[
\dot{T}_l = 0 + v_{T_l}
\]
2.2 Speed sensors

The speed sensor used for the ABS system in a vehicle consists of a toothed wheel, which rotates with the same speed as the wheel. A magnetic or optic pick-up senses the passage of the teeth, and generates an oscillating signal with one period per tooth passage. By use of e.g. a Schmidt-trigger, this signal can be transformed into a pulse train. In the ABS system, this signal is filtered and used for speed estimation. In this paper, it is assumed that the pulsating signal is available directly, and counted. The current angular position of the wheel is then calculated as:

$$\theta_l = \frac{2\pi}{N} \# \text{pulses}$$

(8)

where $N$ is the number of teeth per revolution and $\# \text{pulses}$ is the value of the pulse counter.

It should be noted that the angular resolution of a typical ABS-sensor is lower than the backlash size. In this paper, the used resolution is 0.13 rad, while the backlash size is $2\alpha \approx 0.06$ rad, i.e. a factor 2.25.

The speed sensor for the engine has the same function as the wheel sensors, so a pulsating signal is assumed available here too. However, the resolution of this sensor is better: It has more teeth and, at lower gears, the engine has higher speed than the wheels.

3 Wheel and engine position estimation

In this section, the position of the wheels, $\theta_l$, and the engine, $\theta_m$, are estimated separately. The estimates are then used to calculate the total shaft displacement, which may be of interest, e.g. for control of driveline oscillations (without considering the backlash) (Chen, 1997), (Fredriksson et al., 2002). The results presented here are also prerequisites for the estimators in the next section.

The "standard" use of pulse encoders for speed measurement is to count the number of pulses during a fixed sampling interval, and then calculate the speed. This approach is too slow for this application, and there will be a quantization error of one pulse in the measurement. Therefore, two alternative estimation methods are described in this section.

3.1 Continuous Kalman filter

In the first filter, the input is assumed to be white noise, $v_a$, which models jerk as a random walk process. Noise can also be added to the other states. The position measurement, $y$, used in the simulations is in the form of (8), i.e. a continuous signal with discrete steps. However, in the derivation of the filter, the quantization error is (wrongly) treated as a
white noise disturbance, \( w \). This assumption is further dealt with in the next subsection.

\[
\begin{align*}
\dot{\theta} &= \omega + v_{\theta} \\
\dot{\omega} &= a + v_{\omega} \\
\dot{a} &= v_{a} \\
y &= \theta + w
\end{align*}
\] (9-12)

On state space form:

\[
\begin{align*}
\dot{x} &= Ax + v \\
y &= Cx + w
\end{align*}
\] (13)

where

\[
x = \begin{bmatrix} \theta & \omega & a \end{bmatrix}^T, \quad v = \begin{bmatrix} v_{\theta} & v_{\omega} & v_{a} \end{bmatrix}^T
\]

\[
A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}
\] (14-15)

For this model a Kalman filter is designed:

\[
\begin{align*}
\dot{\hat{x}} &= A\hat{x} + K(y - C\hat{x}) \\
\hat{y} &= C\hat{x}
\end{align*}
\] (16)

where \( K \) is the Kalman filter gain. The noise covariance matrices \( Q \) and \( R \) for the process noise, \( v \), and the measurement noise, \( w \), respectively, is chosen as:

\[
Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R_{\text{engine}} = \rho_m, \quad R_{\text{wheels}} = \rho_l
\] (17)

The simulation of a backlash traverse is shown in Figure 2 and Figure 3. The first figure shows the estimation error for wheel and engine positions. This should be compared to the unfiltered quantization noise with an amplitude of 0.065 rad. The second figure shows the total shaft displacement estimate, which is calculated as \( \theta_d = \dot{\theta}_m / i - \dot{\theta}_l \).

It can be demonstrated that removal of the acceleration state will result in a stationary estimation error when estimating a constant acceleration.

### 3.2 Continuous Kalman filter with event based updates

The continuous filter above treats the quantized position signal as continuous with white noise added. If, instead, information about the exact time instants for the pulses is used, a more accurate estimate is expected. This estimation is achieved by an event based sampling method: A continuous wheel/engine model is used for prediction, and discrete position updates are made when a pulse is detected (at non-equal time intervals). The Continuous-Discrete Kalman filter can be formulated as in (Jazwinski, 1970):
Figure 2: Position estimation error for engine and wheels using separate continuous Kalman filters. As a comparison, the total backlash gap is indicated by the straight horizontal lines. Solid: Wheel position error, $\hat{\theta}_l - \theta_l$. Dashed: Engine position error, scaled with the gear ratio, $(\hat{\theta}_m - \theta_m)/i$.

Figure 3: Total shaft displacement estimation using separate Kalman filters for engine position and wheel position. Solid: 3rd order Kalman filter. Dashed: True displacement.
Pulses arrive at times \( t_k, \; k = 1, \ldots \). At a pulse observation, the system states, \( \hat{x} \), and the estimation error covariance matrix, \( P \), are updated according to:

\[
\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_{k|k-1}(y_k - C\hat{x}_{k|k-1}) \tag{18}
\]

\[
P_{k|k} = P_{k|k-1} - K_{k|k-1}CP_{k|k-1} \tag{19}
\]

where the Kalman gain, \( K \), is given by:

\[
K_{k|k-1} = P_{k|k-1}C^T(CP_{k|k-1}C^T + R)^{-1} \tag{20}
\]

Between the pulse observations \((t_k \leq t < t_{k+1})\), \( \hat{x} \) and \( P(t) \) evolve continuously from \( \hat{x}_{k|k} \) to \( \hat{x}_{k+1|k} \) and from \( P_{k|k} \) to \( P_{k+1|k} \):

\[
\dot{\hat{x}} = A\hat{x} \tag{21}
\]

\[
\dot{P} = AP + PA^T + Q \tag{22}
\]

Using the system matrices \( A, C \) from (15) and \( Q, R \) retuned from (17), the resulting filter performance is radically improved, see Figure 4 for the total shaft displacement estimate, and the estimation error. In Figure 5, the prediction-measurement update process is visualized.

The total shaft displacement, \( \theta_d \), can be estimated with high quality. However, there is yet no information about the position in the backlash gap, \( \theta_b \). This is treated in the next section.
4 Backlash position estimation

In order to have a good estimate of the backlash position, the backlash has to be included in the model, which will then be nonlinear, and therefore an Extended Kalman filter, EKF, is derived here.

In this paper, the backlash gap size, $\alpha$, is considered to be a known parameter, which is a fairly strong assumption. In the framework used in this paper, estimation of the backlash size is possible, which is described in (Lagerberg and Egardt, 2003).

The nonlinear powertrain model can be written as:

\[
\begin{align*}
\dot{x} &= f(x, u, v) \\
y &= g(x, w)
\end{align*}
\]  

(23)

It will be shown in the sequel that, exploiting the structure of the nonlinear backlash model, the powertrain model can be written as a system switching between two linear modes, called backlash mode (bl) and contact mode (co). The state-space model (23) can therefore be written as:

\[
\begin{align*}
\dot{x} &= \begin{cases} 
A_{co}x + Bu + v, & \text{co-mode} \\
A_{bl}x + Bu + v, & \text{bl-mode}
\end{cases} \\
y &= Cx + w
\end{align*}
\]  

(24)

(25)

To derive the $A$, $B$, $C$ matrices, the powertrain and backlash models from (1-7) are used.
The following state and measurement vectors are used:

\[ x = [ \theta_m \ \omega_m \ \theta_l \ \omega_l \ T_l \ T_m \ \theta_b ]^T \]  \hspace{1cm} (26)

\[ y = [ \theta_m \ \theta_l ]^T \]  \hspace{1cm} (27)

State and measurement noise vectors are defined as:

\[ v = [ v_1 \ \cdots \ v_7 ]^T, \ w = [ w_1 \ w_2 ]^T \]  \hspace{1cm} (28)

Using the definitions of the states above, the backlash dynamics (5) can be rewritten as:

\[ \dot{\theta}_b = \dot{x}_7 = H(x) \hat{=} \begin{cases} \max(0, h(x)) & x_7 = -\alpha \\ h(x) & |x_7| < \alpha \\ \min(0, h(x)) & x_7 = \alpha \end{cases} \]  \hspace{1cm} (29)

where \( h(x) \) is linear:

\[ h(x) = ax = [ k/ci \ 1/i \ -k/c \ -1 \ 0 \ 0 \ -k/c ] x \]  \hspace{1cm} (30)

In the derivation of the Extended Kalman filter, the nonlinear process model (23) is linearized, which in this case is equal to linearizing \( H(x) \):

\[ a_7 = \frac{\partial H}{\partial x} = \begin{cases} 0 & ax < 0 \ x_7 = -\alpha \ \text{co} \\ a & ax \geq 0 \ x_7 = -\alpha \ \text{bl} \\ a & |x_7| < \alpha \ \text{bl} \\ 0 & ax > 0 \ x_7 = \alpha \ \text{co} \\ a & ax \leq 0 \ x_7 = \alpha \ \text{bl} \end{cases} \]  \hspace{1cm} (31)

so the nonlinearity only consists of two distinct modes, each linear.

The seventh row of the \( A \)'s in (24), \( a_7 \), has the value \( a \) or 0, depending on the five sets of conditions in (31). A value of \( a \) corresponds to the backlash being open (no contact), or in contact, but moving towards opening (bl-mode). A value of 0 corresponds to persistent contact (co-mode).

Since the shaft torque, \( T_s \), shows up in the state equations for \( \omega_m \) and \( \omega_l \), also \( \omega_b = \dot{\theta}_b \) does. Therefore the same nonlinearity as in \( a_7 \) will also show up in the second and fourth rows of \( A \). Remaining rows of \( A \) are equal in both modes, as is \( B \) and \( C \), yielding the
following matrices:

\[
A_{co} = \begin{bmatrix}
  0 & 1 & 0 & 0 & 0 & 0 \\
  -\frac{k}{J_{m1}} & -\frac{c}{J_{m1}} & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 \\
  k & \frac{c}{J_{m2}} & k & \frac{c}{J_{m2}} & 0 & 0 \\
  -\frac{k}{J_{l1}} & -\frac{c}{J_{l1}} & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]  (32)

\[
A_{bl} = \begin{bmatrix}
  0 & 1 & 0 & 0 & 0 & 0 \\
  0 & -\frac{b_m}{J_m} & 0 & 0 & 0 & \frac{1}{J_m} \\
  0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & -\frac{b_l}{J_l} & -\frac{1}{J_l} & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 \\
  \frac{k}{c} & \frac{1}{c} & -\frac{k}{c} & -1 & 0 & 0 \\
\end{bmatrix}
\]  (33)

\[
B = \begin{bmatrix}
  0 & 0 & 0 & 0 & 1/\tau_{eng} & 0 \\
\end{bmatrix}^T
\]  (34)

\[
C = \begin{bmatrix}
  1 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 & 0 \\
\end{bmatrix}
\]  (35)

A continuous EKF uses a nonlinear process model for prediction, and its linearization for the calculation of \(K\) and \(P\). It is formulated as (Jazwinski, 1970):

\[
\dot{\hat{x}} = f(\hat{x}, u) + K(y - h(\hat{x}, u))
\]  (36)

\[
K = PC(\hat{x})R^{-1}
\]  (37)

\[
\dot{P} = A(\hat{x})P + PA(\hat{x})^T - PC(\hat{x})R^{-1}C(\hat{x})P + Q
\]  (38)

In general, an EKF cannot use the stationary solution to the Riccati equation. However, since the model only switches between two linear modes, an EKF based on two stationary linear gains are used here:

\[
\dot{x} = \begin{cases}
  A_{co}\hat{x} + Bu + K_{co}(y - C\hat{x}), & \text{co-mode} \\
  A_{bl}\hat{x} + Bu + K_{bl}(y - C\hat{x}), & \text{bl-mode}
\end{cases}
\]  (39)

\[
\dot{y} = C\hat{x}
\]  (40)

where \(K_{co}\) and \(K_{bl}\) are designed for their respective cases, and the \(A, B,\) and \(C\)-matrices are taken from (32-35). The control signal is delayed with the same time delay as in the engine model, \(u(t) = u_{control}(t - L_e)\).

### 4.1 Evaluation

The performance of the EKF is evaluated both by simulations and on real vehicle data.
Figure 6: Total shaft displacement. Solid: Extended Kalman Filter estimate (almost on top of true value). Dashed: True displacement. The estimation error is also plotted.

The continuous-discrete filters on wheel and engine, described in Section 3.2, are used as pre-filters to the EKF. The inputs to the EKF is therefore \( \hat{\theta}_m \) and \( \hat{\theta}_l \), the pre-filter estimates, as well as the control signal, \( u \). Figure 6 shows the total shaft displacement estimate, which is improved as compared to Figure 4. The backlash position estimate is seen in Figure 7, where also the influence of the pre-filters is seen. As a robustness check of this filter, it is designed for 1.5 times the nominal values of \( \tau_{eng} \) and \( L_e \).

To further validate the performance of the EKF, measurements in a real vehicle were performed. The measurements were made with high accuracy, and to emulate measurement with lower quality, random errors, simulating sensor tooth position imperfections, were added to the measured signals. The pre-filter - EKF combination had the low quality signals as inputs, and the high quality measurements were taken as the "true" total shaft displacement. The same signals were also used to simultaneously estimate the backlash gap size, as described in (Lagerberg and Egardt, 2003). Figure 8 shows the total shaft displacement estimates and in Figure 9, the backlash position estimate is seen.

5 Conclusions

In this paper, an extended Kalman filter for the backlash position is developed. From the simulations, it is seen that the estimated position follows the true position very well, although some of the model parameters were given with a 50 % error. Evaluation on real vehicle data also shows that the filter works as expected. Event based Kalman filters for wheel and engine position estimation is also described. These can be used to calculate the total shaft displacement, which can be useful for control of driveline oscillations.
**Figure 7:** Backlash position estimate. Solid: Extended Kalman Filter estimate (almost on top of true value). Dashed: True value. Dash-dotted: Estimate without pre-filter. The estimation error is also plotted for the two cases.

**Figure 8:** Total shaft displacement with measured data. Solid: Extended Kalman Filter estimate (almost on top of "true" value). Dashed: "True" displacement. Dash-dotted: Estimate without pre-filter. The estimation error is also plotted for the two cases (solid and dash-dotted).
By using the position estimators as pre-filters, the backlash position filter performance is improved. The results indicate that the described estimation methods can be useful in powertrain applications, and they may also be applicable to other systems with backlash. Further application of the estimators to real vehicle data is under investigation.

Use of the estimators in closed loop control for backlash compensation is a natural next step.

### Acknowledgements

This work was supported by the Volvo Research and Educational Foundations, the Swedish Automotive Research Program and the Volvo Corporation.

### References


Paper V

Estimation of Backlash with Application to Automotive Powertrains

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Estimation of Backlash with Application to Automotive Powertrains

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Abstract

In rotating systems, backlash imposes limitations on the quality of control. Automotive powertrains is an example where this is a well-known limitation. In order to increase the controller performance, knowledge of the backlash properties (size and current position) is needed. In this paper, nonlinear estimators for backlash size and position are described, both based on Kalman filtering theory. The size and position estimators are combined, and the result is an estimate of the state of the backlash. This information can be used in feedback control of the rotating system. Simulation and experimental results show that the estimates are of high quality, and robust to modeling errors.

1 Introduction

In rotating mechanical systems, backlash is almost always present, originating e.g. from play between gears. When the systems are controlled, one of the limitations of the control loop is the ability to compensate for the backlash. In order to design a control system with high performance, both the backlash size and the current position in the backlash are important inputs. This paper describes methods for estimation of these quantities.

One application of special interest is the control of automotive powertrains. When the driver goes from engine braking to acceleration or vice versa, the backlash is traversed, and potentially uncomfortable "shunt and shuffle" phenomena are experienced. Therefore, engine control systems must compensate for the backlash (De La Salle et al., 1999), (Lagerberg, 2001). Thus, backlash estimation in automotive powertrains is the focus of this paper, although the results are applicable to a wider range of rotating systems.

Not much is written on backlash estimation in rotating systems. Reports on position estimation is almost non-existent. In (Tao and Kokotovic, 1996), size estimation comes as part of adaptive control methods for specific system structures; backlash on system input
or output. These model structures are however not applicable for modeling of rotating systems. In (Hovland et al., 2002), backlash size was estimated for an industrial robot arm. An offline method is described, where parameter estimation yields shaft torque as function of shaft displacement, i.e. a plot of the shaft stiffness and backlash characteristics. In (Nordin et al., 2001), a method suitable for rotating systems is described, in which a sinusoid is added to the control signal. A gap size parameter is then adjusted until the gains of a model plant and the physical plant coincide.

In automotive powertrain applications, the backlash size varies between vehicle individuals and with wear, but also with temperature. Therefore, the size estimator should be running continuously, and hence no experiments such as adding sinusoids are allowed. The only excitation needed for the estimators presented in this paper is normal driving, with accelerations and retardations (engine braking).

The temperature dependence becomes clear if the flexible engine suspension is considered as a part of the powertrain backlash. However, this is not explicitly taken into account in the powertrain model used here.

In an automotive powertrain, backlash and shaft flexibility results in an angular position difference between wheels and engine. The principle is to estimate these contributions based on position information from wheel and engine sensors. The engine control signal is also used.

The data used to evaluate the estimators are obtained from simulation of a backlash compensation controller, described in (Lagerberg and Egardt, 2002). There, it is assumed that the signals needed for feedback are available directly, from ideal sensors. Experimental data from a test vehicle is also used for evaluation of the size estimator. The intention of this paper is to describe the estimators that have to be used when the controller is to be implemented in a vehicle.

Next section describes the powertrain model and the sensor properties. In Section 3, an online method for size estimation based on Kalman filtering theory is described. In Section 4, estimation of the current position in the backlash gap is briefly reviewed. It has earlier been reported in (Lagerberg and Egardt, 2003). There, the size was assumed a known parameter, a limitation which is removed by use of the results presented here.

2 System models

The following powertrain and backlash models represent the "true" system, i.e. measurement data is generated from a simulation of this model (Lagerberg and Egardt, 2002). The engine and wheel sensor models are then used to generate noisy (quantized) "measurement" data. The models are also used for the Kalman filters in this paper.
2.1 Powertrain model

The powertrain model is representative for a passenger car on the first gear, at low speed. A two-inertia model is used, where one inertia represents the engine flywheel (motor). The other inertia represents the wheels and vehicle mass (load). A gearbox is located close to the engine inertia.

The following notation is used, see also Figure 1: The indices \( m \) and \( l \) refer to motor and load respectively. \( J_m, J_l \) [kgm\(^2\)] are moments of inertia and \( b_m, b_l \) [Nm/(rad/s)] are viscous friction constants. The latter model lubricants, road friction and air drag. In this paper they are set to zero, but are included for the sake of generality. \( k \) [Nm/rad] is the shaft flexibility and \( c \) [Nm/(rad/s)] is the shaft damping. \( T_m, T_g, T_s, T_l \) [Nm] are torques at the engine output, at the gearbox input, at the gearbox output and the load input, and the road load respectively. \( u \) [Nm] is the requested engine torque. \( i \) [rad/rad] is the gearbox ratio. 2\( \alpha \) [rad] is the backlash gap size. \( \theta_m \) and \( \theta_l \) [rad] are the angular positions of motor and load. \( \theta_1, \theta_2 \) and \( \theta_3 \) [rad] are the angular positions of the indicated positions on the shaft. In the sequel, \( \hat{\cdot} \) will denote an estimated variable.

The equations for the inertias and the gearbox are defined by:

\[
J_m \ddot{\theta}_m + b_m \dot{\theta}_m = T_m - T_g \\
J_l \ddot{\theta}_l + b_l \dot{\theta}_l = T_s - T_l \\
T_g = T_s / i, \theta_3 = \theta_l, \theta_1 = \theta_m / i
\]

A flexible shaft with backlash is connecting the gearbox and the load inertia. The shaft torque is modeled as in (Nordin et al., 2001). This model is more physically correct than the traditionally used dead-zone backlash model. The shaft torque is given by:

\[
T_s = k(\theta_d - \theta_b) + c(\omega_d - \omega_b)
\]

where \( \theta_d = \theta_1 - \theta_3 = \theta_m / i - \theta_l \) is the total shaft displacement, and \( \theta_b \approx \theta_2 - \theta_3 \) is the position in the backlash. With \( \alpha \) denoting half the backlash size, the backlash position is
governed by the following dynamics:

\[
\dot{\theta}_b = \begin{cases} 
\max(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)) & \theta_b = -\alpha \\
\dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b) & |\theta_b| < \alpha \\
\min(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)) & \theta_b = \alpha 
\end{cases}
\] 

(5)

The engine is modeled as an idealised torque generator. Its dynamics is of first order with time constant \(\tau_{\text{eng}}\) and time delay \(L_e\):

\[
\dot{T}_m = \left( u(t - L_e) - T_m(t) \right) / \tau_{\text{eng}}
\]

(6)

The road load is modeled as a random walk process, with \(v_{T_l}\) white noise:

\[
\dot{T}_l = 0 + v_{T_l}
\]

(7)

### 2.2 Speed sensors

The speed sensor used for the ABS system in a vehicle consists of a toothed wheel, which rotates with the same speed as the wheel. A magnetic or optic pick-up senses the passage of the teeth, and generates an oscillating signal with one period per tooth passage. By use of e.g. a Schmidt-trigger, this signal can be transformed into a pulse train. In the ABS system, this signal is filtered and used for speed estimation. In this paper, it is assumed that the pulsating signal is available directly, and counted. The current angular position of the wheel is then calculated as

\[
\theta_l = \frac{2\pi}{N} \#_{\text{pulses}}
\]

(8)

where \(N\) is the number of teeth per revolution and \(#_{\text{pulses}}\) is the value of the pulse counter.

It should be noted that the angular resolution of a typical ABS-sensor is lower than the backlash size. In this paper, the used resolution is 0.13 rad, while the backlash size is \(2\alpha \approx 0.06\) rad, i.e. a factor 2.25.

The speed sensor for the engine has the same function as the wheel sensors, so a pulsating signal is assumed available here, too. However, the resolution of this sensor is better: It has more teeth and, at lower gears, the engine has higher speed than the wheels.

### 3 Backlash size estimation

The fundamental idea for backlash size estimation is to use position measurements from the speed sensors at engine and wheel, and use the position difference as a measure of the backlash size. The engine control signal is also assumed available.
To estimate the backlash size, first a backlash-free model of the powertrain is derived. As a second step, the introduction of backlash can be seen as a varying offset in the position sensors. The linear model is augmented, and a switching Kalman filter results. The two steps are described below.

3.1 Kalman filter for a backlash-free powertrain model

If, instead of the backlash model (4-5), the shaft is modeled as having no backlash, the shaft torque depends linearly on the engine and wheel positions and velocities:

\[ T_s = k(\theta_1 - \theta_3) + c(\omega_1 - \omega_3) \]  

(9)

Using the powertrain model (1-3),(6-7), the process model can be written on state space form:

\[
\begin{align*}
\dot{x} &= Ax + Bu + v \\
y &= Cx + w
\end{align*}
\]  

(10, 11)

where

\[
x = \begin{bmatrix} \theta_m & \omega_m & \theta_l & \omega_l & T_i & T_m \end{bmatrix}^T
\]

(12)

\[
y = \begin{bmatrix} \theta_m & \theta_l \end{bmatrix}^T
\]

(13)

are the state and measurement vectors,

\[
v = [v_1 \cdots v_6]^T, \ w = [w_1, w_2]^T
\]

(14)

are the process and measurement noise vectors, and

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-k/J_m & -c/J_m & -k/J_m & c/J_m & 0 & 1/J_m \\
0 & 0 & 0 & 1 & 0 & 0 \\
k/J_i & c/J_i & -k/J_i & -c/J_i & 1/J_i & 0 \\
0 & 0 & 0 & 0 & 0 & 1/J_eng \\
0 & 0 & 0 & 0 & 0 & 1/J_eng \\
\end{bmatrix}
\]

(15)

\[
B = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1/\tau_{eng}
\end{bmatrix}
\]

(16)

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\]

(17)
3.2 Switched Kalman filter for a powertrain with backlash

The model above will yield a steady-state error in the position difference between engine and wheel. For the system in either positive or negative contact, the error will be constant and related to the backlash size, $\alpha$, but also to a possible offset, $\theta_o$, between the zero position of the two sensors. To make a correction, two offset parameters are introduced: $\theta_{o+}$ and $\theta_{o-}$, valid for positive and negative contact respectively. The relationship between the parameters is:

$$\theta_{o+} = \theta_o + \alpha$$  \hspace{1cm} (18)
$$\theta_{o-} = \theta_o - \alpha$$  \hspace{1cm} (19)

The offset parameters are modeled as random walk processes, augmented as states in (10) and subtracted from the wheel position state in the measurement equation (11).

A switched Kalman filter is used to estimate the offset parameters, such that the positive offset parameter is estimated (and used in the measurement equation) for positive contact, and correspondingly for negative contact. During the backlash traverse, none of the offset parameters should be updated by the filter, and hence a wait-mode is introduced. The switching is based on detection of sign changes of the control signal, and the new estimation mode is entered after some waiting time. The backlash size estimator becomes:

$$\dot{x} = A^* \dot{x} + B^* u + K^* (y - C^* \dot{x})$$  \hspace{1cm} (20)
$$\dot{\alpha} = (\dot{\theta}_{o+} - \dot{\theta}_{o-})/2$$  \hspace{1cm} (21)

where

$$x = [\theta_m \omega_m \theta_l \omega_l T_l T_m \theta_{o+} \theta_{o-}]^T$$  \hspace{1cm} (22)
$$y = [\theta_m \theta_l]^T$$  \hspace{1cm} (23)

$$A^* = \begin{bmatrix} A & \vdots & \vdots \\ \vdots & 0 & 0 \\ 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 \end{bmatrix}$$  \hspace{1cm} (24)

$$B^* = [0 0 0 0 1/\tau_{eng} 0 0]^T$$  \hspace{1cm} (25)

$$C^* = \begin{cases} C_+, & \text{positive estimation-mode} \\ C_-, & \text{negative estimation-mode} \end{cases}$$  \hspace{1cm} (26)

$$K^* = \begin{cases} K_+, & \text{positive estimation-mode} \\ K_-, & \text{negative estimation-mode} \\ K_0, & \text{wait-mode} \end{cases}$$  \hspace{1cm} (27)

$$C_+ = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$  \hspace{1cm} (28)
The Kalman gains are calculated for the linear model \((A^*, C_+)\), with the last (8th) state removed to make the system observable. The elements of the resulting (reduced) \(K_+\) are then rearranged to yield \(K_-\) and \(K_0\). Wait-mode \(+\) in (26) means that this \(C\)-value is used when waiting to enter the positive estimation-mode, and correspondingly for the negative mode.

The control signal is delayed with the same time as in the engine model, \(u(t) = u_{\text{control}}(t - L_e)\).

### 3.3 Simulation results

The test data for estimation comes from a simulation of a backlash compensation controller, (Lagerberg and Egardt, 2002), following a stepwise vehicle acceleration reference. The total shaft displacement, \(\theta_d\), is included in Figure 2, to show how the backlash is traversed. The figure also shows that the size estimate converges after approximately 6 traverses. The remaining fluctuations are due to the measurement noise (quantization) and "bumps" when switching estimator mode. The nature of the switching logics is seen in Figure 3. The offset parameters are estimated when the corresponding estimation mode is active. The parameter estimation is interrupted when the shaft displacement is beginning to move towards the backlash gap. The estimation is continued when the displacement has been stabilized with some margin to the backlash gap, to ensure a stable contact.

To check the robustness to modeling errors, the shaft stiffness, \(k\), is increased by 10\% in the estimation model (relative to the nominal model which is used for data generation).
In Figure 4, it is seen that this results in an increase in the estimated backlash size by approximately 10%.

The values $\tau_{\text{eng}}$ and $L_e$ are also increased by 50%, but this doesn’t influence the estimate. This is natural, since these parameters do not influence the static properties of the model, such as the backlash size.

3.4 Experimental results

To further validate the performance of the size estimator, measurements in a real vehicle were performed. Figure 5 shows the estimated backlash size. As a way to manually estimate the backlash size, one of the driven wheels were lifted from the ground, and rotated by hand. On the first gear, this backlash size estimate was $\alpha \approx 0.024$ rad. The same measurement data was also used to simultaneously estimate the backlash gap position, as described below. More details on the experimental results can be found in (Lagerberg and Egardt, 2004).

4 Backlash position estimation

In order to estimate the backlash position, the backlash has to be included in the model. The model will then be nonlinear, and therefore an Extended Kalman filter, EKF, is de-
**Figure 3:** Upper plot: Switching of the size estimator. The offset parameters, $\theta_o+$ and $\theta_o-$ are also shown. Solid: Estimator mode. A value of 0.1 indicates contact (positive or negative). Dashed: Offset parameters. The true backlash gap is indicated by horizontal lines. Lower plot: The total shaft displacement indicates which mode the estimator is in.

**Figure 4:** Estimated backlash size with 10% error in shaft stiffness. Solid: Backlash estimate, $\hat{\alpha}$. Dashed: True backlash size, $\alpha$. 
Figure 5: Backlash size estimate, $\hat{\alpha}$ [rad]. A "settled" sequence of $\hat{\alpha}$ is shown in the top left corner.

derived here. The complete powertrain model from (1-6) is used. The following state and measurement vectors are used:

$$x = \begin{bmatrix} \theta_m & \omega_m & \theta_l & \omega_l & T_l & T_m & \theta_b \end{bmatrix}^T$$  \hspace{1cm} (33)$$

$$y = \begin{bmatrix} \theta_m & \theta_l \end{bmatrix}^T$$  \hspace{1cm} (34)$$

State and measurement noise vectors are defined as:

$$v = \begin{bmatrix} v_1 & \cdots & v_7 \end{bmatrix}^T, w = \begin{bmatrix} w_1 & w_2 \end{bmatrix}^T$$  \hspace{1cm} (35)$$

Using the definitions of the states above, (5) can be rewritten as:

$$\dot{x}_7 = H(x) = \begin{cases} 
\max(0, h(x)) & x_7 = -\alpha \\
h(x) & |x_7| < \alpha \\
\min(0, h(x)) & x_7 = \alpha 
\end{cases}$$  \hspace{1cm} (36)$$

where $h(x)$ is linear:

$$h(x) = ax = \begin{bmatrix} k/c & 1/i & -k/c & -1 & 0 & 0 & -k/c \end{bmatrix} x$$  \hspace{1cm} (37)$$

Linearizing $H(x)$ yields:

$$a_7 = \frac{\partial H}{\partial x} = \begin{bmatrix} 0 & ax < 0 & x_7 = -\alpha & \text{co} \\
a & ax \geq 0 & x_7 = -\alpha & \text{bl} \\
a & |x_7| < \alpha & x_7 = \alpha & \text{bl} \\
0 & ax > 0 & x_7 = \alpha & \text{co} \\
a & ax \leq 0 & x_7 = \alpha & \text{bl} 
\end{bmatrix}$$  \hspace{1cm} (38)$$
so the nonlinearity only consists of two distinct modes, each linear. The nonlinear pow-


ertrain model can therefore be written on state space form, switching between two linear

modes called backlash mode (bl) and contact mode (co). The seventh row of $A$, ($a_7$), will


have the value $a$ or 0, depending on the five sets of conditions in (38). Rows 2 and 4 will


also depend on the nonlinearity, while the remaining rows of $A$ are equal in both modes,
as is $B$ and $C$, yielding the following matrices:

\[
A_{co} = \begin{bmatrix}
-\frac{k}{J_m i^2} & -\frac{c}{J_m i^2} & 0 & 0 & 0 & 0 \\
\frac{k}{J_f i} & -\frac{c}{J_f} & -\frac{k}{J_f} & -\frac{c+b_k}{J_f} & -\frac{1}{J_f} & 0 & -\frac{k}{J_f} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}
\] (39)

\[
A_{bl} = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-b_m}{J_m} & 0 & 0 & 0 & \frac{1}{J_m} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-1}{\tau_{eng}} & 0 \\
0 & \frac{k}{c} & 0 & -\frac{k}{c} & -1 & 0 & 0 \\
0 & \frac{k}{c} & 0 & -\frac{k}{c} & -1 & 0 & 0 
\end{bmatrix}
\] (40)

\[
B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1/\tau_{eng} & 0 
\end{bmatrix}^T
\] (41)

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 
\end{bmatrix}
\] (42)

An Extended Kalman filter based on this model will be a filter switching between two

linear modes:

\[
\dot{x} = \begin{cases}
A_{co} \dot{x} + Bu + K_{co}(y - C \dot{x}), & \text{co-mode} \\
A_{bl} \dot{x} + Bu + K_{bl}(y - C \dot{x}), & \text{bl-mode}
\end{cases}
\] (43)

\[
\dot{\theta}_b = \dot{x}_7
\] (44)

where $K_{co}$ and $K_{bl}$ are designed for their respective cases, and the $A$, $B$, and $C$-matrices
are taken from (39-42). The control signal is delayed with the same time as in the engine
model, $u(t) = u_{\text{control}}(t - L_e)$.

4.1 Simulation results

Figure 6 shows how the size and position estimators work together. The position filter has
a much higher bandwidth than the size filter. The backlash size signal is low pass filtered
before it is used by the position estimator.

The behaviour of the position estimator in combination with size estimation is seen in
Figures 7 and 8. Figure 8 shows two backlash traverses in detail. The first is taken before
Figure 6: Interconnection of the size and position estimators. The position estimator states are useful for a backlash compensating controller.

the size estimator has converged, while the second shows the converged estimator. For all simulations in this paper, the position estimator is designed for 1.5 times the nominal values of $\tau_{eng}$ and $L_e$. Without this model error, $\theta_b$ and $\hat{\theta}_b$ would coincide in the plots. For example, the lag in the second part of Figure 8 is due to this model error. A size estimation error originating from a 10% error in shaft stiffness, see Figure 4, has the same effect on the position estimate as an unconverged size estimate. See the first plot in Figure 8. Experimental results, similar to those for the size estimator above can be found in (Lagerberg and Egardt, 2004).

5 Conclusions

This paper describes a size estimator and a position estimator for backlash in rotating systems, both based on switched Kalman filters.

From the simulation results, it is seen that the estimated size converges to the true value after a few backlash traverses, despite that some of the model parameters were given with a 50% error. It is most sensitive to model errors in static parameters, exemplified by the shaft stiffness.

Simulations of the position estimator shows that it follows the true position in the backlash gap with high accuracy. This estimator has a higher bandwidth than the size estimator, and it may be increased further, by use of an event based Kalman filter as pre-filter on the position measurement signals. This is described in (Lagerberg and Egardt, 2003).
Figure 7: Backlash size and position estimates. Solid: Backlash position estimate. Dash-dotted: Backlash size estimate. Dashed: True backlash size.

Figure 8: Backlash size estimate, together with position estimate; close-up on a single backlash traverse. Solid: Backlash size estimate and backlash position estimate. Dashed: True backlash position and true backlash size. Dash-dotted: Position estimation error.
The results indicate that the described estimation methods can be useful in powertrain applications, and they may also be applicable to other systems with backlash.

The estimation model can be improved by inclusion of the engine suspension. This may be of importance for capturing the engine motion, which contribute to the "shunt an shuffle" phenomena.

A natural next step is to use the estimators in closed loop control for backlash compensation, which is currently under investigation.

Acknowledgements

This work was supported by the Volvo Research and Educational Foundations, the Swedish Automotive Research Program and the Volvo Car Corporation.

References


Paper VI

Estimation of Backlash in Automotive Powertrains — an Experimental Validation

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Estimation of Backlash in Automotive Powertrains — an Experimental Validation

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Abstract

In automotive powertrains, backlash imposes well-known limitations on the quality of control and hence the driveability. In order to increase the controller performance, knowledge of the backlash properties (size and current position) is needed. In previous papers, nonlinear estimators for backlash size and position are described, based on Kalman filtering theory. The size and position estimators are combined, and the result is an estimate of the state of the backlash. In this paper, experimental results from vehicle tests show that the estimates are of high quality, and hence useful for improving backlash compensation functions in the powertrain control system.

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Keywords: Backlash, Estimators, Extended Kalman filters, Automotive, Tests.

1 Introduction

In automotive powertrains, backlash is always present, originating e.g. from play between gears. When the powertrain is controlled, one of the limitations of the control loop is the ability to compensate for the backlash. In order to design a control system with high performance, both the backlash size and the current position in the backlash are important inputs. In two previous papers, (Lagerberg and Egardt, 2003a; Lagerberg and Egardt, 2003b), methods for estimation of these quantities are described. This paper describes an experimental validation of the earlier presented estimators. The purpose is to verify that estimation is possible in a car with standard sensors, and to investigate the achievable quality of the estimates.

When the driver goes from engine braking to acceleration or vice versa, so called tip-in/tip-out maneuvers, the backlash is traversed, and potentially uncomfortable "shunt and shuffle" phenomena are experienced. Therefore, engine control systems must compensate for the backlash (De La Salle et al., 1999; Lagerberg, 2001).
Not much is written on backlash estimation in rotating systems. Reports on position estimation is almost non-existent. Size estimation is treated e.g. in (Tao and Kokotovic, 1996; Hovland et al., 2002; Nordin and Bodin, 1995). Most of these results require excitation of the system in order to estimate the backlash size. In the automotive powertrain application, the backlash size varies between vehicle individuals and with wear, but also with temperature. Therefore, both the size and position estimator should be running continuously. The only excitation needed for the estimators presented in this paper is normal driving, with accelerations and retardations (engine braking).

In an automotive powertrain, backlash and shaft flexibility results in an angular position difference between wheels and engine. The principle is to estimate these contributions based on position information from wheel and engine sensors. The engine control signal is also used.

Next section describes the powertrain model and defines the notation used throughout the paper. In Section 3, the experiments are described. Sections 4-5, present the used estimators and the obtained results.

---

2 Powertrain model

The Kalman filters in this paper are based on the following powertrain model, see Figure 1. It is representative for a passenger car on the first gear, at low speed. A two-inertia model is used, where one inertia represents the engine flywheel (motor). The other inertia represents the wheels and vehicle mass (load). A gearbox is located close to the engine inertia.

The following notation is used, see also Figure 1: The indices $m$ and $l$ refer to motor and load respectively. $J_m, J_l$ [kgm$^2$] are moments of inertia and $b_m$ and $b_l$ [Nm/(rad/s)] are viscous friction constants. $k$ [Nm/rad] is the shaft flexibility and $c$ [Nm/(rad/s)] is the shaft damping. $T_m, T_g, T_s$ and $T_l$ [Nm] are torques at the engine output, at the gearbox input, at the gearbox output and the load input, and the road load respectively. $u$ [Nm] is the requested engine torque. $i$ [rad/rad] is the gearbox ratio. $2\alpha$ [rad] is the backlash gap size. $\theta_m$ and $\theta_l$ [rad] are the angular positions of motor and load. $\theta_1, \theta_2$ and $\theta_3$ [rad] are...
the angular positions of the indicated positions on the shaft. In the sequel, \( \hat{\cdot} \) will denote an estimated variable.

The equations for the inertias and the gearbox are defined by:

\[
\begin{align*}
J_m \ddot{\theta}_m + b_m \dot{\theta}_m &= T_m - T_g \\
J_l \ddot{\theta}_l + b_l \dot{\theta}_l &= T_s - T_l
\end{align*}
\]

\( T_g = T_s / i, \theta_3 = \theta_l, \theta_1 = \theta_m / i \) (3)

A flexible shaft with backlash is connecting the gear and the vehicle mass inertia. The shaft torque is modeled as in (Nordin et al., 1997). This model is more physically correct than the traditionally used dead-zone backlash model. The shaft torque is given by:

\[
T_s = k(\theta_d - \theta_b) + c(\omega_d - \omega_b)
\]

(4)

where \( \theta_d \equiv \theta_1 - \theta_3 = \theta_m / i - \theta_l \) is the total shaft displacement, and \( \theta_b \equiv \theta_2 - \theta_3 \) is the position in the backlash. With \( \alpha \) denoting half the backlash size, the backlash position is governed by the following dynamics:

\[
\dot{\theta}_b = \begin{cases} 
\max(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)) & \theta_b = -\alpha \\
\dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b) & |\theta_b| < \alpha \\
\min(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)) & \theta_b = \alpha
\end{cases}
\]

(5)

The engine is modeled as an idealised torque generator. Its dynamics is of first order with time constant \( \tau_{eng} \) and time delay \( L_e \):

\[
\dot{T}_m = \frac{(u(t - L_e) - T_m(t))}{\tau_{eng}}
\]

(6)

The road load is modeled as a random walk process, with \( v_{T_l} \) white noise:

\[
\dot{T}_l = 0 + v_{T_l}
\]

(7)

3 Data collection

3.1 Experimental setup

The measurement data used in this paper were collected in a prototype Volvo car. The car was equipped with the development version of the engine control system, ECS, in order to collect data. Among other signals, the ECS delivered the engine speed and a calculated signal for the engine torque with 10 ms sampling interval.

In parallel to the ECS data, a sampling oscilloscope was connected to the engine speed sensor, and to the speed sensors at the driven wheels. These sensors are the original ones, normally mounted in this car type. The engine sensor has 58 teeth per revolution and a gap where two teeth are missing. The purpose of the gap is for the ECS to know the exact position of the engine pistons. The wheel sensors have 48 teeth, and no missing teeth. The speed sensor signals were sampled with 2 \( \mu s \) interval.
3.2 Experiments

With the car equipped as described above, a series of experiments were performed. In each measurement sequence, a number of tip-in and tip-out maneuvers were executed. The standard shunt and shuffle compensators in the ECS were turned off or on. The results presented here are mainly from experiments without these compensators.

As a way to manually estimate the backlash size, one of the driven wheels were lifted from the ground, and rotated by hand. On the first gear, this backlash size estimate was $2\alpha \approx 0.048$ rad. It is interesting to note that this is less than the angular resolution of the wheel position sensor: $2\pi/48 \approx 0.13$ rad.

3.3 Data pre-processing

After the experiments, the data were pre-processed before they could be used by the backlash estimators.

The engine speed sensor signal is a sinusoidal voltage with one period per tooth. This signal is run through a Schmidt-trigger, which outputs a vector of the absolute times for each pulse, with a time resolution better than 2 $\mu$s. The missing teeth are virtually inserted, by linear interpolation from the teeth before and after the tooth gap. The angular position, $\theta_m$, is incremented by $2\pi/60$ rad for each pulse.

The wheel speed sensor signal is treated in the same way as the engine sensor signal, however no tooth gap has to be "filled". The angular position, $\theta_l$, is incremented by $2\pi/48$ rad for each pulse. A general expression for the engine and wheel positions is:

$$\theta_{m,l} = \frac{2\pi}{N_{m,l}} \#\text{pulses}$$

(8)

where $N_{m,l}$ is the number of teeth per revolution and $\#\text{pulses}$ is the value of the pulse counter.

The backlash estimators use the engine and wheel position sensor signals together with the engine control signal. Since the control signal is collected by the ECS, the two measurements are synchronized by use of the ECS engine speed signal. The ECS time values are shifted so that the ECS and oscilloscope speed signals match.

3.4 Disturbed data

As seen in Figure 2, the signal quality obtained with the sampling oscilloscope is very high. If $\theta_d$ is plotted only at the time instants when pulses from the wheel sensors arrive, the signal looks very good. However, when plotted also for the time instants from the engine sensor, there will be a quantization error with the magnitude of a wheel sensor increment, compare with (8).
Figure 2: "Raw" measurement data: Upper plot: Total shaft displacement $\theta_d$ [rad]. The lower graph is taken only at wheel pulse instants, while the upper graph is taken at engine pulse instants and moved 0.2 rad upwards for visibility. Lower plot: Engine torque $T_m$ [Nm].

Apart from the quantization error, no noise disturbances etc. can be seen. Also, no objective truth about the backlash position is available. To emulate measurements with lower quality, random errors, simulating sensor tooth position imperfections, are added to the measured signals. The disturbed, and quantized, signal is used as input to the filters described here, while the high quality measurements (at wheel instants) are taken as the "true" total shaft displacement, $\theta_d$.

4 Backlash size estimation

4.1 Size estimator

The backlash size estimator used here was described in (Lagerberg and Egardt, 2003b), and is only summarized here. The principle is to first model a backlash-free system. For such a system, the backlash can be seen as an offset between the position sensors at the engine and wheels. The offset will have two different values, one at positive contact, and one at negative contact. The proposed estimator tracks these two offset values, $\theta_{o+}$, $\theta_{o-}$.

A switched Kalman filter is used to estimate the offset parameters, such that the positive offset parameter is estimated (and used in the measurement equation) for positive contact, and correspondingly for negative contact. During the backlash traverse, none of the offset parameters should be updated by the filter, and hence a wait-mode is introduced. The switching is based on detection of sign changes of the control signal, and the new estima-
The backlash size estimator becomes:

\[ \dot{x} = A^* \dot{x} + B^* u + K^* (y - C^* \dot{x}) \quad (9) \]
\[ \hat{\alpha} = (\hat{\theta}_{o+} - \hat{\theta}_{o-})/2 \quad (10) \]

where

\[ x = [\theta_m \ \omega_m \ \theta_l \ \omega_l \ T_l \ T_m \ \theta_{o+} \ \theta_{o-}]^T \quad (11) \]
\[ y = [\theta_m \ \theta_l]^T \quad (12) \]
\[ A^* = \begin{bmatrix} \vdots & \vdots \\ A & 0 \\ 0 & 0 \\ 0 & 0 \\
-k_{J_m^2} \ J_m \ J_i & -c/2 + b_m \ J_m^2 \ J_i & k_{J_m^2} \ J_i & c_2 \ J_i & 0 & 1 / J_i & 0 \end{bmatrix} \quad (13) \]
\[ A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -k_{J_m^2} \ J_m \ J_i & -c/2 + b_m \ J_m^2 \ J_i & k_{J_m^2} \ J_i & c_2 \ J_i & 0 & 1 / J_i & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (14) \]
\[ B^* = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1/\tau_{eng} & 0 & 0 \end{bmatrix}^T \quad (15) \]
\[ C^* = \begin{cases} C_+, \text{ positive estimation-mode or wait-mode} \\ C_-, \text{ negative estimation-mode or wait-mode} \end{cases} \quad (16) \]
\[ K^* = \begin{cases} K_+, \text{ positive estimation-mode} \\ K_-, \text{ negative estimation-mode} \\ K_0, \text{ wait-mode} \end{cases} \quad (17) \]
\[ C_+ = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix} \quad (18) \]
\[ C_- = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix} \quad (19) \]
\[ K_+ = \begin{bmatrix} k_{11} & k_{12} \\ \vdots & \vdots \\ k_{61} & k_{62} \\ k_{o1} & k_{o2} \\ 0 & 0 \end{bmatrix} \quad (20) \]
\[ K_- = \begin{bmatrix} k_{11} & k_{12} \\ \vdots & \vdots \\ k_{61} & k_{62} \\ 0 & 0 \\ k_{o1} & k_{o2} \end{bmatrix} \quad (21) \]
The Kalman gains are calculated for the linear model $(A^*, C_+)$, with the last (8th) state removed to make the system observable. The elements of the resulting (reduced) $K_+$ are then rearranged to yield $K_-$ and $K_0$. Wait-mode $+$ in (16) means that this $C$-value is used when waiting to enter the positive estimation-mode, and correspondingly for the negative mode.

4.2 Results

The estimator derived in the previous subsection is used to estimate the backlash size from the measurement data described in Section 3. In Figure 3, the settling of $\hat{\alpha}$ is seen. Since the estimator needs a number of backlash traverses for the estimation, the same measurement sequence of 33 seconds has been rerun six times. There is some remaining fluctuation in the estimate, which emanate from model uncertainties.

For three different measurement sequences, the following values of the backlash size, $\hat{\alpha}$, were found: 0.025, 0.026 and 0.027 rad. The difference between the sequences is on the order 5%. The estimated backlash size should be compared with the manually estimated value, as described in Section 3: $\alpha \approx 0.024$ rad. In Figure 4, the Kalman Filter switching behaviour is seen, for a startup from zero in both $\hat{\theta}_{o+}$ and $\hat{\theta}_{o-}$.
Backlash position estimation

5.1 Position estimator

Backlash position estimation is the problem of tracking the current position in the backlash, $\theta_b$. The estimator used here was described in (Lagerberg and Egardt, 2003a). There, it is shown that the nonlinear powertrain model switches between two linear modes called backlash mode, (bl), and contact mode (co). A switched Kalman filter is therefore derived:

\[
\dot{\hat{x}} = \begin{cases} 
A_{co} \hat{x} + Bu + K_{co}(y - C\hat{x}), & \text{co-mode} \\
A_{bl} \hat{x} + Bu + K_{bl}(y - C\hat{x}), & \text{bl-mode} 
\end{cases}
\]  
(23)

\[
\hat{\theta}_b = \hat{x}_7
\]  
(24)

where

\[
x = \begin{bmatrix} \theta_m & \omega_m & \theta_i & \omega_i & T_i & T_m & \theta_b \end{bmatrix}^T
\]  
(25)

\[
y = \begin{bmatrix} \theta_m & \theta_i \end{bmatrix}^T
\]  
(26)
The Kalman filter gains $K_{co}$ and $K_{bl}$ are designed for their respective cases. The mode switches in (23) are made based on the conditions in (5).

To increase the bandwidth of the presented estimator, an event based Kalman filter was proposed in (Lagerberg and Egardt, 2003a) as pre-filter on the quantized position signals. The pre-filter is also used in this paper.

The size and position estimators will be running simultaneously, and are interconnected as shown in Figure 5, where also the pre-filters are shown. The position filter has a much higher bandwidth than the size filter. The backlash size signal is also low pass filtered before it is used by the position estimator.

5.2 Results

When the position estimator is applied to the measured data, the total shaft displacement is estimated as seen in Figure 6. The effect of using the pre-filters is clearly seen.

Finally, the estimated position in the backlash is seen in Figure 7. Since no true value is available, the following approximation is used for comparison:

$$\theta_{b, true} = \min(\hat{\alpha}, \max(-\hat{\alpha}, \theta_{d, true}))$$

i.e. the total shaft displacement limited by the estimated backlash gap size.

The estimation error is less than 10% of the backlash size, and the error in contact time is approximately 1 ms.
Figure 5: Interconnection of the size and position estimators. The position estimator state vector, $\hat{x}$, may be useful for a backlash compensating controller.

Figure 6: Total shaft displacement estimate, $\hat{\theta}_d$. Solid: Extended Kalman Filter estimate (almost on top of "true" value). Dashed: "True" displacement, as defined in Section 3.4. Dash-dotted: Estimate without pre-filter. The estimation error is also plotted for the two cases (solid and dash-dotted). The simultaneously estimated backlash size, $\hat{\alpha}$, is indicated by straight horizontal dashed lines.
6 Conclusions

The results presented in the previous sections show that both backlash size and current position is possible to estimate with high precision by the suggested estimators.

Only sensors of standard type are used, together with an engine control signal which is already available in the control system. This means that the proposed estimation principle is possible to use in vehicles without extra sensors etc.

The results are encouraging, and as a natural next step, the estimators will be incorporated in closed loop control for backlash compensation.

The small fluctuations in size estimate, due to model uncertainty may be eliminated by using a slightly more complex powertrain model, which captures e.g. wheel slip and engine suspension flexibility.

Acknowledgements

This work was supported by the Volvo Research and Educational Foundations, the Swedish Automotive Research Program and Volvo Car Corporation. The authors wish to thank all helpful staff at the Volvo Car Corporation for the practical help in performing the presented experiments.
References


