Sliding Mode Control of Multivariable Nonlinear Systems with Unknown Control Direction Applied to the Visual Servoing Problem

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Abstract—An output-feedback sliding mode controller based on monitoring functions was recently introduced for linear uncertain single-input-single-output (SISO) systems with unknown control direction. Here, generalization is achieved to include nonlinearities depending on unmeasured states and to deal with multivariable systems. The strategy is based on output-feedback unit vector control to generate the sliding mode and a monitoring scheme to reduce a priori information about the plant high frequency gain matrix $K_p$, usually required by other methods. Global stability properties of the closed-loop system and exact output tracking are proved. Experimental results with a robot visual servoing system using an uncalibrated camera illustrate the robustness and viability of the proposed scheme.

I. INTRODUCTION

The design of output-feedback control of uncertain systems without knowing the sign of the high frequency gain, i.e. control direction, has been an instigating problem since the early 1980s [1]. In the adaptive control literature, the so called Nussbaum gain [2] has been used to design stable systems under this relaxed assumption, including the multivariable case [3]-[5]. Nevertheless, this approach is of arguable practical interest, due to the large transients and lack of robustness that may result [1], [6]. More recently, an output-feedback tracking sliding mode control (SMC) for SISO uncertain linear plants with unknown control direction was introduced in [7]. In lieu of the Nussbaum gain [2], the control sign was adjusted based on monitoring functions.

In this paper, we extend the controller of [7] to nonlinear multiple-input-multiple-output (MIMO) plants with relative degree one using a unit vector model-reference sliding mode control approach. Here, the nonlinear terms are allowed to be state dependent, including strong (e.g., polynomial) nonlinearities, and unmatched w.r.t. the control input.

The main motivation to use a unit vector [8], [9] instead of the vector “sign(·)” switching function, is that a less restrictive prior knowledge of the plant high frequency gain (HFG) matrix $K_p$ is required, when compared to that obtained using some direct norm-bounds on the uncertainty of the control matrix [10] or the positive definiteness property usually required by other methods [11], [12]. Indeed, if a unit vector controller is used, the only requirement about the HFG matrix is that $-K_p$ is Hurwitz and this condition is necessary and sufficient for the attractiveness of the sliding surface as proved in [8], [9].

In this work, $K_p$ is unknown and allowed to be uncertain. Inspired by the recent developments in supervisory control schemes [13] and the spectrum-unmixing sets referred in [14], we propose a switching mechanism that chooses a suitable static pre-compensator matrix $S_q$ in a finite index set of matrices $Q$ through an appropriate monitoring function, such that the unit vector control law pre-multiplied by $S_q$ can guarantee that $-K_pS_q$ is Hurwitz for some $S_q$, $q \in Q$. Global asymptotic stability with respect to a compact set and exact output tracking are demonstrated.

Application to the visual servoing problem with uncertainties in the camera calibration parameters illustrates the effectiveness of the proposed multivariable controller in real-world conditions. In particular, the usual restriction of the camera orientation angle $\psi \in (-\frac{\pi}{2}, \frac{\pi}{2})$, presented in [15]-[19], can be easily removed.

II. NOTATION AND TERMINOLOGY

The following notation and basic concepts are employed:

- ISS means Input-to-State-Stable [20] and classes $K, K_\infty$ functions are defined as usual [21, pp. 144].
- The Euclidean norm of a vector $x$ and the corresponding induced norm of a matrix $A$ are denoted by $|x|$ and $|A|$. The maximum and minimum eigenvalues of $A$ are denoted as $\lambda_m(A)$ and $\lambda_m(A)$, respectively.
- The symbol “$s$” represents either the Laplace variable or the differential operator “$d/dt$”, according to the context. The output of a linear system with transfer function matrix $H(s)$ and input $u$ is written $H(s)u$. Pure convolution $h(t)*u(t)$ is denoted by $H(s)u$, with $h(t)$ being the impulse response of $H(s)$.
- Filippov’s definition for the solution of discontinuous differential equations is assumed [22].

III. PROBLEM FORMULATION

This paper considers the global tracking problem of MIMO nonlinear systems transformable into

$$\dot{\eta} = \phi_0(\eta, y, t),$$

$$\dot{y} = K_p u + \phi_1(\eta, y, t),$$

where $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^m$ is considered as the measured output and the states $\eta \in \mathbb{R}^{n-m}$ of the $\eta$-subsystem, referred to as an “inverse system”, are not assumed to be measurable. The uncertain functions $\phi_0$ and $\phi_1$ are piecewise continuous in $t$ and locally Lipschitz continuous in the others arguments. For each solution of (1)–(2) there exists a maximal time interval of definition given by $[0, t_M)$, where $t_M$ may be finite or infinite.
It is further considered that the plant control direction is unknown (and constant) in the sense that all uncertain parameters of the HFG matrix $K_p$ belong to some compact set $\Omega_p$. In $\Omega_p$, it is only assumed that:

(A1) (i) $\text{det}(K_p) \neq 0$, (ii) there exists a known constant $c > 0$ such that $|K_p^{-1}| \leq c$ and (ii) there exists a finite index set $Q$ of known matrices $S_q \in \mathbb{R}^{m \times m}$ such that $-K_p S_q$ is Hurwitz for some $q \in Q$.

According to (A1), we focus the simplest relative degree one case amenable to Lyapunov design. The case of general relative degree with unknown control direction will be left for a future work. As already commented above, the Hurwitz condition is necessary and sufficient for the attractiveness of the sliding surface in the case of unit vector sliding mode control [8], [9].

This assumption significantly relaxes the usual requirement of positive definiteness and symmetry of $K_p$ [11], [12]. Symmetry is a non generic property. It can be easily destroyed by arbitrarily small uncertainties in $K_p$. Moreover, if $K_p$ is positive definite, then this implies that $-K_p$ is Hurwitz but the converse is not true.

For the SISO case, (A1) can be interpreted as follows: the first two conditions indicate that the scalar $K_p \neq 0$ can be positive or negative, i.e., the control direction is unknown. Moreover, in this case the index set is $Q = \{0, 1\}$ and a scalar $S_q (S_0 = -1, S_1 = 1)$ is required to make $-K_p S_q$ negative.

Here, a switching mechanism is provided for cycling through the elements of the finite index set $Q$ [14].

**Global Tracking Problem**

The aim is to find a dynamic control law $u$, via output feedback and without the knowledge of the plant control direction, to drive the output tracking error

$$e(t) = y(t) - y_m(t)$$

exponentially to zero (exact tracking), starting from any plant/controller initial conditions and maintaining uniform closed-loop signal boundedness, in spite of the uncertainties. The desired trajectory $y_m(t)$ is assumed to be generated by the following reference model:

$$\dot{y}_m = A_m y_m + r, \quad A_m = -\text{diag} \{\gamma_1, \ldots, \gamma_m\}$$

where $r, y_m \in \mathbb{R}^m, \gamma_i > 0 (i = 1, \ldots, m)$ and $r(t)$ is assumed piecewise continuous and uniformly bounded.

In order to achieve the control objective our strategy requires a norm observer for the state $\eta$ of the inverse system (1), according to the following definition and assumption.

**Definition 1:** A first order norm observer for system (1) is a SISO dynamic system of the form ($y$ is the plant output):

$$\dot{\eta} = -\lambda_o \eta + \varphi_o(y, t),$$

with input $\varphi_o(y, t)$ and output $\eta$, such that: (i) $\lambda_o > 0$ is a constant, (ii) $\varphi_o(y, t)$ is a non-negative function, continuous in $y$ and piecewise continuous in $t$, satisfying $\varphi_o \leq \Psi_o(|y|) + k_o$, for some $\Psi_o \in \mathcal{K}$ and some constant $k_o \geq 0$ and (iii) for each initial states $\eta(0)$ and $\bar{\eta}(0)$

$$|\eta(t)| \leq |\bar{\eta}(t)| + k_o (|\bar{\eta}(0)| + |\eta(0)|) e^{-\lambda_o t},$$

$$\forall t \in [0, t_M),$$

with some constant $k_o > 0$.

(A2) The inverse system (1) admits a known norm observer (5) with $\varphi_o$ and $\lambda_o$ known.

It is well known that, in the time-invariant case, if the inverse system (1) is ISS [20] then it admits such norm observer and the plant is minimum-phase.

In the appendix, two cases are given where (5) can be implemented for the time-varying inverse system. The first case incorporates a class of nonlinearities $\phi_0$ where a linear growth condition is required only w.r.t. the unmeasured state $\eta$. The other one, adapted from [23], illustrates a case where strong polynomial nonlinearities in $\eta$ are allowed. In both cases, (1) possess an ISS-like property w.r.t. an appropriate function of $y$ and $t$.

In order to obtain a norm bound for $\varphi_1$ in (2), we additionally assume that:

(A3) There exists a known real valued non-negative function $\varphi_1(|\eta|, y, t)$, class $\mathcal{K}_\infty$ and locally Lipschitz in $|\eta|$, continuous in $y$ and piecewise continuous in $t$ such that $|\phi_1(\eta, y, t)| \leq \varphi_1(|\eta|, y, t)$.

Note that, (A3) is not restrictive since $\varphi_1$ is assumed continuous in $\eta$. Furthermore, no particular growth condition is imposed on the bounding function $\varphi_1$.

**From Tracking to Regulation Problem**

From (2)–(4), the $e$-dynamics can be written as

$$\dot{e} = A_m e + K_p (u - u^*)$$

where

$$u^* := K_p^{-1} (-\phi_1 + A_m y + r).$$

Then, the global tracking problem can be reformulated as the regulation problem described as follows. Find an output-feedback sliding mode control law $u$ in such a way that, for all initial conditions $(\eta(0), e(0), \bar{\eta}(0))$: (i) the solutions of (2), (5) and (7) are bounded and (ii) $e(t)$ tends exponentially to zero as $t \to \infty$.

The ideal control $u^*$ (8) is considered as a matched input disturbance in (7). From (A1)–(A3), it can be norm bounded by available signals

$$|u^*| \leq c (\varphi_1(2|\bar{\eta}|, y, t) + |A_m y + r| + \pi_1,$$

modulo the exponential decaying term $\pi_1 := k_1 (|\bar{\eta}(0)| + |\eta(0)|) e^{-\lambda_o t}$, where $k_1 > 0$ is a constant, $c$ is given in (A1) and $\pi_1$ comes from the exponential term in (6). To develop this inequality we have used the fact that $\varphi_1$ is locally Lipschitz in its first argument and $\psi(a+b) \leq \psi(2a) + \psi(2b)$, $\forall a, b \geq 0$ and $\forall \psi \in \mathcal{K}$. 

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IV. OUTPUT-FEEDBACK SLIDING MODE CONTROLLER

This section presents a first generalization of the proposed controller in [7] for a class of MIMO nonlinear plants with unknown control direction and relative degree one.

Let \( q^* \) be the unknown index of the index set \( Q \), given in (A1), for which the corresponding unknown matrix \( S = S_q \), assures that \(-K_p S\) is Hurwitz. Thus, the Lyapunov equation \((K_p S)^TP + P(K_p S) = I\) has a solution \( P = P^T > 0\).

Now, if the control direction is known (\( q^* \) is known) one can apply the unit vector control (UVC) law [9]

\[
    u = -S q(e, \eta(t)) \frac{e}{|e|}, \tag{10}
\]

to (7) and verify that, if the modulation function \( \varphi \) satisfies

\[
    \varphi \geq c_d u^*(t) + \delta, \quad \delta \geq 0, \tag{11}
\]

modulo the exponential decaying term \( c_d \pi_1 \), then the time derivative of \( V = e^TP e \) along the solutions of (7) satisfies:

\[
    \dot{V} \leq -2\lambda_m V + c_d \pi_1, \quad \forall t \in [t_i, t_M],
\]

for any \( t_i \in [0, t_M] \), where \( 0 < \lambda_m < \min\{\gamma_i\} \), \( c_d \) is an appropriate constant and \( \pi_1 \) comes from (9).

Then, by using the comparison lemma ([22]), one has:

\[
    |e(t)| \leq |\zeta(t)|, \quad \forall t \in [t_i, t_M], \tag{12}
\]

where

\[
    \zeta(t) := |e(t_i)|e^{-\lambda_m(t-t_i)} + \pi_2,
\]

with \( \pi_2 := \Psi_2[|\eta(0)| + |\eta(0)|]e^{-\lambda_c t}, \quad \Psi_2 \in \mathcal{K} \) and \( 0 < \lambda_c < \min\{\lambda_o, \lambda_m\} \) (see [9, Lemma 1] for details).

The major problem is that \( q^* \) is unknown, thus we cannot implement the UVC law (10).

In [7], a switching scheme based on monitoring function was developed to cope with the lack of knowledge of the control direction. Only linear SISO plants with relative degree one were considered. In that case, \( K_p \) was a scalar and after a finite number of changes in the control sign \((S_q = \pm 1)\), the correct control direction could be detected. For MIMO nonlinear plants, the UVC law is redefined as

\[
    u = -S q(e, \eta(t)) \frac{e}{|e|}, \quad \forall t \in [0, t_M], \tag{13}
\]

where \( \varphi \) satisfies (11) and a switching mechanism also based on a monitoring function is used to decide when the static pre-compensator matrix \( S_q \) [13] should be switched within the collection of matrices with \( q \in Q \).

V. SWITCHING SCHEME AND MONITORING FUNCTION

We now construct the monitoring function \( \varphi_m \) based on the norm bound for \( e \) given in (12). Reminding that the (12) holds once the matrix \( S_q \) is correct \((S_q = S)\), it seems natural to use \( \zeta \) as a benchmark to decide whether a switching of \( S_q \) is needed, i.e., the switching occurs only when (12) is violated. However, since \( \pi_2 \) is not available for measurement we consider the following function, defined in the interval \([t_k, t_{k+1})\), to replace \( \zeta \):

\[
    \varphi_k(t) = |e(t_k)|e^{-\lambda_m(t-t_k)} + a(k)e^{-\lambda_c t}, \tag{14}
\]

where the switching time \( t_k \) sets the change of index \( q \in Q \), thus cycling through the \( S_q \) matrices and \( a(k) \) is any positive monotonically increasing unbounded sequence.

The monitoring function \( \varphi_m \) can thus be defined as

\[
    \varphi_m(t) := \varphi_k(t), \quad \forall t \in [t_k, t_{k+1}) \subset (0, t_M). \tag{15}
\]

Note that from (14) and (15), one has \(|e(t_k)| < \varphi_k(t_k)\) at \( t = t_k\). Hence, the switching time \( t_k \) is defined by

\[
    t_{k+1} := \begin{cases} \min\{t > t_k : |e(t)| = \varphi_k(t)\}, & \text{if it exists,} \\ t_M, & \text{otherwise}, \end{cases} \quad \tag{16}
\]

where \( k \in \{0, 1, \ldots\} \) and \( t_0 := 0 \) (see Fig. 1). The following inequality is directly obtained from definition (15)

\[
    |e(t)| \leq \varphi_m(t), \quad \forall t \in [0, t_M). \tag{17}
\]

Fig. 1 illustrates the tracking error norm \(|e|\) as well as the monitoring function \( \varphi_m \).

VI. STABILITY RESULTS

In order to fully account for all initial conditions, let

\[
    z^T(t) := [z^0(t), e(t)], \quad z^0(t) := [\eta(0), |\eta(0)|]e^{-\gamma t}, \tag{18}
\]

where \( z^0 \) denotes the transient state [9] and \( \gamma > 0 \) is a generic constant. The main result is now stated.

**Theorem 1**: Assume that (A1)–(A3) hold. Consider the error equation (7) with UVC law (13) and monitoring function (14)-(15). If the modulation function satisfies (11), then the control direction switching stops. The complete error system, with state \( z(t) \), is globally asymptotically stable w.r.t. a compact set independent of the initial conditions and ultimately exponentially convergent to zero. Moreover, all signals in the closed loop system remain uniformly bounded and if \( \delta > 0 \) in (11), then the sliding mode at the manifold \( e = 0 \) is reached in some finite time.

**Proof**: See Appendix I.

**Remark 1**: [Measure Zero Set] We know that if \(-K_p S_q\) is Hurwitz all trajectories of the system converge to the origin of the error state space [9, Lemma 1]. Moreover, if \(-K_p S_q\) is not Hurwitz, then for almost every initial condition (i.e., except for a set of measure zero) the system trajectories diverge unboundedly or do not converge to the origin. This is a contradiction, since if the switching stops, according to Theorem 1, the state must converge to the origin. Then, almost always, the ultimate matrix \( S_q \) selected is such that \(-K_p S_q\) must be Hurwitz.
Remark 2: [Compact sets] Note that the results of stability with respect to a compact set, not necessarily small, accounts for the initial transient while the monitoring function has not yet stopped. This means that, even if the initial errors are very small, the initial transient may not be correspondingly small.

VII. VISUAL SERVOING APPLICATION

To illustrate the applicability of the proposed monitoring switching scheme, we consider a simple case where the plant is a MIMO integrator without inverse subsystem. The proposed control scheme is used to solve the visual servoing control problem for a robot manipulator using a fixed and uncalibrated camera.

A. Visual Servoing Kinematic Control

Firstly, one considers the kinematic control problem for a robot manipulator. In this approach, the end-effector position \( x \in \mathbb{R}^n \) is given by the forward kinematics map \( x = \mu(\theta) \), where \( \theta \in \mathbb{R}^m \) is the manipulator joint angle vector. The differential kinematics equation can be obtained from the time derivative of the forward kinematics map given by

\[
\dot{x} = J(\theta) \dot{\theta},
\]

where \( J(\theta) = \frac{\partial \mu}{\partial \theta} \in \mathbb{R}^{n \times m} \) is the manipulator Jacobian. Then, considering \( \dot{\theta}_i \) as the control input \( v_i \) \( (i = 1, \cdots , m) \) one obtains the following control system

\[
\dot{x} = J(\theta) v.
\]

A cartesian control law \( u \) can be transformed to joint control signals by using

\[
v = J(\theta)^{-1} u,
\]

provided that \( u \) does not drive the robot manipulator to singular configurations.

Now, the visual servoing control problem for a robot manipulator is considered. In this context, the visual servoing approach is used to provide closed-loop position control for the robot end-effector. Let \( y \in \mathbb{R}^2 \) be the end-effector position in the image frame and \( y_m \in \mathbb{R}^2 \) be the desired time-varying trajectory for a target feature fixed on the arm tip. Then, the control goal can be described by

\[
y \to y_m(t), \quad e = y - y_m \to 0,
\]

where \( e \in \mathbb{R}^2 \) is the image error. Here, one considers that the robot manipulator performs planar movements in the cartesian space. Hence, without loss of generality, \( n = m = 2 \) and \( x \in \mathbb{R}^2 \). Then, considering a monocular fixed CCD camera with optical axis perpendicular to the robot frame, the camera/workspace transformation [25] can be represented by

\[
y = K_p x + y_0,
\]

with

\[
K_p = \frac{f_0}{f_0 + z_0} \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{bmatrix},
\]

where \( y_0 \) is a constant term, which depends on the position of the camera frame with respect to the robot frame, \( K_p \) is the camera/workspace transformation matrix and considers the camera orientation angle \( \psi \) (or camera misalignment) with respect to the robot frame, \( f_0 \) is the camera focal length, \( z_0 \) is the depth from the camera frame to the robot workspace (in general \( z_0 \gg f_0 \)), and \( \alpha_1, \alpha_2 > 0 \) are the scaling factors of the camera [pixel/mm].

Thus, the cartesian control problem in the image frame is described from (22) by

\[
\dot{y} = K_p u.
\]

Then, based on (23) and considering a feedforward and proportional control law given by

\[
u = K_p^{-1} [\dot{y}_m + K(y_m - y)],
\]

one has that the image error dynamics is governed by \( \dot{e} + K e = 0 \). Hence, by a proper choice of \( K \) as a positive definite matrix, \( e \to 0 \) exponentially as \( t \to \infty \).

However, by assuming that the intrinsic and extrinsic parameters of camera model are uncertain (uncalibrated camera), the camera/workspace transformation matrix \( K_p \) is also uncertain. Therefore, the control law (24) does not guarantee asymptotic tracking of the desired trajectory, since the closed-loop system cannot be linearized. In this context, some adaptive schemes were proposed in order to cope with the uncertainties in the camera calibration parameters [15]-[19]. Nevertheless, it is well-known that the adaptive strategies can lead to bad transient behavior and lack of robustness with respect to unmodeled dynamics. Moreover, in these approaches the camera orientation angle \( \psi \) must be restricted to the range \( (-\frac{\pi}{2}, \frac{\pi}{2}) \).

In what follows, the combination of the proposed sliding mode approach and the switching scheme based on the monitoring function is applied to circumvent the above problems.

B. Visual Servoing based on Sliding Mode Approach

Here, the proposed control scheme is applied to solve the visual servoing control problem for a robot manipulator in the presence of uncertainties in the camera parameters and under any camera misalignment angle.

To illustrate the disturbance rejection property of the proposed scheme, we have added an artificial disturbance \( \delta(\theta) = [\theta_1^2 \theta_2^2]^T \) to the control input \( u \), by using the measurement \( \theta \) obtained from the robot encoders. Then, from the inverse kinematics map and (22), the cartesian control problem in the image frame can be rewritten as (2) with an appropriate \( \phi_1(y,t) \).

The monitoring function \( \varphi_m \) (14)-(15) is used to switch the matrix \( S_q \) in (13). The finite set of matrices \( S_q, q \in Q = \{0, 1, 2, 3\} \) can be chosen as

\[
S_0=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, S_1=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, S_2=\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, S_3=\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.
\]

For any camera misalignment angle \( \psi \), \( -K_p S_q \) is Hurwitz for some \( S_q \) and therefore, the usual restriction \( |\psi| < \frac{\pi}{2} \) can be removed.
VIII. Experiments and Results

This section describes the experimental setup and presents the results obtained in the experimental tests, which illustrate the remarkable performance of the proposed control scheme.

A. Experimental setup

The experimental results were obtained by implementing the proposed controller on a 6-DOF Zebra Zero robot manipulator (Integrated Motions, Inc.). The dynamic effects are negligible in this robot due to its large gear ratios and a high gain velocity control loop.

A KP-D50 CCD camera (Hitachi, Ltd.) with a lens of length \(f_0 = 6\) [mm] was mounted in front of the Zebra Zero (see Fig. 2 for the camera point of view). The average depth from the image plane to the robot workspace was \(z_0 = 1\) [m]. The extracted visual feature are the centroid coordinates of a red disc fixed on the robot wrist. The images of 640×480 [pixel] are acquired using a Meteor frame-grabber (Matrox, Ltd.) at 30 frames per second (FPS).

![Experimental station](image)

Fig. 2. Experimental station.

The visual servo controller is coded in C language and executed in 35.0 \(m/s\) on a 200 MHz Pentium Pro processor with 64 Mbyte RAM using Linux OS. The joint velocity command generated by the visual servoing control law feeds the Zebra Zero ISA board which closes a velocity loop using an HCTL1100 microcontroller (HP Inc.) working in the Zebra Zero ISA board which closes a velocity loop with 64 Mbyte RAM using Linux OS. The joint velocity command generated by the visual servoing control law feeds the Zebra Zero ISA board which closes a velocity loop with 64 Mbyte RAM using Linux OS. The joint velocity command generated by the visual servoing control law feeds the Zebra Zero ISA board which closes a velocity loop with 64 Mbyte RAM using Linux OS. The joint velocity command generated by the visual servoing control law feeds the Zebra Zero ISA board which closes a velocity loop with 64 Mbyte RAM using Linux OS. The joint velocity command generated by the visual servoing control law feeds the Zebra Zero ISA board which closes a velocity loop with 64 Mbyte RAM using Linux OS. The joint velocity command generated by the visual servoing control law feeds the Zebra Zero ISA board which closes a velocity loop with 64 Mbyte RAM using Linux OS. The joint velocity command generated by the visual servoing control law feeds the Zebra Zero ISA board which closes a velocity loop with 64 Mbyte RAM using Linux OS. The joint velocity command generated by the visual servoing control law feeds the Zebra Zero ISA board which closes a velocity loop with 64 Mbyte RAM using Linux OS. The joint velocity command generated by the visual servoing control law feeds the Zebra Zero ISA board which closes a velocity loop with 64 Mbyte RAM using Linux OS. The joint velocity command generated by the visual servoing control law feeds the Zebra Zero ISA board which closes a velocity loop with 64 Mbyte RAM using Linux OS. The joint velocity command generated by the visual servoing control law feeds the Zebra Zero ISA board which closes a velocity loop with 64 Mbyte RAM using Linux OS.

The image processing in RGB format is performed on a subwindow 100×100 [pixel] wide. The first estimation of the centroid coordinates is performed off-line using an ad-hoc Graphical User Interface developed in Tcl/Tk language [26] as shown in Fig. 2. During the task execution, the feature is computed using the image moments algorithm [27]. Due to noise sensitivity, the proportional gain in the velocity loop is not high enough to eliminate the steady state error due to gravity effects. This disturbance was identified off-line using a least squares method and effectively compensated [28].

B. Experimental results

The experimental tests are performed without regarding any calibration procedure. The desired trajectory \(\gamma_m\) is generated by the model (4), with \(\gamma_1 = \gamma_2 = 1\), and \(R = [r_1, r_2]\) with reference signals

\[
\begin{align*}
 r_1 &= y_1(0) + c_a R [1 - \cos(\omega_r t)], \\
 r_2 &= y_2(0) + c_b R [\sin(\omega_r t)],
\end{align*}
\]

where \(y^T(0) = [y_1(0), y_2(0)]\) is the initial position of the centroid coordinates in the image frame, \(c_a\) and \(c_b\) are constant parameters which determine the movement direction, \(R\) and \(\omega_r\) are the ratio and the angular velocity of the reference trajectory, respectively. In the experiments, the robot manipulator has to perform the tracking of a circular trajectory specified in the image frame with \(R = 40\) [pixel] and \(\omega_r = \frac{\pi}{5}\) [rad/s]. Other parameters are \(y_1(0) = 330\) [pixel], \(y_2(0) = -275\) [pixel] and \(c_a = c_b = 1\). The camera parameters are: \(\psi \approx \pi\) [rad], \(\alpha_1 = 119\) [pixel/mm] and \(\alpha_2 = 102\) [pixel/mm]. The monitoring function \(\varphi_m\) is obtained from (14)-(15) with \(a(k) = k + 1\), \(\lambda_m = \lambda_r = 1\). In addition, a constant of 15 was added to \(\varphi_m\) to reduce spurious modifications in the control direction estimate due to the measurement noise. It is well known that the measurement noise can cause control chattering. However, it can be alleviated by using the boundary layer method [10] in the UVC law.

The modulation function was implemented in order to satisfy (11) and an upper bound for \(d(\theta)\) was obtained by using the inverse kinematics map. All test cases were designed to avoid Jacobian singularities in (20).

Fig. 3 shows the time history of the monitoring function \(\varphi_m\) and the error norm \(|e|\). The feedback control was initialized with the matrix \(S_0\) (which is not the correct matrix for \(\psi = \pi\)). Then, the camera misalignment angle was modified for \(\psi \approx \pi/2\). Note that, at the fourth switching \((k = 4)\) throughout the index set \(Q\), the correct \(S_0\) matrix (for \(\psi = \pi/2\)) is selected (\(-K_pS_0\) is Hurwitz) and thereafter \(|e|\) → 0. Fig. 4 describes the time history of the image error \(e\) and the control signal \(u\), respectively. Note that, the asymptotic convergence of the error to a small residual set is evident. The target trajectory is illustrated in Fig. 5, where one notices that the tracking is achieved even for \(|\psi| > \pi/2\) and changes in \(\psi\) artificially introduced during the experiment. In spite of kinematics uncertainties due to arm flexibility and backlash, the proposed sliding mode controller guarantees excellent performance and robustness.

IX. Conclusions

An output-feedback model-reference sliding mode controller was developed for uncertain multivariable nonlinear systems with unknown high frequency gain matrix and relative degree one, generalizing the controller in [7]. The resulting controller based on monitoring function leads to global asymptotic stability with respect to some compact set and ultimate exponential convergence of the tracking error to zero with improved transient response in contrast to the Nussbaum gain approach. The proposed controller was successfully tested with a robotics visual servoing experiment.

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In this paper we have assumed that one can obtain a norm observer of the form (5) for the state \( \eta \) of the inverse system \( \dot{\eta} = \phi_0(\eta, y, t) \), given in (1). In this section, we characterize a class of MIMO nonlinear plants and give a simple example for which such norm observer can be implemented.

In both cases, one can obtain a storage function \( V(\eta) \) satisfying \( \alpha(|\eta|) \leq V(\eta) \leq \alpha(\|\eta\|) \), with \( \alpha(\sigma) = \Delta \sigma^2 \), \( \tilde{\alpha}(\sigma) = \hat{\lambda} \sigma^2 \) and \( \Delta, \hat{\lambda} \) known such that, \( \forall t \in [0, T] \),
\[
\frac{\partial V}{\partial \eta} \phi_0(\eta, y, t) \leq -\alpha(|\eta|) + \gamma(y, t),
\]
where \( \alpha \in \mathcal{K} \) and \( \gamma \) are known and \( \gamma \) is a non-negative function, continuous in \( y \) and piecewise continuous in \( t \) satisfying \( \gamma \leq \Psi_\gamma(|y|) + k_\gamma \), for some \( \Psi_\gamma \in \mathcal{K} \) and constant \( k_\gamma \geq 0 \). Note that, it assures that the inverse system (1) has an ISS-like property from \( \gamma \) to \( \eta \).

Moreover, the class-\( \mathcal{K} \) function \( \alpha_1 := \alpha \circ \alpha^{-1} \) is such that, \( \alpha_1(\|\sigma\|/\sigma) \) is a non-negative increasing function in the interval \( \sigma \in (0, \infty) \) and the norm observer (5) can be obtained as follows.

The above property of the function \( \alpha_1 \) guarantees that \( \alpha_1(\|\sigma\|/\sigma) > 2\lambda \sigma, \forall \sigma \geq e \), for any \( e > 0 \) and \( \lambda < \alpha_1(e)/(2e) \). In particular, if \( \alpha_1(\|\sigma\|/\sigma) > 2\lambda \sigma, \forall \lambda > 0 \), one can take \( e = 0 \), i.e., \( \alpha_1(\|\sigma\|/\sigma) > 2\lambda \sigma, \forall \sigma > 0 \) and \( \lambda < \alpha_1 \).

Consider the notation \( \hat{V} = \frac{\partial V}{\partial \eta} \phi_0(\eta, y, t) \) and choose \( \lambda \), as described above, for any given \( e \geq 0 \). From (27), one can write
\[
\hat{V} \leq -\alpha_1(V) + \gamma(y, t) \text{ or, equivalently, } \hat{V} \leq -2\lambda V + [2\lambda V - \alpha_1(V)] + \gamma(y, t).
\]

Now, given any \( V \), either \( V \leq \epsilon \) or \( V > \epsilon \). Hence, either \( \hat{V} \leq -2\lambda V + [2\lambda V - \alpha_1(\|\epsilon\|)] + \gamma \) or \( \hat{V} \leq -2\lambda V + \gamma \), leading to the conclusion that \( \hat{V} \leq -2\lambda V + [2\lambda V - \alpha_1(\|\epsilon\|)] + \gamma \). Therefore, by using comparison theorems [21], one can conclude that
\[
\hat{V} \leq e^{-2\lambda t} \gamma_1(y(t), t) + V(0)e^{-2\lambda t},
\]
where \( \gamma_1 = \gamma + 2\lambda e + \alpha_1(e) \) is known. Finally, one can implement a norm observer of the form (5) for the state \( \eta \), with \( \varphi_0(y, t) = \sqrt{\gamma_1(y, t)/\Delta} \) and \( \lambda_0 = \lambda \), by applying the function \( \alpha^{-1} \) in both sides of the last inequality.

A. Inverse system with linear growth condition w.r.t. the unmeasured state

Consider the class of nonlinear MIMO plants (1)–(2) with the function \( \phi_0 \) given by
\[
\phi_0(\eta, y, t) = A_0 \eta + \tilde{\phi}_0(\eta, y, t),
\]
where \( A_0 \) and \( \tilde{\phi}_0 \) can be uncertain. We consider that all parametric uncertainties belong to some compact set \( \Omega_0 \) such that the necessary uncertainty bounds to be defined in what follows are available for design.

In \( \Omega_0 \), we assume that: (i) \( A_0 \) is Hurwitz, (ii) there exist known positive constants \( c_0, c_1, c_2 \) such that \( |P| < c_0, \lambda_M(P) < c_1 \) and \( \lambda_0(P) > c_2 \), where \( P = P^T > 0 \) is the solution of \( A_0^T P + PA_0 = -I \) and (iii) there exist a known constant \( \mu \geq 0 \) and a known function \( \varphi_0 \) such that
\[
|\tilde{\phi}_0| \leq \mu |\eta| + \varphi_0(y, t),
\]
where \( \varphi_0 \) is non-negative, continuous in \( y \), piecewise continuous in \( t \) and satisfies \( \varphi_0 \leq \Psi_0(|y|) + k_0 \), for some \( \Psi_0 \in \mathcal{K} \) and constant \( k_0 \geq 0 \). Now, using the quadratic function \( V(\eta) = \eta^T P\eta \) one can obtain inequality (27) with
\[
\alpha(|\eta|) = \frac{|\eta|^2}{2} \quad \text{and} \quad \gamma(y, t) = \frac{8c_0^2}{1 - 2c_0\mu} \varphi_0^2(y, t),
\]
provided \( \mu < 1/(2c_0) \). Thus, \( \alpha_1(\sigma) = \sigma/(2\lambda_M(P)) \) and defining \( c_3 = 8c_0^2/(1 - 2c_0\mu) \), the norm observer (5) can be implemented with \( \lambda_\alpha < 1/(4c_1) \) and \( \varphi_0(y, t) = \sqrt{c_3/c_2^2 \tilde{\phi}_0(y, t)} \).
B. Inverse system with no particular linear growth condition w.r.t. the unmeasured state

To illustrate that the applicability of the proposed strategy is not restricted to nonlinear plants with $\phi_0(\eta, y, t)$ affinely norm bounded in the unmeasured state $\eta$, as in (28) and (29), we consider the simple case adapted from [23], where $\phi_0 \in IR$ is given by

$$\phi_0(\eta, y, t) = -\eta^5 - \eta^2 |y|.$$  

(30)

In this case, the $\eta$-dynamics is ISS with an ISS Lyapunov function $V(\eta) = \eta^2/2$. Indeed, one has $V = -\eta^6 - \eta^3 y^2$ and, consequently, $V \leq -\eta^2/2 + y^2/2$. Hence, one can obtain inequality (27) with $\alpha(\eta) = \eta^2/2$ and $\gamma(y, t) = y^2/2$. In this case, $\alpha(\sigma) = \sigma^3/2$ and the norm observer (5) can be implemented with $\lambda_0 < \epsilon^2/4$ and $\dot{\varphi}_0(y, t) = \sqrt{y^2/2 + 2\lambda_0 \epsilon + \epsilon^3/2}$. 

**APPENDIX II**

**PROOF OF THEOREM**

In what follows, $k_i > 0$ are constants not depending on the initial conditions and $\Psi_i(\cdot) \in K$. 

The control direction switching stops: Suppose that $S_\kappa$ in (13) switches without stopping, $V \in [0, t_M)$. Then, $\alpha(k_i)$ in (14)-(15) increases unboundedly as $k_i \to \infty$. Thus, there is a finite value $k = \kappa$ such that $\alpha(k) \geq \tau_2(0) - k_0 S_\kappa$ is Hurwitz. In this case, $\dot{\varphi}_m(t) = \zeta(t), \forall t \in [t_i, t_{i+1})$, with $\zeta$ in (12). Moreover, $\zeta$ is a valid upper bound for $|e|$. Hence, no switching will occur after that for $t \to t_M$ which leads to a contradiction. Therefore, $\varphi_m$ (15) has to stop switching after some finite $k = N$ for $t \in [0, t_M)$. 

Stability w.r.t. a compact set: It is not difficult to conclude that $N$ can be related to $|z(0)|$, since $\tau_2(0) \leq k_i |z(0)|$ by definition. Indeed, one can write $N \leq \Psi_3(|z(0)| + k_0)$. Thus, one has $\alpha(N) \leq \Psi_2(|z|) + k_3$ and, from (17) and (18), we can obtain $z(t) \leq \Psi_2(|z(0)| + c_2), \forall t \in [0, t_M)$, where $c_2$ is a positive constant. Thus, given $R > c_2$, for $|z(t)| < R_0$ with $R_0 \leq \Psi_3^{-1}(R - c_2)$, one has that $|z(t)|$ is bounded away from $R$ as $t \to t_M$. This implies that $z(t)$ is uniformly bounded and cannot escape in finite time, i.e., $t_M = +\infty$. Hence, stability with respect to the ball of radius $c_2$ is guaranteed for $z(0)$ in the $R_0$-ball. Since $R$ and thus $R_0$ can be chosen arbitrarily large, global stability is concluded. 

Closed loop signal boundedness and ultimate exponential convergence to zero: Since the control direction switching stops and $\varphi_m$ converges to zero exponentially, then, one concludes (independently of whether a Hurwitz matrix $-K_0 S_\kappa$ is selected at $k = N$ or not) that $e(t)$ and $z(t)$ in (18) will converge to zero at least exponentially. Reminding that $\gamma_m$ is uniformly bounded then $y = e + \gamma_m$ and, from (A2), we can conclude that all closed loop system signals are also uniformly bounded. Also, from [9, Proposition 1], one can further conclude that $e$ becomes identically zero after a finite time, provided that $\delta > 0$ in (11). 

**REFERENCES**


