XPath, transitive closure logic, and nested tree walking automata

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Codd (1972) proposes first-order logic (FO) as yardstick of expressive power. He shows that his relational algebra is expressively complete for FO: it can express every FO-definable query.

Our aim: a theory of Codd completeness for XPath.
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XPath

- XPath expressions navigate through XML documents from one node to another. In other words, they define binary relations.

- The four basic moves in the tree:
  - ↑ (go to the parent of the current node)
  - ↓ (go to a child of the current node)
  - ← (go to the previous sibling of the current node)
  - → (go to the next sibling of the current node)

- These can be combined with node tests and various operations such as composition (/) and union (∪).
  (which operations are allowed differs per XPath dialect.)

- Example: ↑/↓ denotes the binary relation
  \[\{(n_{\text{start}}, n_{\text{end}}) \mid \text{either } n_{\text{end}} \text{ is a sibling of } n_{\text{start}}, \text{ or } n_{\text{end}} = n_{\text{start}} \neq \text{root}\}\]
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XPath is used for selecting nodes in an XML document tree.

XPath lies at the core of the XML querying and processing languages XQuery and XSLT.

Here, we are interested in the tree navigation power of XPath. We ignore arithmetical- and string operations etc. ⇒ we study the “navigational (logical) core” of XPath.
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XML documents are finite sibling-ordered trees whose nodes are labeled with atomic information (tag, attribute-value pairs, string content).

- We model atomic information by a set $\Sigma$ of node labels.
- So, an XML document is a structure $T = (\text{dom} T, R_\downarrow, R_\rightarrow, L)$ where
  - $\text{dom} T$ is the set of nodes,
  - $R_\downarrow$ and $R_\rightarrow$ are the ‘child’ and ‘next sibling’ relations, and
  - $L : N \to \wp(\Sigma)$.

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Core XPath 1.0 (the navigational core of XPath 1.0) has two types of expressions:

- **Path expressions**
  \[ \alpha ::= d \mid d^+ \mid . \mid \alpha/\beta \mid \alpha \cup \beta \mid \alpha[\phi] \quad (d \in \{↑, ↓, ←, →\}) \]

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  \[ \phi ::= p \mid \lnot \phi \mid \phi \land \psi \mid \langle \alpha \rangle \quad (p \in \Sigma) \]

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Contribute to a theory of **Codd completeness for XPath**:

(a) what are natural yardsticks of expressive power for XML?
   - **first-order logic (FO)**
     By analogy to relational DBs
   - **monadic second-order logic (MSO)**
     PTime query evaluation (data complexity),
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   - **first-order logic + monadic transitive closure (FO(MTC))**
     Lies in-between FO and MSO

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- The descendant relation <
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MSO extends FO with set quantification (∀X, ∃X).

FO(MTC) extends FO with transitive closure for binary relations, denoted by \([TC_{xy} \phi]\).

On trees, \(FO \subsetneq FO(MTC) \subsetneq MSO\) (cf. Pothoff 1994)
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\[ R(x, y, z) \]

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Some XPath extensions were proposed that are expressively complete for MSO (e.g., with fixed point operators). None is very attractive as a practical language.

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Examples of queries using transitive closure

- “Retrieve all nodes at even distance from the root”: \(/(^{↓}/^{↓})^{*}\). Can’t be expressed without transitive closure.

- (In a directory structure) “Retrieve all files reachable from the current folder by repeatedly selecting non-hidden subfolders”.
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Reasons for adding TC to XPath

- DTDs can be expressed inside XPath using TC, which makes query containment relative to a DTD a special case of query containment. (Marx 2004; Fan et al. 2005)

- TC enables query rewriting for recursive XML views (Fan et al. 2006).

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**Theorem**

Regular XPath(W) is equally expressive as FO(MTC).
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“the current node has an even number of descendants”

can nicely be expressed using $\ast$ and $W$:

- Let $(\alpha \text{ while } \phi)$ be shorthand for $(.[\phi]/\alpha)^\ast$
- Let $\text{suc}$ be shorthand for
  $$\downarrow[\neg \langle \leftarrow \rangle] \cup .[\neg \langle \downarrow \rangle]/(\uparrow \text{ while } \neg \langle \rightarrow \rangle)/\rightarrow$$
  (“go to the successor in depth first left-to-right ordering”).
- Then $W\langle(\text{suc/\text{suc}})^\ast[\neg \langle \text{suc} \rangle]\rangle$ expresses the intended node test.

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Outline of difficult direction ($\text{FO(MTC)} \subseteq \text{Regular XPath}(W)$):

1. May assume binary branching trees.

2. May assume that $\text{FO(MTC)}$ formulas are in a normal form: allow only TCs of the form

$$\left[ \text{TC}_{xy} \phi(x, y, u, v) \land u < x, y \land v \not< x, y \right](u, v)$$

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Outline of the proof that $\text{FO(MTC)} \subsetneq \text{MSO}$

- Main ingredient: an automata model that captures exactly $\text{FO(MTC)}$. 
Tree automata

A standard (bottom-up) tree automaton in action:

Essentially computes a fixed point.

Are tree automata a “natural” generalization of string automata?

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![Tree walking automaton diagram]

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A nested twa (of degree \( k \)) is a twa \( A \) with finitely many sub-automata (of degree \(< k\)), that can perform these tests:

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How do nested twa compare to pebble twa?

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  (NB: we do not know whether $FO(posMTC) \subsetneq FO(MTC)$)

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- FO(MTC) is a natural yardstick of expressive power on trees, strictly in between FO and MSO.

- Regular XPath(W) ≡ FO(MTC)

- All our results generalize to infinite trees.

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