Multiphase Mixed-Integer Optimal Control Framework for Aircraft Conflict Avoidance

Manuel Soler, Maryam Kamgarpour, Claire Tomlin, Ernesto Staffetti

Abstract—This paper formulates the problem of aircraft conflict avoidance as a multiphase mixed-integer optimal control problem. In order to find optimal maneuvers, accurate models of aircraft nonlinear dynamics and flight envelop constraints are used. Wind forecast and obstacles in airspace due to hazardous weather are included. The objective is to design aircraft maneuvers that ensure safety while minimizing fuel consumption. The solution approach is based on conversion of the multiphase mixed-integer optimal control problem into a mixed-integer nonlinear programming problem. Two case studies for the Airbus 320 aircraft illustrate the approach.

I. INTRODUCTION

The air traffic system is safety critical and large scale. Its operations are vulnerable to traffic fluctuations and weather hazard uncertainties. Additionally, there are increasing concerns over its environmental impact through fuel consumption. The Air Traffic Management (ATM) system has historically grown along highly structured airspace with various restrictions on aircraft trajectories in order to ensure safety, predictability and ease of control by human operators. With the projected increase in air traffic, it becomes critical to develop algorithms to ensure safety with respect to nearby aircraft and hazardous weather while optimizing performance of individual aircraft. The proposed Trajectory Based Operations (TBO) concept, pursued in Europe and the United States [7], will provide the capabilities, decision support tools and automation to manage aircraft movement by trajectory. Moreover it will give aircraft flexibility to fly trajectories that meet their own interests, such as minimizing fuel consumption, but always under the agreement of Air Traffic Control (ATC).

Therefore, automation tools need to be designed to fly optimal and accurate 4D (space and time) trajectories. The design process must consider that the trajectories are implemented under supervision of pilots and in communication with ATC. In this work, we propose an optimal control framework, consistent with operations of air traffic controllers, pilots and autopilots, for strategic (20 minutes to one hour) de-conflicted trajectory planning.

A natural framework to capture the interactions of discrete flight modes and procedures with continuous aircraft state is a hybrid dynamical system [25]. The control input for a hybrid system has a discrete component, which is the sequence of the discrete modes, and two continuous components, which are the duration of each mode and the input driving each mode. A main challenge in the hybrid optimal control problem is finding the optimal sequence of modes in a computationally efficient manner.

Numerical algorithms for addressing the hybrid optimal control problem based on the extensions of the Maximum Principle for hybrid systems [24] have been derived [2], [21]. These algorithms are computationally intense as the number of state dimensions or constraints grow. The dynamic programming principle has also been extended for hybrid systems [5], and methods based on approximate dynamic programming have been derived [11]. In practice, unless additional assumptions are made, such as linearity, computation based on dynamic programming is limited to problems with small state dimensions. One can solve the hybrid optimal control using a hierarchical procedure: at one level, the mode sequence is fixed and the optimal switching times between modes and inputs controlling each mode are determined and, at another level, the mode sequence is varied through either brute force [27], hamming distance [21] or computation of gradient of cost function [6], [8]. Despite all research, efficient algorithms for optimal control of general nonlinear hybrid dynamical systems and applications to realistic engineering problems remain limited.

In aerospace engineering, hybrid optimal control problems with a known mode sequence have been referred to as multiphase optimal control problems. In [22], [23], the multiphase optimal control was formulated for civilian aircraft trajectory optimization, taking into account accurate nonlinear dynamics. It was shown that the fuel consumption could be significantly reduced. The hierarchical hybrid optimal control approach of [8] was applied to single aircraft trajectory planning with accurate nonlinear dynamics and airspace constraints [13]. Inspired by the improvements in 4D trajectory planning and fuel consumption through including accurate aircraft dynamics, here we extend our previous work in [22], [23], [8], [13] to develop a hybrid optimal control framework for aircraft conflict avoidance and hazardous weather avoidance.

Aircraft conflict detection and resolution has been a subject of study for several decades. Discussing all of the previous research is beyond the scope of this paper. We provide a brief review and we refer the readers to [14] for an
excellent survey. Conflict detection methods predict conflict by projecting aircraft trajectory in future time. Depending on the time horizon, nominal, worst-case or probabilistic projections may be used. For conflict resolution, previous work has focused on prescribing discrete maneuvers consistent with air traffic procedures. Hybrid system reachability has been used for designing provably safe maneuvers under worst-case trajectory prediction [25]. Conflict detection methods predict conflict by projecting aircraft trajectory in future time. Depending on the time horizon, nominal, worst-case or probabilistic projections may be used. For conflict resolution, previous work has focused on prescribing discrete maneuvers consistent with air traffic procedures. Hybrid system reachability has been used for designing provably safe maneuvers under worst-case trajectory prediction [25]. Conflict detection and resolution algorithm based on mixed integer linear program (MILP for short) have been published [18], [1]. The optimal control framework has been used to find minimum-time safe trajectories [16]. In most previous work, due to the complexity of problem, conflict resolution through speed, heading or altitude maneuvers are considered separately and an accurate fuel consumption model is not included.

The contributions of this paper are threefold: First, we formulate the conflict free trajectory planning as a hybrid optimal control problem. By including accurate nonlinear aircraft dynamics and constraints we can obtain 4D trajectories while minimizing fuel consumption. In addition, by including hybrid modes of flight, we ensure the methods can integrate with the current procedures implemented by pilots, autopilots and air traffic controllers. Second, to develop a numerical algorithm, we propose a method for casting the hybrid optimal control problem as a multiphase mixed-integer optimal control problem (MIOPC for short) [20]. We describe how the MIOPC can be transcribed as a mixed-integer nonlinear programming problem (MINLP for short) [9], [4] and solved via Branch & Bound techniques. Third, we apply our formulation and solution approach to an Airbus 320 aircraft model to optimize trajectories for collision and hazardous weather avoidance.

The paper is organized as follows. In Section II, we describe the hybrid optimal control problem. In Section III, we develop the solution approach. In Section IV, we describe aircraft dynamics and flight constraints. In Section V, we solve two realistic trajectory planning problems. In Section VI, we summarize our results and directions of future work.

II. HYBRID OPTIMAL CONTROL PROBLEM

Aircraft motion has the characteristic of a switched system due to different flight modes. A switched system is a hybrid dynamical system for which there are no state resets (jumps) at switching times. The dynamics of the switched system are described by a set of differential equations

\[
\dot{x}(t) = f_q(x(t), u(t)), \quad q \in Q := \{1, 2, \ldots, N_q\},
\]

where \( x \in \mathbb{R}^n \) represents the continuous state, \( f_q : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n \), and \( N_q \) represents the number of discrete modes. The input \( u \) belongs to a subset of compact set in \( \mathbb{R}^m \).

A switching sequence \( \sigma \) is defined as the timed sequence of active dynamical systems, or modes, as follows:

\[
\sigma = [(t_0, q_0), (t_1, q_1), \ldots, (t_N, q_N)],
\]

where \( N+1 \) represents the number of phases, \( t_0 \leq t_1 \leq \cdots \leq t_N \leq t_{N+1} \) are the switching times, and \( q_i \in Q \) for \( i = 0, 1, \ldots, N \). The pair \( (t_i, q_i) \) for \( 1 \leq i \leq N \) indicates that at time \( t_i \) the dynamics change from mode \( q_{i-1} \) to \( q_i \). Thus, in the time interval \( [t_i, t_{i+1}) \), referred to as the \( i \)th phase, the state evolution is governed by the vector field \( f_{q_i} \).

The state and input are subjected to inequality and equality constraints for each mode \( q \in Q \), compactly represented as

\[
h_q(x(t), u(t)) \leq 0, \quad \text{for } q \in Q,
\]

where in the above \( h_q : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^{N_c} \). The hybrid optimal control problem is then formulated as follows: Consider the system described by the differential equations (1) and the constraints (3), with initial condition \( x_0 \in \mathbb{R}^n \) at \( t_0 \). Find a switching sequence \( \sigma \) and an input \( u \), that fulfill Equations (1) and (3) and minimize the cost function:

\[
J(\sigma, u) = \phi(x(t_{N+1})) + \sum_{i=0}^{N} \int_{t_i}^{t_{i+1}} L_{q_i}(x(t), u(t))dt.
\]

In the above, \( \phi \) is the final cost and the integral term is the running cost. The initial time \( t_0 \) is given while the final time \( t_{N+1} \) is an optimization variable. We assume \( \forall q \in Q \) the functions \( f_q \), \( h_q \), \( \phi \), and \( L_q \) are Lipschitz and differentiable and their derivatives are also Lipschitz in their arguments.

III. SOLUTION APPROACH

Our approach is summarized as follows: First, we convert the problem to one in which the mode duration in each phase is fixed. Then, we introduce binary control variables for each mode and each phase to formulate a MIOPC. Finally, we apply a collocation discretization rule [10] to convert the dynamic equations of the system into constraints and thus formulate a MINLP. We now describe each step.

A. Multiphase Mixed-Integer Optimal Control Formulation

1) Conversion of variable switching times to fixed switching times: The hybrid optimal control problem above with a fixed sequence of modes \( (q_0, q_1, \ldots, q_N) \) is referred to as a multiphase optimal control problem. Without loss of generality, assume \( t_0 = 0 \). First, introduce new state variables \( x_{n+1+i} \) corresponding to the switching times \( t_i, i = 1, \ldots, N + 1 \) and with dynamics \( \dot{x}_{n+1+i} = 0 \). Then, introduce a new independent variable \( \tau \in [0, N + 1] \) as follows:

\[
t = \begin{cases} 
  x_{n+1} \tau & \tau \in [0, 1] \\
  x_{n+i+1}(\tau - i) - x_{n+i}(\tau - i - 1) & \tau \in [i, i+1],
\end{cases}
\]

where \( 1 \leq i \leq N \). Next, define \( \hat{f}_{q_i} : \mathbb{R}^{n+N+1} \times \mathbb{R}^m \to \mathbb{R}^n ; 
\)

\[
\hat{f}_{q_i} = \begin{cases} 
  x_{n+1}f_{q_0} & i = 0 \\
  (x_{n+i+1} - x_{n+i})f_{q_i} & i = 1, \ldots, N
\end{cases}
\]

Let \( (\cdot)' \) denote the derivative of \( (\cdot) \) with respect to \( \tau \). The differential and algebraic constraints become

\[
{x'}(\tau) = \hat{f}_{q_i}(x(\tau), u(\tau)), \quad \tau \in [i, i+1]
\]

\[
h_{q_i}(x(\tau), u(\tau)) \leq 0, \quad \tau \in [i, i+1]
\]

Similarly, the cost function in Eq. (4) can be written as a function of the new independent variable \( \tau \). Through this transformation, the phase durations are fixed. Thus, for a fixed mode sequence, the transformed problem is a conventional optimal control problem in the extended state \( \bar{x} = (x_1, x_2, \ldots, x_{n+N+1}) \) and new time variable \( \tau \).
2) Binary variables for the mode sequence: To determine the mode sequence, we introduce binary vectors \( w_i \in \{0, 1\}^{N_q} \) for each phase \( i = 0, 1, \ldots, N \). Here, \( w_{qi} = 1 \) indicates that mode \( q \) is active in the interval \([t_i, t_{i+1}]\), or equivalently, in \([i, i+1]\) in the transformed problem. Let \( f : \mathbb{R}^{n+N+1} \times \mathbb{R}^m \times \{0, 1\}^{N_q} \rightarrow \mathbb{R}^{n+1} \) be defined as \( \sum_{q=1}^{N_q} w_{qi} f_q \).

Similarly, define \( \hat{h} \) and \( \hat{L} \). Let \( w = (w_0, \ldots, w_N) \in \{0, 1\}^{N_q \times (N+1)} \) denote all binary variables. Combining the time transformation and state extension of the previous section with the binary mode sequence variables, the hybrid optimal control problem can be written as a MIOCP:

\[
\begin{align*}
\min & \quad J(w, u) = \phi(x(N+1)) \\
\text{s. t.} & \quad \hat{x}'(\tau) = \hat{f}(\hat{x}(\tau), u(\tau), w_i), \\
& \quad \hat{h}(\hat{x}(\tau), u(\tau), w_i) \leq 0, \\
& \quad \sum_{q=1}^{N_q} w_{qi} = 1, \\
& \quad i = 0, 1, \ldots, N.
\end{align*}
\]

B. Numerical Solution

1) Conversion to a MINLP: Consider fixing the binary vectors \( w_{qi} \) for \( i = 0, \ldots, N, q \in Q \). This is equivalent to fixing the sequence of modes. The MIOCP becomes a conventional optimal control problem due to the transformation of Section III-A.1. With a time discretization of dynamics, for example through Trapezoidal, or Simpson rule, the optimal control problem can be converted to a nonlinear program (NLP for short). This method is called direct collocation [10]. It has been widely used for solving optimal control problems in aircraft and aerospace applications due to its computational efficiency [3]. We briefly describe it here.

Let \( N_s \) denote the number of discretization samples for each phase. Let \( k = 0, \ldots, N_s \cdot (N + 1) \). Define \( x^k := x(i + \frac{k}{N_s}), u^k := u(i + \frac{k}{N_s}) \), for \( l = 0, \ldots, N_s - 1 \). With a typical collocation scheme, the differential constraint (1) is represented by \( N_s \cdot (N + 1) \) nonlinear constraints:

\[
c(x^k, x^{k+1}, u^k, u^{k+1}, x_{n+1}, w_i) = 0,
\]
where in the above, \( k = iN_s, \ldots, (i+1) \cdot N_s - 1 \) and the function \( c \) depends on the numerical integration rule. In addition, the algebraic constraints are enforced at the sample points. The running cost is approximated as the summation of \( L_{qi} \), evaluated at the sample points. The optimization variables are now the inputs and states at sample points.

If the sequence of modes is allowed to vary, the NLP becomes a MINLP due to binary decision variables. In all generality, MINLP is an undecidable problem but if one assumes that the feasible region is bounded, which is the case here, it is NP-Hard. Although bounded MINLP problems can be solved in theory, they are one of the most challenging problems in computational optimization, in particular, when the nonlinearities are not convex, as it is the case for the aircraft dynamics and constraints. A simple algorithm for determining the mode sequence could be to enumerate all possible values for \( w_{qi} \), solve the associated optimal control problems and pick the best solution. Unfortunately, this method is impractical for more than a few binary variables. A common approach to address larger problems is to do an implicit enumeration via the Branch & Bound algorithm [15]. We provide a sketch of this method to stress the particularities that arise in applying it to solve the MIOCP.

2) Branch & Bound Algorithm with Collocation Method: Branch & Bound is a divide-and-conquer method. The problem is divided by partitioning the set of feasible solutions into smaller and smaller subsets. The conquering is done by computing bounds on the cost of the best feasible solution in each subset and discarding subsets whose lower bound exceeds a known feasible solution. Branch & Bound is an exact algorithm when the bound used in each subproblem is a valid lower bound. In our case, to find a lower bound for the MIOCP, we relax the binary variables, that is, let \( w_{qi} \in \{0, 1\} \) and solve the associated NLP. However, obtaining a true lower bound on the value of MIOCP is a difficult task due to presence of nonconvex dynamics and constraints in the NLP. Thus, our solution is heuristic.

IV. AIRCRAFT DYNAMICS

In order to plan fuel optimal aircraft trajectories, it is common to consider a 3 degree of freedom dynamic model that describes the point variable-mass motion of the aircraft over a spherical flat-earth model. We consider a symmetric flight, that is, we assume there is no sideslip and all forces lie in the plane of symmetry of aircraft. Wind is included due to its considerable effects on fuel consumption. The equations of motion of the aircraft are:

\[
\begin{align*}
(R + h_e) \cos \theta \dot{\lambda} &= V \cos \gamma \cos \chi + V_{wx}, \\
(R + h_e) \dot{\theta} &= V \cos \gamma \sin \chi + V_{wy}, \\
h_e &= V \sin \gamma + V_{wz}, \\
\dot{\mu} &= -\eta \dot{\gamma}, \\
mV \dot{\chi} &= L \sin \mu, \\
mV \dot{\gamma} &= L \cos \mu - mg \cos \gamma.
\end{align*}
\]

In the above, the three kinematic equations (Eq. (5a-5c)) are expressed in a ground based reference frame \((x_e, y_e, z_e)\) and the three dynamic equations (Eq. (5e-5g)) are expressed in an aircraft-attached reference frame \((x_w, y_w, z_w)\) as shown in Fig. 1. The states are: \( \lambda, \theta, h_e \) referring to the aircraft 3D position (longitude, latitude, altitude); \( V, \chi, \gamma \) referring to the true airspeed, heading angle, and flight path angle respectively. \( R \) is the radius of earth; \( m \) is aircraft mass, \( \eta \) is the speed dependent fuel efficiency coefficient. \( V_{wx}, V_{wy}, V_{wz} \) are components of the wind, \( T \) is the thrust, and \( \mu \) is the bank angle. Lift \( L = C_L S q \) and drag \( D = C_D S q \) are the components of the aerodynamic force, \( S \) is the reference wing surface area and \( \dot{q} = \frac{1}{2} \rho V^2 \) is the dynamic pressure.
A parabolic drag polar $CD = CD_0 + KC^2$, and a standard atmosphere are assumed. $CL$ is a known function of the angle of attack $\alpha$ and the Mach number. The aircraft position in 2D is approximated as $x_\alpha = \lambda(R+h_\alpha)\cos \theta$ and $y_\alpha = (R+h_\alpha)\theta$. The bank angle $\mu$, the engine thrust $T$, and the coefficient of lift $C_L$ are the inputs, that is, $u(t) = (T(t), \mu(t), C_L(t))$. For further details on aircraft dynamics, please refer to [12].

A. Flight Modes

In the en-route portion of flight, aircraft fly straight line segments connecting waypoints. To avoid conflict, the aircraft may be required to deviate from their nominal paths. In terms of air traffic control, these deviations are characterized by maneuvers which may consist of heading, speed, or altitude changes. We consider flight maneuvers as modes of the switched system. A 3D flight plan can be subdivided into a sequence of modes pertaining to flights in a vertical or horizontal plane. We characterize the maneuvers by three modes of control speed (mode 1), control heading (mode 2), and control altitude (mode 3). These maneuvers are routinely used in current air traffic control practice since they are easily implemented by autopilots [25]. We neglect the vertical component of wind $V_{wh}$, due to its low influence.

1) **Control speed:** The aircraft flies with variable speed but constant heading. The bank angle $\mu$ is set to zero. The engine thrust is the input, that is, $u(t) = T(t)$.

2) **Control heading:** The speed is held constant while the heading can change. The input is $\mu$, that is $u(t) = \mu(t)$.

In above two modes $\gamma$, $\dot{\gamma}$, and $\dot{h}$ are set to zero. Thus, the following algebraic constraint is present: $L \cos \mu = mg$.

3) **Control altitude:** We consider vertical climb/descent, so that the bank angle $\mu$ is set to zero. Without loss of generality, we let $\chi = 0$. The thrust and the lift coefficient are the inputs, that is, $u(t) = (T(t), C_L(t))$.

B. Constraints

1) **Flight Envelop:** These constraints are derived from the geometry of the aircraft, structural limitations, engine power, and aerodynamic characteristics. We use the BADA performance limitations model and parameters [17]:

$$
0 \leq h(t) \leq \min[h_{M0}, h_{u(t)}], \quad \gamma_{\min} \leq \gamma(t) \leq \gamma_{\max}, \quad M(t) \leq M_{M0}, \quad m_{\min} \leq m(t) \leq m_{\max}, \quad V(t) \leq \bar{a}_1, \quad \dot{V}(t)V(t) \leq \bar{a}_m, \quad 0 \leq C_L(t) \leq C_{L_{\max}}, \quad T_{\min}(t) \leq T(t) \leq T_{\max}(t), \quad \mu(t) \leq \bar{\mu}.
$$

In the above, $h_{M0}$ is the maximum reachable altitude, $h_{u(t)}$ is the maximum operative altitude at a given mass (it increases as fuel is burned); $M(t)$ is the Mach number and $M_{M0}$ is the maximum operating Mach number; $C_{L_{\max}}$ is the minimum speed coefficient, $V_{s}(t)$ is the stall speed and $V_{M0}$ is the maximum operating calibrated airspeed; $\bar{a}_m$ and $\bar{a}_1$ are respectively the maximum normal and longitudinal accelerations for civilian aircraft. Note that several flight envelop constraints are nonconvex. $T_{\min}$ and $T_{\max}$ correspond, respectively, to the minimum and maximum available thrust. $\bar{\mu}$ corresponds to the maximum bank angle due to structural limitations.

2) **Unsafe Airspace Regions:** A good representation of convective hazard to aviation is high values of Vertically Integrated Liquid (VIL) water content measurements [19]. The VIL values are available from a forecast product over a gridded airspace. To capture regions with high VIL values as no-fly zones in airspace in an algorithmically friendly manner, minimum-volume bounding ellipsoids can be used. Let $M \in \mathbb{R}^{3 \times 3}$ denote the eccentricity matrix of the ellipsoid and $\varepsilon \in \mathbb{R}^{3}$ denote its center. A hazardous weather avoidance constraint is then written as

$$
([x, y, h] - c)^T M ([x, y, h] - c) \geq 1.
$$

3) **Aircraft Collision Constraint:** Two aircrafts are required to be separated by a distance of $R_c$ nautical miles in the horizontal plane or $H$ feet in altitude. Let $(x^i, y^i, h^i)$ denotes the Cartesian position of aircraft $i = 1, 2$. The collision avoidance constraint is written as

$$
||[x^1, y^1] - [x^2, y^2]||_2 \geq R_c \lor |h^1 - h^2| \geq H.
$$

Note that both of the airspace constraints are nonconvex.

V. CASE STUDIES

We consider two case studies: conflict avoidance in 2D and hazardous weather avoidance in 3D for a BADA Airbus 320 model [17]. In both problems optimal maneuvers have to be designed to minimize fuel consumption. The problems are formulated as MIOPCs as described above. For discretization of the dynamics we use Hermite-Simpson collocation scheme [10], a yet accurate scheme but not to complex so it results solvable. The resulting MINLP has a large number of nonconvex equality and inequality constraints and is sparse. A suitable MINLP solver for such problem is Bonmin [4], which implements an NLP based Branch & Bound algorithm, and uses IPOPT [26] as the NLP solver. Bonmin and IPOPT are open-source solvers available from COIN-OR1.

A. **Aircraft Conflict Avoidance**

We consider two aircraft in approx. thirty minute portion of their flights between two meter fixes (waypoints). Aircraft 1 initial and final fixes are $[\lambda_{0}^{1}, \theta_{0}^{1}, h_{0}^{1}] = [-2^\circ, 40^\circ, 11000\text{m}]$ and $[\lambda_{0}^{2}, \theta_{0}^{2}, h_{0}^{2}] = [2^\circ, 40^\circ, 11000\text{m}]$ respectively. The collision avoidance distance is set to $R_c = 5 \text{ nmi}$. Aircraft 2 has reverse initial and final fixes. Thus a midway collision is predicted in the straight path between waypoints and the intended trajectories must be modified and agreed with the ATC. Given that this is a mid-term trajectory planning there is sufficient time for conflict resolution in the horizontal plane. We consider the horizontal modes of control speed and control heading and assume there is a maximum of two phases for each aircraft. Thus, we introduce 4 binary variables for each aircraft. We further assume that initiation of the maneuvers is coordinated. The cost function is the total fuel consumption of both aircraft expressed in Lagrange form, i.e.:

1http://www.coin-or.org
\[
N_q \sum_{q=1}^{N} \sum_{i=0}^{i+1} \left( \int_{i}^{i+1} w_1 q_i \cdot m_1^q(\tau) d\tau + \int_{i}^{i+1} w_2 q_i \cdot m_2^q(\tau) d\tau \right).
\]

The final meter fixes are added as final state constraints. Aircraft are constrained to overfly their final fixes within a 2-minute time window of [1700, 1820] seconds.

Wind forecast is provided by the National Oceanic and Atmospheric Administration\(^2\) by means of GRIB (Gridded Binary) format files. The wind forecast of October the 20th, 2010 has been considered. We convert the tabular data into analytic functions by means of nonlinear regression.

In the optimal solution both aircraft require only one phase: aircraft 1 uses a control speed maneuver, while aircraft 2 uses a control heading maneuver. The results are summarized in Table I. Here, 2D free refers to original infeasible trajectories. The relevant control inputs and states, and the aircraft paths are shown in Figures 3(a), 3(b), and 2 respectively. In both the intended and modified trajectories aircraft 1 uses less fuel due to tailwind.

### Table I

**Optimized Conflict Avoidance**

<table>
<thead>
<tr>
<th>mode sequence</th>
<th>Intended trajectory</th>
<th>Modified trajectory</th>
</tr>
</thead>
<tbody>
<tr>
<td>aircraft 1</td>
<td>2D free</td>
<td>aircraft 1</td>
</tr>
<tr>
<td>aircraft 2</td>
<td>2D free</td>
<td>aircraft 2</td>
</tr>
<tr>
<td>total fuel (kg)</td>
<td>806</td>
<td>806</td>
</tr>
<tr>
<td>flight time (sec)</td>
<td>1328</td>
<td>1617</td>
</tr>
</tbody>
</table>

Fig. 2. Aircraft collision avoidance paths in Google Earth

**B. Aircraft Hazardous Weather Avoidance**

In this case study, two ellipsoids represent unsafe regions of airspace due to convective weather. The ellipsoids’ centers are located at \(c^1 = [0^\circ, 40^\circ, 11700 \text{ m}]\), \(c^2 = [0^\circ, 40^\circ, 11000 \text{ m}]\) and their eccentricities are \(M^1 = 0.00413\), \(M^2 = 0.0153\), where \(I_3\) is the identity matrix in \(\mathbb{R}^{3 \times 3}\). The aircraft initial and final fixes are \([\lambda_0, \theta_0, h_0] = [-2^\circ, 40^\circ, 11000 \text{m}]\) and \([\lambda_f, \theta_f, h_f] = [2^\circ, 40^\circ, 11000 \text{m}]\) respectively. Thus, the intended trajectory intersects the hazardous weather and the 4D trajectory must be modified and agreed with ATC. The aircraft can be in 3 modes of control speed, control heading or control altitude. We assume a maximum of 3 phases to resolve the conflict. In this example, wind is not taken into account. The cost function is fuel consumption as in the previous case.

In the optimal solution the aircraft requires only two phases: a control altitude followed by a control speed mode. The results are summarized in Table II. The relevant control inputs and states are shown in Figures 5(a), 5(b) respectively. As can be seen in Fig. 4, the aircraft gradually climbs up to avoid the first obstacle until it reaches the second obstacle, at which time it flies tangentially below this obstacle. It then climbs up again to minimize fuel consumption and finally returns to the initial altitude, where it switches to mode 1 and reduces velocity towards the desired meter fix. The modified trajectory does not result in substantial modifications of fuel consumption or the arrival time at the final meter fix.

### Table II

**Optimized Hazardous Weather Avoidance**

<table>
<thead>
<tr>
<th>mode sequence</th>
<th>Intended trajectory</th>
<th>Modified trajectory</th>
</tr>
</thead>
<tbody>
<tr>
<td>aircraft 1</td>
<td>3D free</td>
<td>aircraft 1</td>
</tr>
<tr>
<td>aircraft 2</td>
<td>3D free</td>
<td>aircraft 2</td>
</tr>
<tr>
<td>total fuel (kg)</td>
<td>871</td>
<td>873</td>
</tr>
<tr>
<td>switching time (sec)</td>
<td>1490</td>
<td>(1420, 1488)</td>
</tr>
</tbody>
</table>

Fig. 4. 3D hazardous weather avoidance trajectory

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\(^2\)http://www.noaa.gov/
VI. DISCUSSION AND FUTURE WORK

In both cases, to assess the Bonmin solutions, we tried all combinations of the integer variables and solved each resulting NLP problem. It was found that Bonmin gives the same local optimal solution as the minimum of all trials. Also, the algorithm neglects the spurious phases by setting the duration of such phases to zero, i.e., \( t_i = t_{i+1} \). Thus, the algorithm indirectly determines the optimal number of phases. The resulting solution includes the 4D trajectory, the evolution of state variables and the optimal control inputs. The 4D trajectory would ensure satisfaction of time window constraints as required in the TBO concept of flight. The control inputs are useful for optimizing autopilots. The running time for the case studies were below thirty minutes on a 2.56 GHz laptop with 4 GB RAM. Thus, online computation for mid-term trajectory modifications is feasible.

The combination of maneuvers with accurate nonlinear flight dynamics and constraints will allow airlines, in coordination with ATC, to automate strategic trajectory planning and fly the most cost efficient profile, even in the presence of unexpected events. The aim of the authors in the future is to improve computational efficiency of the algorithm and implement the algorithm for several air traffic scenarios.

REFERENCES


