# THE LOSS FACTOR AS A MEASURE OF MECHANICAL DAMPING

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***ABSTRACT.*** *The problem of damping representation and measurement is investigated. Among the many parameters found in literature, the most comprehensive is loss factor $\eta$. Several definitions of $\eta$ are feasible, but in linear problems they all should reduce to the ratio of the in-phase and quadrature parts of the associated complex modulus. This work surveys measuring methods for materials and structures with respect to the way they express $\eta$, either by approximated or correct expressions. Since some commonly used techniques such as the Oberst method do not follow the definition of $\eta$, special care is required when dealing with damping values obtained by different methods and in different environments. On the whole, nonhomogeneous values must be expected, owing to the physical differences among the phenomena that enable damping measurement.*

## NOMENCLATURE

| | |
|---|---|
| $\eta$ : | Loss factor |
| $E'$: | Storage modulus |
| $E''$: | Loss modulus |
| $\varphi$ | Loss angle |
| $D$ | Energy dissipated per cycle |
| $W$ | Vibration energy |
| $m$ : | Mass |
| $k'$ : | Stiffness |
| $k''$ : | Imaginary part of complex stiffness |
| $X_0$: | Oscillation amplitude |
| $\omega_n$: | Natural frequency. |
| $Q$ : | Figure of merit |
| $\zeta$: | Damping ratio |
| $i$ : | Imaginary unit |
| $F$: | Force modulus |
| $H(\omega)$ : | Frequency Response Function |
| $h(t)$ : | Impulse Response Function |

## INTRODUCTION

The dynamic responses and sound-transmission characteristics of structures are essentially determined by their properties of mass and stiffness, which are responsible for the energy stored in the system, and damping, which is responsible for loss of energy from the system. Of the three properties, damping is the least understood and the most difficult to predict since, unlike mass and stiffness, it cannot normally be inferred from simple static measurements. In addition, owing to the fact that damping has insignificant effects on wholly steel structures, the major problems connected with measuring and representing damping have often been neglected, while in the case of highly damped systems such as the trimmed panels used in automobiles, the classic approach has proved woefully inadequate. Hence, there is a need for more advanced analytical and experimental tools to enable designers to account for damping in dynamic analysis.

A distinction must be made between damping as a global characteristic of a given structure and damping as a specific property of a material. The former entails hysteresis and other phenomena such as friction in joints, while the latter refers to the hysteresis typical of some materials subjected to cyclical loading.

Three kinds of effects enable the evaluation of damping:

1. Reduction of vibration amplitudes at resonances
2. Temporal decay of free vibrations
3. Spatial attenuation of forced vibrations.

Of paramount importance is the choice of the most general parameter for describing damping in relation to any of the possible environments. According to many authors **[1-3]**, the loss factor η is the most suitable index. The loss factor was originally introduced as a measure of intrinsic damping of viscoelastic materials:

$$\eta = \frac{E''}{E'} = tg\varphi \qquad (1)$$

that is the ratio between the imaginary and real parts of a complex modulus. In prescribing procedures for measuring damping in elastomers, ISO Standard 6721 **[4]** specifies (1) as for defining the loss factor. Alternatively, a definition in terms of energy concepts can be used. With respect to steady-state oscillation:

$$\eta = \frac{D}{2\pi W} \qquad (2)$$

where *D* stands for the energy dissipated per cycle (or equivalently the amount of energy to be provided to maintain steady-state conditions). If *D* could be removed uniformly along a cycle (which, however, is not very likely), then *D*/2π could be interpreted as the energy loss per radian **[5]**. *W* represents the energy associated to the vibration, i.e., it is a term defining stored energy which admits several feasible definitions.

The loss factor is undoubtedly the most general of the measurement indexes. Whereas other indexes such as the damping ratio ζ are defined on the grounds of the linear single degree of freedom (SDOF) viscous model, the loss factor can be successfully applied in case of nonlinear systems, using (2) rather than (1). (For more details on nonlinear damping phenomena, **[2]** is recommended.) η can be used in material testing or in evaluating a composite structure. It directly measures dissipation, with no reference to the physical mechanisms involved—undoubtedly, a recognition of its generality.

Let us now take a closer look at the term *W* to define energy of vibration. According to Kerwin and Ungar **[6]**, this term is unambiguous only for lightly damped structures, for which the total energy does not vary markedly during a cycle. For these systems, η could be even extended to freely decaying vibration, since *W* and *D* remain virtually unaltered between cycles. It should be recalled that even in a nondissipative system such as that comprising a mass and spring, the vibration energy in harmonic motion is a function of the instantaneous position:

$$W = \frac{1}{2}k'x(t)^2 + \frac{1}{2}m\dot{x}(t)^2 = \qquad (3)$$

$$= \left(\frac{\omega}{\omega_n}\right)^2 \cdot \left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right] \cdot \frac{1}{2}k'x(t)^2 + \left(\frac{\omega}{\omega_n}\right)^2 \cdot \frac{1}{2}k'X_0^2$$

Only at resonant conditions does *W* become constant:

$$W\big|_{w=w_n} = \frac{1}{2}k'X_{0Res}^2 \quad \text{with} \quad X_{0Res} = X_0\big|_{w=w_n} \qquad (4)$$

In highly damped systems, phase shifts can be expected between the moving parts, thereby increasing complexity in the definition of *W*. It is therefore convenient to use energy deformation term for *W* from among the following definitions **[2]**:

1. Energy stored during loading, from zero to maximum force (**Figure 2a**), or from zero to maximum deformation (**Figure 2b**).
2. Energy released during unloading from maximum deformation point *a* (**Figure 2c**).
3. Maximum deformation energy stored relatively to the nondissipative part[1] of the system (**Figure 2d**).

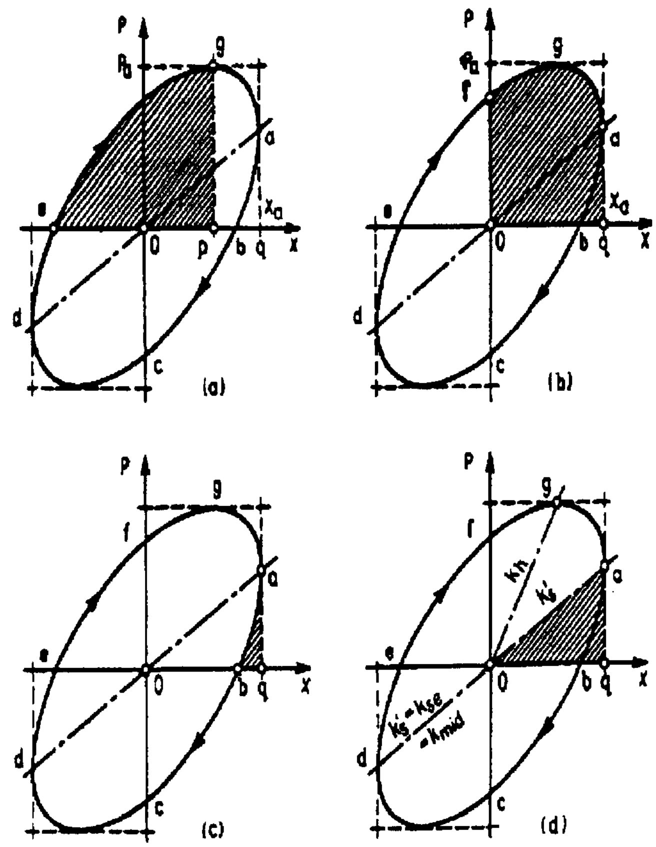


**Figure 2. Graphical representations of energy deformation terms [2].**

When damping is low (η < 0,1), the three values are exceedingly close, whereas for systems comprising elastomers, there are marked differences, and one must be selected. Most authors prefer 3):

$$W = \frac{1}{2}k'X_0^2 \qquad (5)$$

[1] This term is explained in what follows.

Since the kinetic energy is zero, it can also be termed total energy. By this definition, it is implicitly assumed that the stiffness of a composite structure may be described by a complex value and then reduced to an "equivalent spring". This procedure is always licit wherever there is guarantee of linearity. We may now verify the coincidence of (2) and (3):

$$\eta = \frac{D}{2\pi W} = \frac{\pi \cdot k'' \cdot X_0^2}{2\pi \cdot \frac{1}{2} X_0^2 \cdot k'} = \frac{k''}{k'} = \frac{E''}{E'} \qquad (6)$$

Many experimental methods such as the 3 dB method used for materials and structures actually evaluate modal damping in the form of a figure of merit. With these methods, derivation of a frequency response function (FRF) diagram is required **(Figure 3)**.

Interestingly enough, the figure of merit Q does not match the loss factor:

$$Q = \frac{\omega_n}{\omega_2 - \omega_1} \qquad (7)$$

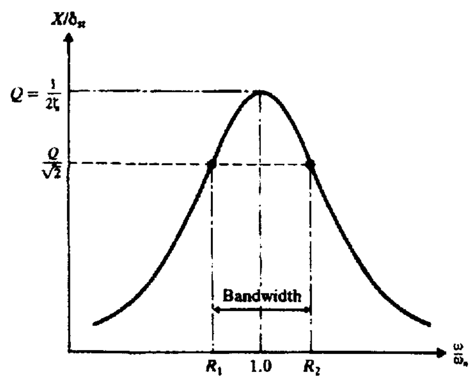


**Figure 3. The 3dB method.**

The commonly provided equivalence:

$$\eta = Q^{-1} \qquad (8)$$

is incorrect. Instead, as pointed out by **[1,7]**:

$$Q^{-1} = \frac{\omega_2 - \omega_1}{\omega_n} = \sqrt{1+\eta} - \sqrt{1-\eta} \qquad (9)$$

Using the proper series expansion for $\eta \ll 1$, we can deduce (8). A 1% error limit on the value of $\eta$ is obtained for:

$$0 \leq \eta \leq 0.28 \qquad (10)$$

These limits are good justification for using (8). Following similar reasoning, we can show that:

$$\eta = 2\zeta \qquad (11)$$

is also incorrect. Note that if we assume (11) to be exact and not an approximation, we would erroneously conclude that the introduction of $\eta$ is redundant.

## EXPERIMENTAL MEASUREMENT OF THE LOSS FACTOR

In investigations of the loss factor, it is advisable to distinguish between methods for materials and structures. The three main classes of methods for determining the loss factor in materials are **[8]**:

1. Valuation of modal loss factor by the 3 dB and modal interpolation methods
2. Free decay methods
3. The Power Input Method (PIM).

The first two classes express damping by a parameter other than $\eta$ and therefore require a conversion. The PIM **[9]**, based on global energy principles, applies (3) to continuously evaluate $\eta$ in the frequency domain, without being limited solely to modal values. It thus should solve the problem of modal overlapping for high damping values ($\eta > 0,1$).

A general classification of methods for measuring damping in elastomers is illustrated in **Figure 4 [10]**.

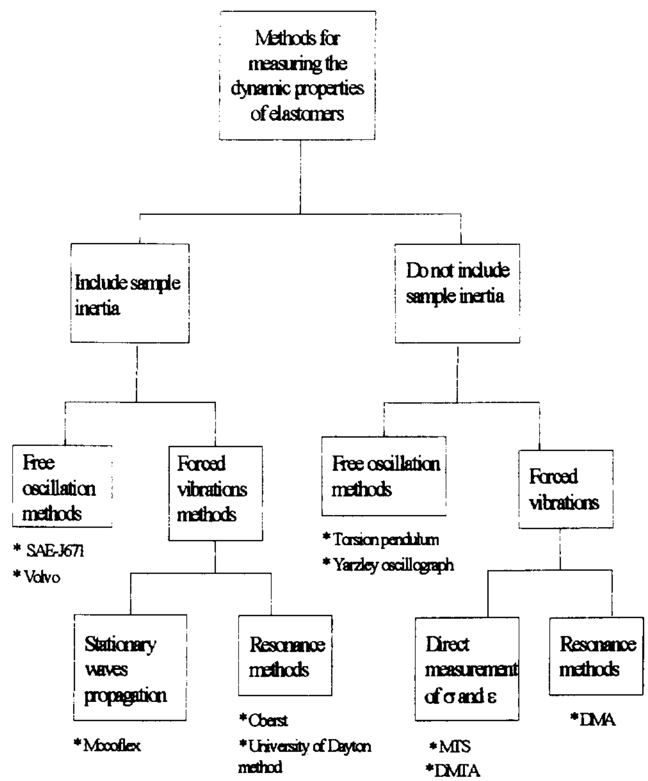


**Figure 4 - Methods for measuring the damping properties in elastomers [10].**

The original loss factor definitions (1) and (2) are followed only by the methods which directly measure the phase shift between stress and strain. In these methods, also referred to as "nonresonance methods" **[11]**, a sinusoidal load (strain) is applied in a special load cell. Since the problems typical of the resonance and decay methods are absent, the nonresonance methods are particularly well-suited for high damping values ($\eta$>0,1). Detailed measurements can be carried out in the glass-rubber transition zone followed by the construction of master diagrams based on the principle of reduced variables.

The other methods express viscoelastic properties to various degrees of approximation. One of the reasons for investigating the constitutive equations of elastomers **[12]** is the need for a description of viscoelasticity in alternative to the complex modulus which is more suitable to time-domain methods. This problem has led some authors **[13, 14]** to investigate the free-response function for the hysteretic model.

The hysteretic model is an elementary SDOF system that for harmonic excitation exhibits a harmonic damping force with a modulus proportional to displacement and a corresponding $\pi/2$ phase shift. The elastic and dissipative effects are accounted for by the use of a complex stiffness, which is allowed to be frequency dependent to take into account the elastomer properties, despite the fact that classic hysteretic model damping should be independent of frequency. The dynamic equilibrium is expressed by:

$$m\ddot{x} + k'(1 + i\eta)\, x = Re\left(Fe^{i\omega t}\right) \qquad (12)$$

with:

$$\eta = k''/k' \qquad (13)$$

The complex term in (12) restricts the validity of the model to harmonic excitation. Strictly speaking, as Crandall **[5]** and Bert **[15]** point out, a time-domain equation with mixed complex and real terms is a mathematical nonsense. It is advisable instead to reason in terms of FRFs:

$$H(\omega) = 1 / [k'(1 + i\cdot\eta) - m\cdot\omega^2] \qquad (14)$$

Jones **[13]** and Milne **[14]** deduced two expressions of the impulse response function for the system by numeric integration of (15):

$$h(t) = (1/2\pi)\int_{-\infty}^{+\infty} H(\omega)\, e^{j\omega t} d\omega \qquad (15)$$

These results could be applied to free decay problems so that more precise correlations between $\eta$ and other indexes (logarithmic decrement, decay rate) could be worked out.

The most common materials measuring method variously known as the Oberst method, after the German scholar who first proposed it, the "flexural vibration test" **[17]**, and the "resonant beam technique" **[18]**, is described in numerous standards, among them ISO 6721 Part3 **[16]**, DIN 53440 **[17]**, ASTM E 756-93 **[18]**, and BSI 4618 Part 5.4 **[19]**. This widely used method reduces to evaluating the loss factor by the 3 dB method for a laminated sample consisting of a steel bar covered with damping material. The recommended values of $\eta$ generally lie within the following limits:

$$0{,}01 < \eta < 0{,}1 \qquad (15)$$

## CONCLUSIONS

The loss factor has been proposed as the most comprehensive damping index. After reiterating the fundamental distinction between material and structural damping, a broad survey was undertaken of the measuring methods. Many do not make direct use of the loss factor and therefore require approximated conversions in order to obtain it. It is therefore advisable to exercise great care when comparing damping values measured by different methods and in different environments owing to their inevitably nonhomogeneous nature.

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