27th February 2008
BARI-TH/00

On a new observable for measuring the Lense–Thirring effect with Satellite Laser Ranging

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Abstract

It turns out that, for a pair of twin Earth's satellites placed in identical orbits with supplementary inclinations, in addition to the already proposed sum of the residuals of the nodal rates for the LAGEOS–LARES mission, also the difference of the residuals of the perigee rates could be employed in measuring the general relativistic Lense–Thirring effect. Indeed, on one hand, the gravitomagnetic precessions of the perigees of two supplementary satellites in identical orbits differ for a – sign, and, on the other, also for this observable the classical secular perturbations induced by the mismodelling of the even zonal harmonics of the static part of the Earth's gravity field would be cancelled out. Since the eccentricities of the two satellites could be chosen to be equal, contrary to the LAGEOS–LARES project, such cancellation would occur at a very accurate level. Among the time–dependent perturbations, the proposed observable would allow to cancel out the even and odd zonal gravitational tidal perturbations and some non–gravitational perturbations. With a proper choice of the inclination of the two satellites, the periods of all the uncancelled time–dependent perturbations could be made short enough to allow to fit and remove them from the signal over observational time spans of few years. The linear perturbation induced by the terrestrial Yarkovski–Rubincam effect would affect the proposed measurement at a level well below 10^{-3}.

1 Introduction

In its weak–field and slow–motion approximation General Relativity predicts that, among other things, the orbit of a test particle freely falling in the gravitational field of a central rotating body is affected by the so called gravitomagnetic dragging of the inertial frames or Lense–Thirring effect. More precisely, the longitude of the ascending node Ω and the argument of the perigee ω of the orbit [1] undergo tiny precessions [2] (The original papers by Lense and Thirring can be found in english translation in [3])

$$\dot{\Omega}_{\rm LT} = \frac{2GJ}{c^2a^3(1-e^2)^{\frac{3}{2}}}, \tag{1}$$

$$\dot{\omega}_{\rm LT} = -\frac{6GJ\cos i}{c^2a^3(1-e^2)^{\frac{3}{2}}}, \tag{2}$$

in which G is the Newtonian gravitational constant, J is the proper angular momentum of the central body, c is the speed of light *in vacuum*, a, e and i are the semimajor axis, the eccentricity and the inclination, respectively, of the orbit of the test particle.

The first measurement of this effect in the gravitational field of the Earth has been obtained by analyzing a suitable combination of the laser-ranged data to the existing passive geodetic satellites LAGEOS and LAGEOS II [4]. The observable [5] is a linear trend with a slope of 60.2 milliarcseconds per year (mas/y in the following) and includes the residuals of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II[1]. The Lense–Thirring precessions for the LAGEOS satellites amount to

$$\dot{\Omega}_{\rm LT}^{\rm LAGEOS} = 31 \text{ mas/y}, \tag{3}$$

$$\dot{\Omega}_{\rm LT}^{\rm LAGEOS\ II} = 31.5 \text{ mas/y}, \tag{4}$$

$$\dot{\omega}_{\rm LT}^{\rm LAGEOS} = 31.6 \text{ mas/y}, \tag{5}$$

$$\dot{\omega}_{\rm LT}^{\rm LAGEOS\ II} = -57 \text{ mas/y}. \tag{6}$$

The total relative accuracy of the measurement of the solve-for parameter $\mu_{\rm LT}$, introduced in order to account for this general relativistic effect, is 2×10^{-1} [4].

[1] The perigee of LAGEOS was not used because it introduces large observational errors due to the smallness of the LAGEOS eccentricity [5] which amounts to 0.0045.

In this kind of experiment the major source of systematic errors is represented by the aliasing trends due to the classical secular precessions [6] of the node and the perigee induced by the mismodelled even zonal harmonics of the geopotential $\delta J_2,\ \delta J_4,\ \delta J_6, ...$ Indeed, according to the present knowledge of the Earth's gravity field based on EGM96 model [7], they amount to a large part of the gravitomagnetic precessions of interest, especially for the first two even zonal harmonics. In the performed LAGEOS experiment the adopted observable allowed for the cancellation of the static and dynamical effects of δJ_2 and δJ_4. The remaining higher degree even zonal harmonics affected the measurement at a 12.9% level.

In order to achieve a few percent accuracy, in [8] it was proposed to launch a passive geodetic laser-ranged satellite- the former LAGEOS III - with the same orbital parameters of LAGEOS apart from its inclination which should be supplementary to that of LAGEOS.

This orbital configuration would be able to cancel out exactly the classical nodal precessions, which are proportional to $\cos i$, provided that the observable to be adopted is the sum of the residuals of the nodal precessions of LAGEOS III and LAGEOS

$$\delta\dot{\Omega}^{\mathrm{III}} + \delta\dot{\Omega}^{\mathrm{I}} = 62\mu_{\mathrm{LT}}. \tag{7}$$

Later on the concept of the mission slightly changed. The area-to-mass ratio of LAGEOS III was reduced in order to make less relevant the impact of the non-gravitational perturbations and the eccentricity was enhanced in order to be able to perform other general relativistic tests: the LARES was born [9].

Currently, the observable of the LAGEOS–LARES mission is under revision in order to improve the obtainable accuracy [10].

The orbital parameters of LAGEOS, LAGEOS II and LARES are in Tab. 1.

Table 1: Orbital parameters of LAGEOS, LAGEOS II and LARES.

Orbital parameter	LAGEOS	LAGEOS II	LARES
a (km)	12,270	12,163	12,270
e	0.0045	0.014	0.04
i (deg)	110	52.65	70

In this paper we show that the configuration of twin satellites placed in identical orbits with supplementary inclinations can reveal itself more fruitful than that one could have imagined before. Indeed, the sum of the nodes can be supplemented with a new observable given by the difference of the perigees.

The paper is organized as follows. In section 2 we describe such new observable, the impact of the mismodelled static part of the gravitational field of the Earth on it and some possibilities for its practical implementation. In section 3 and 4 we discuss the impact of the non–gravitational and gravitational orbital perturbations on the proposed measurement. Section 5 is devoted to the conclusions.

2 A new perigee–only observable

The concept of a couple of satellites placed in identical orbits with supplementary inclinations could be fruitfully exploited in the following new way.

An inspection of eq. (2) and of the explicit expressions of the rates of the classical perigee precessions induced by the even zonal harmonics of the geopotential [11] suggests to adopt as observable the difference of the residuals of the perigee precessions of the two satellites

$$\delta\dot{\omega}^{i} - \delta\dot{\omega}^{\pi-i} = X_{\rm LT}\mu_{\rm LT}, \tag{8}$$

so to obtain a secular trend with a slope of $X_{\rm LT}$ mas/y. Indeed, on one hand, the Lense–Thirring perigee precessions depend on $\cos i$, contrary to the nodal rates which are independent of the inclination, so that, by considering the relativistic effect as an unmodelled force entirely adsorbed in the residuals, in eq. (8) they sum up. On the other, it turns out that the classical even zonal perigee precessions depend on even powers of $\sin i$ and on $\cos^2 i$, so that they cancel out exactly in eq. (8). It may be interesting to notice that the proposed observable of eq. (8) is insensitive to the other general relativistic feature which affects the pericenter of a test body, i.e. the gravitoelectric Einstein precession. Indeed, as it is well known [2], it does not depend on the inclination of the orbital plane.

In regard to a practical application of such idea, we note that the LAGEOS–LARES mission would be unsuitable because the perigee of LAGEOS is not a good observable due to the notable smallness of the eccentricity of its orbit. For the sake of concreteness, we could think about a LARES II which should be the supplementary companion of LAGEOS II. In this case we would have a gravitomagnetic trend with a slope of -115.2 mas/y (which is almost twice that of the LAGEOS–LARES node–only mission). Moreover, since the magnitude of the eccentricity of LAGEOS II is satisfactory in order to perform relativistic measurements with its perigee, the LARES II, contrary to the LAGEOS–LARES mission, could be inserted in an orbit with the same eccentricity of that of LAGEOS II. So, the cancellation of the classical secular precessions would occur at a higher level than in the LAGEOS–LARES node–only observable [10]. Of course, a careful analysis of the time–dependent gravitational [12] and, especially, non–gravitational perturbations (see [13] for the radiative perturbations and [14] for the thermal, spin–dependent perturbations), to which the perigee is particularly sensitive, contrary to the node, is needed in order to make clear if also for such perturbations some useful cancellations may occur, and to which extent the uncancelled perturbations may affect the proposed measurement.

3 The non–gravitational perturbations

3.1 The radiative perturbations

3.1.1 The direct solar radiation pressure

According to [13], the direct solar radiation pressure does not induce any secular trend on the perigee rate: its signature is long–periodic. Its effect on the difference of the perigee rates of two supplementary satellites amounts to[2]

$$\begin{aligned}\dot\omega^{i}_{\rm SRP}-\dot\omega^{\pi-i}_{\rm SRP} &= \frac{3A_{\odot}\cos i}{4nae}\left\{\cos\varepsilon\left[\cos(\Omega-\lambda+\omega)+\cos(\Omega+\lambda-\omega)\right]-\right.\\ &- \left.(1-\cos\varepsilon)\left[\cos(\Omega+\lambda+\omega)+\cos(\Omega-\lambda-\omega)\right]\right\}. \end{aligned} \tag{9}$$

In it $A_{\odot}$ is the acceleration induced by the direct solar radiation pressure. In the case of a supplementary configuration based on LAGEOS II the harmonic $\cos(\Omega+\lambda-\omega)$ would induce serious troubles for the proposed measurement of the Lense–Thirring effect. Indeed, on one hand its period amounts to 4,244 days, i.e. 11.6 years, on the other, even by assuming a 0.5% mismodelling in $A_{\odot}$ [13], the mismodelled amplitude of the perigee rate amounts to 609

[2] In deriving eq. (9) it has been accounted for the fact that for a couple of supplementary satellites the classical rate of the node changes sign because it depends on $\cos i$, while the rate of perigee remains unchanged because it depends on $\cos^2 i$ and on even powers of $\sin i$ [11].

mas/y, while the Lense–Thirring effect is, for the LAGEOS II supplementary configuration, 115.2 mas/y. The mismodelled amplitude of the perigee perturbation would amount to 750.8 mas. Then, over a reasonable time span of few years it would superimpose to the relativistic signal and its level of uncertainty would vanish any attempts for extracting the gravitomagnetic signature.

The situation ameliorates if we consider a couple of entirely new laser–ranged satellites of LAGEOS type with frozen perigees. This means that the inclination would amount to 63.4 deg, so to make the period of perigee extremely long. A possible orbital configuration could be

Table 2: Orbital parameters of the new supplementary satellites.

a (km)	i (deg)	e	$P(\Omega)$ (days)	$P(\omega)$ (days)	$\dot{\Omega}_{\rm LT}$ (mas/y)	$\dot{\omega}_{\rm LT}$ (mas/y)
12,000	63.4 (116.6)	0.05	$\mp$733.45	269,078.41	33	$\mp$44.4

that in Tab. 2. In it we quote also the Lense–Thirring effects on the node and the perigee. So, the periodicities of the perturbing harmonics would amount to

$$P(\Omega+\lambda-\omega) = P(\Omega+\lambda+\omega) = 729.56 \text{ days}, \quad (10)$$

$$P(\Omega-\lambda-\omega) = P(\Omega-\lambda+\omega) = -243.8 \text{ days}. \quad (11)$$

This is very important because, in this case, over an observational time span of few years the time–dependent perturbations due to the direct solar radiation pressure could be viewed as empirically fitted quantities and could be removed from the signal.

3.1.2 The Earth's albedo

For the Earth's albedo, which induces only long–periodic harmonic perturbations on the perigee rate [13], the same considerations as for the direct solar radiation pressure hold because the periodicities are the same.

3.2 The thermal perturbations

3.2.1 The Yarkovski–Rubincam effect

According to [14], the terrestrial Yarkovski–Rubincam effect induces on the perigee rate both secular and long–periodic perturbations. Of course, in regard to the measurement of the secular Lense–Thirring trend the linear Rubincam effect is the most insidious one.

The genuine secular part of the terrestrial Yarkovski–Rubincam perturbation on the perigee rate is, according to [14]

$$\dot\omega_{\rm Rub\ sec}=\frac{A_{\rm Rub}}{4na}\cos\vartheta\left[1+2\cos^2 i+S_z^2\left(1-6\cos^2 i\right)\right], \tag{12}$$

where $A_{\rm Rub}$ is the acceleration due to the Rubincam effect, ϑ is the thermal lag angle and S_z is the component of the satellite's spin axis along the z axis of a geocentric, equatorial inertial frame.

By assuming, for the sake of generality, for the supplementary satellite a thermal lag angle slightly different from that of its twin, so that $\vartheta^{\pi-i}=\vartheta^{i}+\delta$ with δ small, the Rubincam secular effect on the difference of the perigee rates of a pair of supplementary satellites becomes

$$\begin{aligned}\dot\omega^{i}_{\rm Rub\ sec}-\dot\omega^{\pi-i}_{\rm Rub\ sec} &= \frac{A_{\rm Rub}}{4na}\left\{\left[\left(1+2\cos^2 i\right)+\left(1-6\cos^2 i\right)\left(S_z^{\pi-i}\right)^2\right]\delta\sin\vartheta+\right.\\ &+ \left.\left(1-6\cos^2 i\right)\cos\vartheta\left[\left(S_z^{i}\right)^2-\left(S_z^{\pi-i}\right)^2\right]\right\}.\end{aligned} \tag{13}$$

By assuming $(S_z^i)^2=(S_z^{\pi-i})^2$ and $(S_z^{\pi-i})^2=1$, eq. (13) reduces to

$$\dot\omega^{i}_{\rm Rub\ sec}-\dot\omega^{\pi-i}_{\rm Rub\ sec}=\frac{A_{\rm Rub}}{4na}\left(2-4\cos^2 i\right)\delta\sin\vartheta. \tag{14}$$

According to [14], for LAGEOS II $A_{\rm Rub}=-6.62\times10^{-10}$ cm s^{-2}, so that

$$\frac{A_{\rm Rub}}{4na}=-1.8\ {\rm mas/y}. \tag{15}$$

By assuming $\vartheta=55$ deg, as for LAGEOS, and a mismodelling of 20% on $A_{\rm Rub}$, eq. (14), for an orbital supplementary configuration based on LAGEOS II, yields a mismodelled linear trend of $\delta\times0.1$ mas/y, so that the relative error in the Lense–Thirring measurement would amount to $\delta\times(1.4\times10^{-3})$.

On the other hand, by assuming $\delta\sim0$ deg and $(S_z^i)^2\neq(S_z^{\pi-i})^2$, eq. (13) reduces to

$$\dot\omega^{i}_{\rm Rub\ sec}-\dot\omega^{\pi-i}_{\rm Rub\ sec}=\frac{A_{\rm Rub}}{4na}\left(1-6\cos^2 i\right)\cos\vartheta\left[\left(S_z^{i}\right)^2-\left(S_z^{\pi-i}\right)^2\right]. \tag{16}$$

With the same assumption as before for LAGEOS II, eq. (16) yields $\left[\left(S_z^{\rm L2}\right)^2-\left(S_z^{\rm LR2}\right)^2\right]\times 0.2$ mas/y with a relative error in the Lense–Thirring measurement of $\left[\left(S_z^{\rm L2}\right)^2-\left(S_z^{\rm LR2}\right)^2\right]\times(1.7\times 10^{-3})$. However, it should be noticed that both δ and $(S_z^i)^2-(S_z^{\pi-i})^2$ could be made very small, so that the presented estimates would become even more favorable. If, e.g. we think about a pair of new, geodetic satellites of LAGEOS type constructed very carefully in the same way and placed in supplementary frozen perigee orbital configuration, it would be quite reasonable to assume their spins as fixed in the inertial space, mainly along the z axis, during the first years of orbital life, as it happened for LAGEOS and LAGEOS II.

According to Tab. 2 of [14], among the periodicities of the harmonic terms $2\dot\omega$ and $4\dot\omega$ are present. For a couple of new supplementary satellites with frozen perigees the mismodelled part of such harmonics, which, in general do not cancel out in the difference of the perigee rates, would resemble aliasing secular trends. However, their impact on the Lense–Thirring measurement should be at the 10^{-3} level because their amplitude is proportional to $\frac{(\delta A_{\rm Rub})}{4na}$ which amounts to 0.3 mas/y, for LAGEOS II, by assuming a mismodelling of 20% in $A_{\rm Rub}$.

3.2.2 The solar Yarkovski–Schach effect

The solar Yarkovski–Schach effect does not induce secular perturbations on the perigee rate [14]. In regard to its long–periodic harmonic terms, which, in general, do not cancel out in the difference of the rates of the perigees of a pair of supplementary satellites, if the frozen perigee configuration would be adopted, their impact was not insidious for the proposed Lense–Thirring measurement. Indeed, as can be inferred from Tab. 4 of [14], their periodicities do not contain any multiple of the perigee frequency, so that, with the proposed configuration, no semisecular terms would affect the signal.

4 The gravitational tidal perturbations

In regard to the orbital perturbations induced by the Earth solid and ocean tides, according to [12], the perigee is particularly sensitive to them, not only to the $l=2$ part of the tidal spectrum, but also to the $l=3$ constituents.

An important role in assessing the impact of the long periodic tidal perturbations on the proposed measurement of the Lense–Thirring effect is played by their frequencies which are given by

$$\dot{\Gamma}_f + (l-2p)\dot{\omega} + m\dot{\Omega}. \tag{17}$$

Recall that [12] for the even constituents $l-2p=0$, while for the odd constituents $l-2p \neq 0$. Moreover, in eq. (17), for a given tidal constituent f, $\dot{\Gamma}_f$ depends only on the luni–solar variables. In order to evaluate correctly the impact of the gravitational time–dependent perturbations on the perigee, it is important to recall that, according to eq. (36) and eq. (50) of [12], their amplitudes are proportional to

$$\frac{(1-e^2)}{e}F_{lmp}\frac{dG_{lpq}}{de} - \frac{\cos i}{\sin i}G_{lpq}\frac{dF_{lmp}}{di}, \tag{18}$$

where $F_{lmp}(i)$ and $G_{lpq}(e)$ are the inclination functions and eccentricity functions, respectively [*Kaula*, 1966].

Fortunately, the proposed combination $\dot{\omega}^i - \dot{\omega}^{\pi-i}$ allows to cancel out the 18.6–year and the 9.3–year tides because they are even zonal perturbations. This is an important feature because their extremely long periods are independent of those of the node and/or the perigee of the satellites to be employed: indeed, they depend only on the luni–solar variables. Moreover, their $l=2,\ m=0$ constituents would have large amplitudes, so that, if not canceled out, they would represent very insidious superimposed biasing trends. In regard to the $l=2$ tesseral $(m=1)$ and sectorial $(m=2)$ tides, both solid and ocean[3], from the fact that their frequencies depend on $\dot{\Omega}$, which, as already previously pointed out, changes sign for a supplementary satellite, and from

$$F_{211} = -\frac{3}{2}\sin i\cos i, \tag{19}$$

$$\frac{dF_{211}}{di} = -\frac{3}{2}\left(\cos^2 i - \sin^2 i\right), \tag{20}$$

$$F_{221} = \frac{3}{2}\sin^2 i, \tag{21}$$

$$\frac{dF_{221}}{di} = 3\sin i\cos i, \tag{22}$$

(23)

[3]For the ocean tides we consider only the prograde constituents.

it turns out that they do affect $\dot{\omega}^{i}-\dot{\omega}^{\pi-i}$. However, this fact would not have a serious impact on the proposed measurement of the Lense–Thirring effect since the periods of such perturbations would not be too long, so that they could be fitted and removed from the signal over an observational time span of some years.

A careful analysis must be performed for the ocean odd tidal perturbations. Fortunately, the odd zonal ($l=3,\ m=0$) tidal perturbations cancel out. Indeed, for them it turns out that

$$\begin{aligned}
F_{300} &= -\frac{5}{16}\sin^3 i, && (24)\\
\frac{dF_{300}}{di} &= -\frac{15}{16}\sin^2 i\cos i, && (25)\\
F_{301} &= \frac{15}{16}\sin^3 i-\frac{3}{4}\sin i, && (26)\\
\frac{dF_{301}}{di} &= \frac{45}{16}\sin^2 i\cos i-\frac{3}{4}\cos i, && (27)\\
F_{302} &= -F_{301}, && (28)\\
\frac{dF_{302}}{di} &= -\frac{dF_{301}}{di}, && (29)\\
F_{303} &= -F_{300}, && (30)\\
\frac{dF_{303}}{di} &= -\frac{dF_{300}}{di}. && (31)
\end{aligned}$$

Moreover, their frequencies are independent of $\dot{\Omega}$. It is important to notice that the same result holds also for the time–dependent perturbation induced on the perigee by the J_3 odd zonal harmonic of the geopotential. This is an important feature because its frequency is $\dot{\omega}$; for a frozen perigee configuration it would represent a very insidious secular perturbation.

In regard to the $l=3$ tesseral ($m=1$) and sectorial ($m=2$) tidal lines, it can be proved that they do affect $\dot{\omega}^{i}-\dot{\omega}^{\pi-i}$ because their amplitudes depend on inclination functions $F_{lmp}(i)$ which depend, among other factors, also on $\cos i$, contrary to the $l=3,\ m=0$ case. Indeed, it turns out

$$\begin{aligned}
F_{311} &= \frac{15}{16}\sin^2 i(1+3\cos i)-\frac{3}{4}(1+\cos i), && (32)\\
\frac{dF_{311}}{di} &= \frac{15}{8}\sin i\cos i(1+3\cos i)-\frac{45}{16}\sin^3 i+\frac{3}{4}\sin i, && (33)\\
F_{312} &= \frac{15}{16}\sin^2 i(1-3\cos i)-\frac{3}{4}(1-\cos i), && (34)\\
\frac{dF_{312}}{di} &= \frac{15}{8}\sin i\cos i(1-3\cos i)+\frac{45}{16}\sin^3 i-\frac{3}{4}\sin i, && (35)
\end{aligned}$$

$$F_{321} = \frac{15}{8}\sin i(1-2\cos i-3\cos^2 i), \tag{36}$$

$$\frac{dF_{321}}{di} = \frac{15}{8}\cos i(1-2\cos i-3\cos^2 i)+\frac{15}{8}\sin i(2\sin i+6\sin i\cos i), \tag{37}$$

$$F_{322} = -\frac{15}{8}\sin i(1+2\cos i-3\cos^2 i), \tag{38}$$

$$\frac{dF_{322}}{di} = -\frac{15}{8}\cos i(1+2\cos i-3\cos^2 i)-\frac{15}{8}\sin i(-2\sin i+6\sin i\cos i), \tag{39}$$

$$\tag{40}$$

Moreover, their frequencies are combinations of the form $\dot{\Gamma}_f \pm \dot{\omega} + \dot{\Omega}$ and $\dot{\Gamma}_f \pm \dot{\omega} + 2\dot{\Omega}$. For an orbital configuration based on LAGEOS II this fact would represent a serious drawback because, as pointed out in [12], the $K_1,\ l=3,\ m=1,\ p=1,\ q=-1$ tidal line induces on the perigee of LAGEOS II a perturbation with nominal amplitude of -1,136 mas and period of 1,851.9 days, i.e. 5.07 years, from the frequency $\dot{\omega}+\dot{\Omega}$. The situation is quite similar to that of the direct solar radiation pressure harmonic with a period of 11.6 years. Instead, as in that case, with a frozen perigee configuration the period of the $K_1,\ l=3,\ m=1,\ p=1,\ q=-1$ tidal line would greatly reduce, so that it could be fitted and removed from the signal over an observational time span of few years. The same holds also for the other $l=3,\ m=1,2$ tidal lines.

5 Conclusions

In this paper we have proposed to consider the difference of the residuals of the perigee rates $\delta\dot{\omega}^i - \delta\dot{\omega}^{\pi-i}$, in addition to the already proposed sum of the residuals of the nodes $\delta\dot{\Omega}^i - \delta\dot{\Omega}^{\pi-i}$, as a new observable for measuring the Lense–Thirring effect with a pair of laser–ranged Earth's satellites in identical orbits with supplementary inclinations. In the well known originally proposed LAGEOS –LARES mission $\delta\dot{\omega}^i - \delta\dot{\omega}^{\pi-i}$ would be unsuitable because the perigee of LAGEOS cannot be measured accurately due to the smallness of its eccentricity.

A careful analysis of the systematic errors induced by the non–gravitational and gravitational orbital perturbations has been carried out. In regard to the gravitational perturbations, also for $\delta\dot{\omega}^i - \delta\dot{\omega}^{\pi-i}$, as for $\delta\dot{\Omega}^i + \delta\dot{\Omega}^{\pi-i}$, the main systematic error induced by the mismodelled even zonal coefficients of the multipolar expansion of the Earth's gravitational field cancels out. Moreover, the even and odd zonal time–varying tidal perturbations do not affect the proposed observable. This is a very important feature because among them there are the very insidious semisecular 18.6–year and 9.3–year tides, whose frequencies are independent of the satellite's orbital configuration because depend only on the luni–solar variables. On the contrary, the tesseral and sectorial tides and most of the non–gravitational time–dependent perturbations do affect $\delta\dot\omega^{i}-\delta\dot\omega^{\pi-i}$.

If an orbital configuration based on LAGEOS II and a twin of its, say LARES II, was adopted it would present some drawbacks because of two uncancelled long–periodic harmonic perturbations which have periods of 5.07 years ($K_1,\ l=3,\ m=1,\ p=1,\ q=-1$ tide) and 11.6 years (direct solar radiation pressure), respectively. Indeed, over observational time spans of few years they would resemble aliasing superimposed trends which could bias the recovery of the linear Lense–Thirring signal. The optimal choice would be the use of a couple of entirely new geodetic laser–ranged satellites of LAGEOS type accurately constructed in an identical manner with a small area–to–mass ratio, so to minimize the impact of the non–gravitational perturbations, and placed in a frozen perigee configuration. In this way there would not be semi–secular effects and all the time–dependent perturbations affecting the proposed observable would have short enough periods so to be fitted and removed from the signal over reasonable time spans.

The terrestrial Yarkovski–Rubincam effect would induce, among other things, an uncancelled, genuine linear perturbation. This fact is very important because it could mimic the relativistic trend and make its measurement impossible due to the related level of mismodelling. By the way, its impact on the proposed measurement of the Lense–Thirring effect would be well below the 10^{-3} level.

In conclusion, the proposal of measuring the Lense–Thirring effect with a supplementary pair of laser–ranged satellites turns out to be enforced because it would be possible to analyze not only $\delta\dot\Omega^{i}+\delta\dot\Omega^{\pi-i}$, as in the originally proposed LAGEOS–LARES proposal, but also $\delta\dot\omega^{i}-\delta\dot\omega^{\pi-i}$, provided that a carefully selected orbital configuration is adopted. Moreover, if the new satellites to be launched had rather eccentric orbits, we would have at our disposal both their perigees, apart from that of the existing LAGEOS II (and, perhaps, of LARES) to perform other gravitational tests concerning, e.g., the realtivistic gravitoelectric perigee advance [15] and the hypotheis of a fifth force [16].

Acknowledgements

I'm grateful to L. Guerriero for his support while at Bari. Special thanks to D.M. Lucchesi for his helpful and important informations on the non–gravitational perturbations on LAGEOS II.

References

[1] Sterne, T. E., *An Introduction to Celestial Mechanics*, 206 pp., Interscience, New York, 1960.

[2] Ciufolini, I., and J. A. Wheeler, *Gravitation and Inertia*, 498 pp., Princeton University Press, New York, 1995.

[3] Lense, J., and H. Thirring, Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie, *Phys. Z., 19*, 156-163, 1918, translated by Mashhoon, B., F. W. Hehl, and D. S. Theiss, On the Gravitational Effects of Rotating Masses: The Thirring-Lense Papers, *Gen. Rel. Grav., 16*, 711-750, 1984.

[4] Ciufolini, I., E. Pavlis, F. Chieppa, E. Fernandes-Vieira, and J. Pérez-Mercader, Test of General Relativity and Measurement of the Lense-Thirring Effect with Two Earth Satellites, *Science, 279*, 2100-2103, 1998.

[5] Ciufolini, I., On a new method to measure the gravitomagnetic field using two orbiting satellites, *Il Nuovo Cimento, 109A, 12*, 1709-1720, 1996.

[6] Kaula, W. M., *Theory of Satellite Geodesy*, 124 pp., Blaisdell Publishing Company, Waltham, 1966

[7] Lemoine, F. G., et al., The Development of the Joint NASA GSFC and the National Imagery Mapping Agency (NIMA) Geopotential Model EGM96, NASA/TP-1998-206861, 1998.

[8] Ciufolini, I., Measurement of Lense–Thirring drag on high-altitude, laser ranged artificial satellites, *Phys. Rev. Lett., 56*, 278-281, 1986.

[9] Ciufolini, I., The Concept of the LARES experiment, in *LARES Phase–A Study*, pp. 16-33, Rome, 1998.

[10] Iorio, L. D.M. Lucchesi, and I. Ciufolini, The LARES mission revisited: an alternative scenario, to be published in *Class. and Quantum Grav.*, preprint gr-qc/0203099, 2002.

[11] Iorio, L., Recent Developments in Testing General Relativity with Satellite Laser Ranging, to be published in *La Rivista del Nuovo Cimento,* preprint gr-qc/0111086, 2002a.

[12] Iorio, L., Earth tides and Lense-Thirring effect, *Celest. Mech., 79*(3), 201-230, 2001.

[13] Lucchesi, D., Reassessment of the error modelling of non–gravitational perturbations on LAGEOS II and their impact in the Lense–Thirring determination. Part I, *Plan. and Space Sci., 49*, 447-463, 2001.

[14] Lucchesi, D., Reassessment of the error modelling of non-gravitational perturbations on LAGEOS II and their impact in the Lense–Thirring determination. Part II, to be published in *Plan. and Space Sci.*, 2002.

[15] Iorio, L., I. Ciufolini and E.C. Pavlis, Measuring the Relativistic Perigee Advance with Satellite Laser Ranging, submitted to *Class. and Quantum Grav.*, preprint gr-qc/0103088, 2002.

[16] Iorio, L., Constraints to a Yukawa gravitational potential from laser data to LAGEOS satellites, *Physics Lett. A, 298*(5–6), 315–318, 2002b.