Blind Channel Estimation with Lower Complexity Algorithm for OFDM System

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Abstract—Fast and reliable channel estimation is very important for wireless communication transmission. This paper addresses blind channel estimation for OFDM systems. A novel One-Cycle blind channel estimation algorithm based on cyclostationarity properties of OFDM signal is proposed in this paper, the new algorithm is formed from using the two related z-transform values of delay parameter. Furthermore, a lower complexity algorithm based on partial spectrum information can further reduce computational complexity. Computer simulation results verify that the performance of the proposed One-Cycle algorithm is superior to that of the Two-Cycle algorithm. Although the lower complexity One-Cycle algorithm has some performance loss in high SNR region, its computational complexity is dramatically reduced

Index Terms—OFDM, Cyclostationarity, Blind channel estimation

I. INTRODUCTION

The growing demand for wireless communications requires reliable and high-rate data transmission over the wireless channel. How to estimate the channel quickly and accurately is one of the key technologies to improve the reliability of information transmission. Generally, there are two kinds of channel estimation method, pilot based channel estimation [1-3] and blind channel estimation [4-9].

Blind estimation methods can be divided into high-order statistical method and second-order statistical method. The former needs a large amount of data samples, has a high complexity, and is difficult to be implemented in practical systems. Gardner first discussed the signal cyclostationarity characteristics and application in communication systems [5]. As the output second-order statistics contains the phase information of the channel due to the cyclostationarity, most non-minimum phase channels can be identified from the second-order statistics of the cyclostationary output sequence [6]. Since then, many elegant solutions such as subspace method in [8] and deterministic approach in [13] have been proposed for blind channel identification and equalization. In [9, 11], Giannakis etc use a precoder to induce cyclostationarity at the transmitter that guarantees blind identifiability of channels with minimal degradation of information rate. Heath and Giannakis [10] proposed a subspace method from using the cyclic correlation of the channel output to blindly estimate the channel in OFDM systems. This method is called Two-Cycle algorithm. In this algorithm, the spectrum information of two different non-zero cycle frequencies is used in the Two-Cycle algorithm for OFDM system, leading to lower utilization of spectrum resources and thus making degradation of the system estimation performance.

In this paper, we use the cyclostationarity induced by the cyclic prefix in the OFDM system for the Inter-Signal Interference (ISI) to develop a One-Cycle blind channel estimation algorithm for identifying the channel in OFDM systems. Different from [10], the new method is formed from analyzing the z-transform of delay variable. This algorithm is called One-Cycle algorithm due to only one cycle frequency of the spectrum information used. Furthermore, a lower complexity algorithm based on partial spectrum information can further reduce the computational complexity.

This paper is organized as follows. An overview of cyclostationarity frequency properties for OFDM system is provided in section II. The related blind channel estimation algorithm is outlined in Section III. The new proposed with lower complexity algorithms are developed in Section IV. Some computer simulation results are given in Section V. Section VI concludes this paper.

II. SYSTEM MODEL AND CYCLOSTATIONARY PROPERTIES

The block diagram for the transmitter of a typical QPSK-OFDM system is shown in Figure.1. At the OFDM transmitter, data are firstly converted from serial to parallel form. Each parallel data transmission is modulated by different carrier frequencies using QPSK or QAM scheme The OFDM modulator takes the M-point IFFT of a block input symbols. A sequence of \( L < M \) symbols named cyclic prefix (see Figure.2) is appended to the beginning of each block in order to avoid inter-block interference (IBI). At the receiver, the data are retrieved by a FFT and, then, demapped with...
corresponding scheme to obtain the estimated data as in Figure.3.

Figure 1. The QPSK-OFDM transmitter

Figure 2. The OFDM frame structure

Figure 3. The QPSK-OFDM receiver

Without timing and frequency offset, the nth sample of the mth OFDM symbol can be described by

\[ x_m(n) = x(nP + m) = \sum_{p=0}^{P-1} x_p(n) e^{j \frac{2\pi}{M} p(m-L)} \quad (n=0,\ldots,N-1) \]  

where \( x_p \) is the pth subcarrier in the mth symbol at the input of the OFDM modulator and the \( \exp(j \frac{2\pi p(-L)}{M}) \) accounting for the cyclic prefix, which is a repetition of the last \( L \) time domain symbols as in Figure.3.

Cyclic extensions are implemented with the cyclic repetition of the part of the IFFT output, as shown in

Figure 4. Therefore periodicity is induced in the second-order statistics of the transmitted signals.

Figure 4. Autocorrelation of a OFDM block at a delay equivalent to one IFFT block Cyclic extensions

With the unit variance, independently and identically distributed (i.i.d) input stream, the transmitted signal autocorrelation appears at delays \( \tau = \pm M \) as a pulse of length \( L \) and period \( P \) with the following form

\[ R(n,\tau) = E[x(n) x(n+\tau)^H] \]

\[ = \delta(M\tau + \delta(\tau-M) \sum_{z=0}^{\frac{L-1}{M}} x(n-z) + \delta(\tau+M) \sum_{z=\frac{P-1}{M}}^{\frac{P-1}{M}} x(n-z)) \]  

(2)

where \( X \) is the transmitted signal and superscript \( H \) stands for conjugate transpose. An example given in Figure.5 for \( M=12 \) and \( L=4 \) shows periodic pulse at \( \tau = \pm 12 \) .

Figure 5. Example illustrating periodicity in transmitted signals autocorrelation

According to (2), the autocorrelation related to time \( n \) of the transmitted signal is a periodic function with period \( P \). Therefore, the time-varying autocorrelation admits a Fourier series representation known as cyclic correlation. Each Fourier coefficient is defined at the k'th cyclic frequency as:

\[ R_k(\tau) = \frac{1}{P} \sum_{n=0}^{P-1} R(n,\tau) e^{-j \frac{2\pi n k}{P}} \]  

(3)

Substituting (2) into (3), the cyclic correlation \( R_k(\tau) \) at a fixed cycle \( k \), \( 0 \leq k < P-1 \), is given by
where

\[ E_i(k) = e^{-\frac{\pi}{P} (l-1)} \sin(\pi kL / P) / \sin(\pi k / P) \] (5)

\[ E_z(k) = E_i(k) e^{-\frac{2\pi Mk}{P}} \] (6)

The cyclic correlation function of the transmitted signal obtained for (2) has the following properties:

(a) an impulse is located at \( \tau = 0 \) and \( k = 0 \)

(b) It has a sinc function at \( \tau = \pm M \), and also is zero value located at value \( k \) with integer period \( P / L \).

As the cyclic correlation of the noise is \( R_v(k, \tau) = \sigma_v^2 \delta(\tau) \delta(k) \), which is zeros for nonzero cycles \( k \). Therefore, we will assume that \( k \neq 0 \) to avoid stationary noise.

From (4), a conclusion can be drawn that the energy of transmitted signal autocorrelation is mainly distributed in \([-M, M] \), while that of receive signal in \([-(M+L), (M+L)] \). In this case, the cycle correlation of receive signal is

\[ R_z(k, \tau) = \sum_{l=0}^{L-1} \sum_{q=-\infty}^{\infty} h(l) h^*(l+\tau-q) R_{r}(n-l,q) + R_{r}(\tau) \] (7)

where \( v \) represents the AWGN noise with autocorrelation \( R_v(\tau) \). Linear times-invariant filtering is known to conserve cyclostationarity property. This is verified in Figure 7. In this example, the received signal is passed through \( h = [1 -0.8 + 0.2 \ 0.6 - 0.3j \ 0.8 - 0.5j \ 0.6 - 0.4j] \). The cyclic correlation of the received signal is zero located at integer of \( P / L \) radio.
\[
S_j(k, z) = \sum_{\tau=-M}^{M} R_j(k, \tau)z^{-\tau}
\]  

(10)

Substituting (9) into (10), the cyclic spectrum for a special cycle \(k \neq 0\) is

\[
S_j(k, z) = \sum_{p=0}^{M+L_h} \sum_{q=0}^{M} H(\tau) (l+\tau-q) R(k, q) e^{-j2\pi \frac{q}{P} \tau} z^{-\tau}
\]

(11)

\[
= \sum_{p=0}^{M} \sum_{q=0}^{M} R(k, q) e^{-j2\pi \frac{q}{P} \tau} \sum_{l=-(M+L_h)}^{M+L_h} h'_l (l+\tau-q) z^{-\tau}
\]

and hence

\[
S_j(k, z) = H(e^{j2\pi \frac{q}{P} \tau} z^{-1}) S_j(k, z) H'(z')
\]

(12)

where (11) was obtained by changing the summation and denoting with

\[
H(z) = \sum_{\tau=-M}^{M} h(\tau)(z)^{-\tau}
\]

the channel’s Z transform and \(S_j(k, z)\) is the cyclic correlation of transmitted signal.

III. THE EXISTING TWO-CYCLIC ALGORITHM

If two different non-zero cycle frequencies \(k_1\) and \(k_2\) available are taken into (12), we can get the linear equations as follows

\[
\begin{align*}
S_j(k_1, z) &= H(e^{j2\pi \frac{k_1}{P} z^{-1}}) S_j(k_1, z) H(z') \\
S_j(k_2, z) &= H(e^{j2\pi \frac{k_2}{P} z^{-1}}) S_j(k_2, z) H(z')
\end{align*}
\]

(13)

and hence

\[
\frac{S_j(k_1, z)}{S_j(k_2, z)} = \frac{H(e^{j2\pi \frac{k_1}{P} z^{-1}}) S_j(k_1, z)}{H(e^{j2\pi \frac{k_2}{P} z^{-1}}) S_j(k_2, z)}
\]

(14)

which is equivalent to

\[
S_j(k_1, z) H(e^{j2\pi \frac{k_1}{P} z^{-1}}) S_j(k_2, z)
\]

\[
= S_j(k_2, z) H(e^{j2\pi \frac{k_2}{P} z^{-1}}) S_j(k_1, z)
\]

(15)

\[
H'(z') \text{ can be canceled by the combination of the two above linear equations and (15) can be rewritten in the polynomial form as}
\]

\[
\sum_{p=M}^{M+L_h} \sum_{q=M}^{M} R(k, q) e^{-j2\pi \frac{q}{P} \tau} \sum_{l=-(M+L_h)}^{M} h'_l (l+\tau-q) z^{-\tau}
\]

\[
= \sum_{p=M}^{M+L_h} \sum_{q=M}^{M} R(k, q) e^{-j2\pi \frac{q}{P} \tau} \sum_{l=-(M+L_h)}^{M} h'_l (l+\tau-q) z^{-\tau}
\]

(16)

In order to estimate the channel, some notations are used. Let \(T_y^k\) denote the \((4M+3L_y+1)\times(2M+L_y+1)\) Toeplitz matrix with first column \([R(k_1, M+L_y) \cdots R(k_1, -L_y) 0 \cdots 0]\) and first row \([R_y(k_1, M+L_y) 0 \cdots 0]\); \(T_x^k\) the \((4M+3L_y+1)\times(2M+L_y+1)\) Toeplitz matrix with first column \([R_x(k_2, M+L_y) \cdots R_x(k_2, -L_y) 0 \cdots 0]\) and first row \([R_x(k_2, M+L_y) 0 \cdots 0]\). Similarly, let \(T_y^k\) denote the \((4M+3L_y+1)\times(2M+L_y+1)\) Toeplitz matrix with first column \([R(k_1, M+L_y) \cdots R(k_1, -L_y) 0 \cdots 0]\) and first row \([R_y(k_1, M+L_y) 0 \cdots 0]\); \(T_x^k\) denote the \((4M+3L_y+1)\times(2M+L_y+1)\) Toeplitz matrix with first column \([R_x(k_2, M+L_y) \cdots R_x(k_2, -L_y) 0 \cdots 0]\) and first row \([R_x(k_2, M+L_y) 0 \cdots 0]\). With as transpose, let \(h=[0 \quad \ldots \quad h_{(M+L_h)}^T], D_h=diag([e^{j2\pi \frac{k_1}{P}} \cdots e^{j2\pi \frac{k_1}{P}}])\) and \(D_h=diag([e^{j2\pi \frac{k_1}{P}} \cdots e^{j2\pi \frac{k_1}{P}}])\).

The coefficients in both sides of (15) satisfy the following equations

\[
T_y^k \times T_x^k \times D_{k_2} \times h = T_y^k \times T_x^k \times D_h \times h
\]

(17)

which is equivalent to

\[
(T_y^k \times T_x^k \times D_{k_2} - T_y^k \times T_x^k \times D_h) \times h = 0
\]

(18)

The channel coefficient vector \(h\) can be uniquely recovered from (18), This forms the Two-Cyclic algorithm.

IV. THE CYCLOSTATIONARITY-BASED ONE-CYCLIC ALGORITHM AND LOWER COMPLEXITY FOR OFDM
Different from the Two-Cyclic algorithm, this paper addresses blind channel identification relying on the One-Cyclic algorithm through selecting the special variable z for the OFDM system.

A. The One-Cyclic algorithm

Firstly, taking variable z respectively as $e^{j2\pi k/P}z^{-1}$ and $e^{j2\pi k/P}z^{2}$ in (12), we generate the following two equations:

\[
egin{align*}
S(k, e^{j2\pi k/P}z^{-1}) &= H(z)S(k, e^{j2\pi k/P}z^{-1})H^H(e^{j2\pi k/P}z) \\
S(k, e^{j2\pi k/P}z^{2}) &= H(e^{j2\pi k/P}z)S(k, e^{j2\pi k/P}z^{2})H^H(e^{j2\pi k/P}z)
\end{align*}
\]

Then canceling $H^H(e^{j2\pi k/P}z^{-1})$ in (19) form the polynomial as

\[
\sum_{m=1}^{M_2} R_l(k, e^{j2\pi m/P}z^{-m}) = \sum_{m=1}^{M_2} R_{l2}(k, e^{j2\pi m/P}z^{-m})^2
\]

Finally, four Toeplitz matrix named $T_1$, $T_2$, $T_3$, $T_4$ cab be constructed, where $T_1$ and $T_2$ are $(4M + 3L_n + 1) \times (2M + L_n + 1)$ -dimensional Toeplitz matrix formed respectively from $R_l(k, \tau)e^{j\pi \tau k/P}$, $R_{l2}(k, \tau)e^{j\pi \tau k/P}$, and $T_3$ and $T_4$ are $(2M + L_n + 1) \times (L_n + 1)$ -dimensional Toeplitz matrix formed respectively in the same way from $R_l(k, q)e^{j\pi \tau \tau k/P}$, $R_{l2}(k, q)e^{j\pi \tau \tau k/P}$.

According to the Toeplitz matrix multiplication polynomial norms [12], we can rewrite (20) in the following matrix format

\[
(T_1^T T_1 - T_2^T T_2 D_k) h = Th = 0
\]

where $D_k = \text{diag}([1, e^{j2\pi k/P}, e^{j4\pi k/P}, ... , e^{j(M-1)\pi k/P}])$ , the impulse response is $h = [h_0, h_1, ..., h(L_n)]^T$. The information of channel can be achieved by solving the linear equations (21).

B. The lower complexity One-Cyclic algorithm

Both the Two-Cyclic algorithm and the One-Cyclic algorithm have the shortcoming of large amount of computing. By analyzing the distribution of energy spectrum function, the lower complexity One-Cyclic algorithm based on partial spectrum information can further reduce the computational complexity.

Equation (4) tells us that the autocorrelation function has a non-zero value only in the M-point during $\tau > 0$, this reveals that the main energy distribution of the autocorrelation function for the received signal is in $[M - L_n, M + L_n]$. Based on this result, (20) can be approximately simplified as

\[
\sum_{l=1}^{L_n}\sum_{l=0}^{L_n} R_l(k, \tau)e^{j\pi \tau k/P}z^{-l} = \sum_{l=0}^{L_n} R_l(k, \tau)e^{j\pi \tau k/P}z^{-l}
\]

Only two Toeplitz matrices named $T_1$, $T_2$ are needed and constructed respectively from $R_l(k, \tau)e^{j\pi \tau k/P}$, $R_{l2}(k, \tau)e^{j\pi \tau k/P}$ with lower dimension as $(3L_n + 1) \times (L_n + 1)$.

With the $T_1$, $T_2$, we can rewrite (22) in the following matrix format

\[
(R_l(k, M)T_1 - R_{l2}(k, M)T_2 D_k) h = Th = 0
\]

In consideration of the system performance, (23) can be expressed as

\[
R_l(k, M) - R_{l2}(k, M)(\hat{\mathbf{D}}_k) h = 0
\]

Finally, the linear equations can be solved by the least-square method.

Compared with the above two kinds of algorithms, the lower complexity One-Cyclic algorithm not only reduce the number of Toeplitz matrix from 4 to 2, but the dimension of the matrices is also from $(4M + 3L_n + 1) \times (2M + L_n + 1)$ to $(3L_n + 1) \times (L_n + 1)$ . Table 1 shows the comparison of the computational complexity among the Two-Cycle algorithm, One-Cyclic algorithm, and the lower complexity One-Cyclic algorithm.

In generally, OFDM signal length $M$ is much larger than the channel delay, so the lower complexity algorithm can greatly reduces the computational complexity of the original algorithm, while performance is not significantly reduced, with high practical value.

| Table 1. The comparison of computational complexity for the three methods |
|-----------------------------|-----------------------------|-----------------------------|
| Matrix dimension           | Matrix number |
| The Two-Cycle algorithm    | $(4M + 3L_n + 1) \times (2M + L_n + 1)$ | 4 |
| The One-Cyclic algorithm   | $(4M + 3L_n + 1) \times (2M + L_n + 1)$ | 4 |
| The lower complexity       | $(3L_n + 1) \times (L_n + 1)$ | 2 |
V. SIMULATION RESULTS

In this section, the performances of the Two-Cycle algorithm, One-Cyclic algorithm, and the lower complexity One-Cyclic algorithm are compared. In order to measure the performance of the three algorithms, we define the mean-square error (MSE) as

\[
\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} \left\| \hat{h}_i - h \right\|^2
\]

which is averaged over N Monte Carlos to evaluate the estimated channel error. The channel impulse response is \( h = [1, -0.8 + 0.2i, 0.6 - 0.3i, 0.8 - 0.5i, 0.6 - 0.4i] \). We used \( M=32, L=8 \). QPSK is used for modulation. All the results are averaged over 1000 independent runs.

In Figure 8 and Figure 9, with \( P=M+L=32+8=40 \), we respectively show the real and image part of the average estimated channel for SNR=15dB versus the number of blocks where '+' , '△' and '□', respectively represent the Two-Cycle algorithm, One-Cyclic algorithm, and the lower complexity One-Cyclic algorithm. The coefficients of the channel estimated respectively from the three algorithms are provided in Table 2.

<table>
<thead>
<tr>
<th>Two-Cycle algorithm</th>
<th>One-Cyclic algorithm</th>
<th>lower One-Cyclic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.145 - 0.0041i</td>
<td>0.9788 + 0.023i</td>
<td>0.6046 - 0.434i</td>
</tr>
<tr>
<td>-0.834 + 0.246i</td>
<td>-0.787 + 0.2038i</td>
<td>-0.8425 + 0.195i</td>
</tr>
<tr>
<td>0.4766 + 0.1635i</td>
<td>0.6341 - 0.2891i</td>
<td>0.6262 - 0.2970i</td>
</tr>
<tr>
<td>0.8335 - 0.6296i</td>
<td>0.7653 - 0.4951i</td>
<td>0.7097 - 0.4734i</td>
</tr>
<tr>
<td>0.5998 - 0.4507i</td>
<td>0.6046 - 0.4343i</td>
<td>0.5620 - 0.4269i</td>
</tr>
</tbody>
</table>

With \( P=M+L=32+8=40 \) and SNR=20, we plot the real part of the channel estimates for Figure 8 and Figure 9. The image part of the channel estimates for Figure 10 shows the absolute value of autocorrelation function for the received signal versus the time delay variable when \( n = 5 \) and \( \text{SNR} = 15\text{dB} \). From the figure we know that the main energy of autocorrelation function is concentrated in \([M - L, M + L] = [28, 36]\) interval. The lower complexity One-Cyclic algorithm is formed based on this property.

The symbol BER versus SNR is displayed in Figure 11, which demonstrates that the performance of the One-Cyclic algorithm is the best among the three algorithms, and is almost the same as the actual channel in the low SNR region, the BER for both the lower complexity algorithm and the One-Cyclic algorithm is less than that of the Two-Cycle algorithm.

The curves of channel mean-square error (MSE) from three algorithms are plotted in Figure 12. It is seen that the Two-Cycle algorithm has the largest MSE, matched well with the BER analysis in Figure 11. It’s very interesting that the lower complexity One-Cyclic algorithm has the lowest MSE in lower SNR, and the One-Cyclic algorithm has the best performance with the high SNR. Further analysis will be discussed in another paper.

VI. CONCLUSION

A blind channel estimation algorithm and its lower complexity algorithm based on cyclostationarity properties induced by the cyclic prefix in OFDM system are proposed in this paper. In the new algorithms, the channel can be identified uniquely through analyzing the spectrum information of send and receive signal at the single frequency. The algorithms have better performance than the previous Two-Cycle algorithm. Furthermore, a lower complexity algorithm is also developed which greatly reduce the computational complexity.
REFERENCES


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