Chapter 1

Bayesian Methods for Aircraft Structural Health Monitoring

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1.1 Introduction

Aircraft structures, whether metallic or composite, are subject to service damage which requires their periodic inspection and maintenance. While taking the aircraft out of service is quite costly, the assurance of structural integrity requires such inspection – and possible repair work based on the inspection results obtained. If damage is detected from an inspection, the decision whether to repair as well as the method of repair must be made on the basis of data relevant to the specific inspection method being used, and with an uncertainty that has been characterized as accurately as possible. This is an example of an application that is amenable to a Dynamic Data Driven Application System (DDDAS) solution. That is, data can be acquired dynamically, and compared to a model of the structure such that damage can be located and a determination made as to whether it requires further inspection and possible repair. Moreover, Bayesian methods allow the characterization of uncertainty, and with the appropriate inference networks they allow conditional probabilities to be determined in terms of what is known about the structure from the model and what is measured during the inspection. The methodology under development allows the accuracy of the model as well as that of the inspection data to be taken into consideration, and uses an iterative approach to improve both the model and the inspection data. That is, inspection data can be used to determine physical constants or variables used in the model (e.g., Young’s modulus, diffusion constants, etc.), and the computational model can be used to improve inspection data (e.g., pose, noise, hysteresis, etc.).
Although we focus here on aircraft structures and inspection sensors, the methods being developed may be applied broadly to any physical system that is modeled and monitored with sensors; including, aircraft, bridges, refineries, etc. Of particular interest in this investigation is the use of ultrasonic sensing of Lamb waves to detect and identify damage in aircraft structures. It is noted that newer generation aircraft use a significant amount of composite materials in both primary and secondary structures. Current ultrasonic sensing systems based on Lamb waves are mostly experimental (see [25] for a very good overview of this topic), and one of our goals is to develop robust methods for structural health monitoring which can then be applied even when there are uncertainties in the measurements, system models and sensor locations, as well as possible time variations of the underlying systems. Some reasons why this is quite challenging include:

- every physical system (e.g., an individual aircraft) requires a customized model,
- the model parameters change over time,
- the idealized model deviates from the physical system (e.g., due to simplifications),
- sensor data is noisy,
- sensor locations are not precisely known.

The overall goal of this work is to advance the DDDAS state-of-the-art by developing a framework in which the data acquired for a specific aircraft allow the most cost effective determination of whether damage has been produced in the structure, and the location of the possible damage. Attaining this capability requires the development of:

- appropriate models (at various levels of resolution),
- effective computational procedures,
- adequate frameworks to characterize uncertainty in the model,
- appropriate sensor systems (perhaps as part of a robotic data acquisition system),
- adequate sensor models,
- uncertainty representations for sensor data,
- methodologies for dynamic interaction of models and sensor data that allow the determination of system properties (e.g., damage) with necessary uncertainty quantification.

The success of this research will lead to a general framework to assess the accuracy of models, and to address the dynamic use of sensor data to determine changes in the system state, and thus dynamic model development as the system changes over time.
1.2 Bayesian Computational Sensor Networks

Computational Sensor Networks (CSN) [12] combine computational models of physical phenomena (e.g., heat flow, ultrasound, etc.) with sensor models to monitor and characterize a variety of systems. Previous work by the authors has shown how CSN can be applied to heat flow [13, 14] and reaction-diffusion model accuracy assessment [15, 16]. Furthermore, a Bayesian Computational Sensor Network (BCSN) was developed in [24]. Figure 1.1 shows the basic layout of a BCSN system. The standard setup is characterized by the gray blocks in the diagram. A structure is monitored using an array of sensors that are bonded or in some situations temporarily attached. The sensor signals are stored in a database and analyzed to extract information about the structure. The lighter blocks at the top and right side of the figure indicate our organization of the solution into a computational model. Such computational models typically involve a set of linear or nonlinear partial differential equations, as well as models of sensor behavior. The sensor models may account for the physics of operation of the sensors and their interaction with the structure as well as noise in the system. Finally, the computational system also monitors or contains information about network topology, communication, latency, quality of service, etc.

The dynamic data driven aspect of the system is indicated by the phenomenon model and its loops, as well as the uncertainty quantification. We focus on these two aspects of the DDDAS problem. The physical structure is monitored by a network of sensors whose data is used to update a computational model of the structure, and the uncertainty of the resulting analysis is quantified. Moreover, the placement of sensors can be performed dynamically to optimize the effectiveness of acquired information or energy consumption.

The major objectives for the project as applied to aircraft structural health monitoring are to develop:

1. Bayesian Computational Sensor Networks that detect and identify structural damage, model physical phenomena and sensors, and characterize uncertainty
in calculated quantities of interest. The uncertainty quantification may involve real-valued as well as logical variables. The sensors employed in this work are piezoelectric ultrasound sensors, even though the techniques developed are also valid for other types of sensors.

2. Active feedback methodologies using model-based sampling regimes. The subgoals here include embedded and active sensor placement, online sensor model validation; and

3. Rigorous uncertainty quantification models for system states, model parameters, sensor network parameters (e.g., locations of sensors, noise) and material damage assessments (location, magnitude, etc.).

This project addresses three of the four DDDAS research components: (1) applications model development, (2) advances in mathematics and statistical algorithms, and (3) application measurement systems and methods. Our overall DDDAS approach is shown in Figure 1.2. This approach is based on the validation, calibration and prediction process as described by Oberkampf [21]. Experiments are used to establish parameters in the computational model, and these in turn affect the result of the validation metric. Both simulations and physical experiments are used to help with experiment design as well as to inform the computational modeling process. When studying parameter estimation methods in simulations, implicit methods are used to represent the phenomenon, whereas an explicit approach is used in the estimation method (e.g., estimation update formulas are based on the explicit time step function at each location). Such an approach provides information about the feasibility and truncation error effects of the explicit numerical method.

The first step in our project extended the existing 1D Bayesian Computational Sensor Networks approach for heat flow to 2-D to establish the adequacy of the approach on a simpler problem than our ultimate goal: ultrasound. In [14] we described the impact of parameter estimation on Model Accuracy Assessment (MAA).
We performed a comparison of seven parameter estimation approaches (Inverse Method, Linear Least Squares (LLS), Maximum Likelihood Estimation (MLE), Extended Kalman Filter (EKF), Particle Filter, Levenberg-Marquardt, and Minimum RMS error) to estimate the value of thermal diffusivity ($k$) associated with heat flow in a 2-D plate. A comparison of these methods was made in terms of the adequacy requirements. An important finding of this work [14] was that the statistics produced by the parameter estimation techniques can be used to characterize the adequacy of the model. The determination is highlighted below, and details are available in [14]. In order to compare the methods, we use both simulated and experimental heat flow data through a 2-D plate. The layout of the experimental apparatus is shown in Figure 1.3. A FLIR T420 high performance IR camera takes a 320x240 pixel array, of which a 170x170 subset samples the aluminum plate. Figure 1.4 shows an example image with heat sources on the left and upper parts of the plate. In order to get smoother results in the parameter estimation methods, the image is averaged down to a 17x17 grid. $\Delta t$ is set to 30 sec with $max_t = 59 \times 30 = 1770$, and $\Delta x = \Delta y = 15.24 / 17$ cm (in simulation experiments, $k$ is set to 0.85). The sample set is then $T_n$ with time step $t = 1, 2, 3, \ldots, 58$:

$$T_n = T(x, y, 1 : t + 1)$$

In the simulations, we used the testing data, $T$, to run experiments for the seven estimation methods to get the value of the thermal diffusivity parameter over 30 trials for each method. The error of the estimate, $k$, is compared between the seven methods according to:

$$k_{\text{error}} = \frac{\| k - k_{\text{est}} \|}{k}$$

Note that this corresponds to finding the computational model parameter. We then use the estimate $k$ to run a new heat flow experiment $S(x, y, t)$ and compare with
the simulated temperature at location \((x,y)\) and time \(t\), and compute the RMS (Root Mean Square) error:

\[
RMS_{\text{error}} = \sqrt{\frac{\sum (T_{x,y,t} - S_{x,y,t})^2}{N}}
\]

where \(N\) is the number of locations times the number of time steps minus 1. If the error is below the specified amount (e.g., average 1 degree C), then the adequacy of the model is demonstrated. Figure 1.5 shows the RMS error for the temperature sequences produced with the respective \(k\) values of the seven methods.

### 1.2.1 Ultrasound-based Damage Assessment

In this chapter, we consider structural health monitoring systems employing piezo-electric ultrasound sensors and actuators. There are two broad classes of such systems – passive and active. Passive systems continuously listen for acoustic emission waveforms that arise from impacts or other events that could result in structural damage. Once the sensor network receives such signals, the computational network will locate the source of the waveforms, i.e., impact or damage location. The system may also determine if damage exists, and characterize the damage based on the signal properties. The second class, which is the focus of this chapter, is active SHM systems.

Active SHM is performed by exciting the structure to be monitored with waveforms produced by an actuating transducer. Signals propagated from each actuator are collected at sensors distributed on the structure. Assuming that we have baseline signals collected from the structure at some time, any change in the structure
(for example, new damage) will result in corresponding changes in the sensor signals. Figure 1.6 shows an example. The bottom left panel displays the sensor signal from a healthy structure. Assuming that new damage was introduced in the structure as shown in the top right panel, we can expect new measurements using the same transducer-sensor pairs to contain reflected components of the excitation waveforms from the boundaries of the damage. The waveform depicted in the bottom right panel describes such a scenario. Based on the properties of the received signals, the damage state of the structure is estimated. In the example of Figure 1.6, one may estimate the time of arrival of the directly propagated waveform and the reflected component. Knowing the velocity of propagation (we assume in this example that the structure is isotropic), we can define an ellipse on which the reflecting boundary lies. This is shown in Figure 1.7. With the help of multiple actuator-sensor pairs, we may then estimate the boundary of the anomaly in the structure. Other methods for locating the damage and characterizing the extent of the damage are also available.

These algorithms are implemented so that automated monitoring of the structure may be achieved. An alternate approach to bonding or embedding sensors on the structure is to employ mobile robotic elements to sense at selected locations on the structure. Such a technique is under exploration in our research. Knowledge of the input wave, time difference between transmission and reception of different components in the sensor waveform, as well as the wave propagation properties of the structure, taken together allow the estimation of damage existence, location and scale.

The basics of robot sensing for structural health monitoring is as follows. A picture of a robot equipped with two sensors used in this work is shown in Figure 1.10. The robot has two ultrasound transducers fixed at a distance L apart as shown in the figure. A set of samples are taken over the surface of the structure, and assuming that parameters characterizing the undamaged structure are available, a baseline model of the sensor signal for each actuator-sensor pair can be estimated. By moving the robot and obtaining several range estimates, the intersection of the ellipses provides an estimate of the damage location. By circumnavigating the
detected damage location, the robot can use the range information to determine the reflecting boundaries of the damage, and thus, its extent.

1.3 Lamb Waves in Structural Health Monitoring

Figure 1.8 lays out the approach to using Lamb waves for SHM. Lamb waves are guided waves that propagate in solid structures. In active SHM systems, Lamb waves may be induced in the structure by ultrasound transducers that may act as actuators and sensors as needed. The propagation takes place in multiple modes. The velocity of each mode at any location of the structure depends on the product of the frequency of excitation and the thickness of the structure at that location. Figure 1.9 displays the phase velocity of different Lamb wave modes in an Aluminum plate. Because of the frequency dependent velocity profiles, the propagation of these modes is dispersive. For a detailed introduction to ultrasound waves, see [23]; there has also been a lot of work in the application of these techniques in SHM (see
1.3. LAMB WAVES IN STRUCTURAL HEALTH MONITORING

Dynamic Data-Driven Structural Health Monitoring

![Lamb Wave-based Structural Health Monitoring](image)

Figure 1.8: Lamb Wave-based Structural Health Monitoring.

![Lamb Wave Dispersion Curves](image)

Figure 1.9: Lamb Wave Dispersion Curves.

[6, 7, 8, 9, 17, 18, 29], as well as a number of Air Force Masters theses on the topic [1, 2, 3, 4, 5, 10, 20, 22, 27]. For an excellent recent study on a data-driven approach, see [11]. Overlapped original and reflected modes (from boundaries or damaged areas) are then separated, and finally damage locations are identified based on this knowledge. Online model accuracy assessment is crucial since the multimodal and dispersive characteristics of Lamb waves may change due to changes in environmental conditions and structural properties. Such changes may result in the failure of static damage localization models, and thus in the DDDAS approach, the models are updated (re-calibrated) in every data collection step.
1.3.1 Modeling Lamb Wave Propagation

The dispersive multi-modal propagation of the excitation signal $x(t)$ from an actuator to a sensor may be modeled as

$$y(t) = \sum_{m=1}^{M} \int_{-\infty}^{\infty} h_m(\tau)x(t-\tau)d\tau$$

where we have assumed that the propagation is linear, $h_m(t)$ represent the dispersion and attenuation characteristics of the structure for the $m$th mode of propagation. Equivalently, we can express the above in the frequency domain as

$$Y(\omega) = \sum_{m=1}^{M} H_m(\omega)X(\omega)$$

where $Y(\omega)$, $H_m(\omega)$ and $X(\omega)$ are the Fourier transforms of $y(t)$, $h_m(t)$ and $x(t)$, respectively. The magnitude response $|H_m(\omega)|$ represents the frequency-dependent attenuation characteristics of the structure. This quantity varies with modes as well as the frequency of excitation. As one would expect, the path length also affects the attenuation of the signals in the structure. The phase response depends on the frequency of excitation through the dispersion curves and the path length. In what follows we describe how a model for the dispersion curves can be built.

The governing partial differential equation for displacement in an isotropic plate with density $\rho$, displacement vector $u$, and Lamé constants $\mu$ and $\lambda$ assuming no body forces is expressed in vector notation as:

$$\rho \ddot{u} = \mu \nabla^2 u + (\lambda + \mu) \nabla (\nabla \cdot u)$$

where $\ddot{u}$ is the acceleration of displacement. The governing longitudinal wave is:

$$\frac{\delta^2 \varphi}{\delta x^2} + \frac{\delta^2 \varphi}{\delta y^2} = \frac{1}{C_L^2} \frac{\delta^2 \varphi}{\delta t^2} \quad (1.1)$$

and the governing shear wave is:

$$\frac{\delta^2 \psi}{\delta x^2} + \frac{\delta^2 \psi}{\delta y^2} = \frac{1}{C_T^2} \frac{\delta^2 \psi}{\delta t^2} \quad (1.2)$$

where $C_L$ and $C_T$ are the longitudinal and transverse wave speeds, respectively. Assuming solutions to the governing equations to be of the form:

$$\varphi = \Phi(y)exp^{i(kx - wt)}$$

$$\psi = \Psi(y)exp^{i(kx - wt)}$$

we can substitute the above expressions into the governing wave equations (Equ 1.1–1.2) to get:

$$\Phi(y) = A_1 \sin(py) + A_2 \cos(py)$$

$$\Psi(y) = B_1 \sin(qy) + B_2 \cos(qy)$$

where $A$ and $B$ are found from the boundary conditions, and:

$$p^2 = \frac{w^2}{C_L^2} - k^2 \quad q^2 = \frac{w^2}{C_T^2} - k^2$$
Further development [23] allows us to obtain the Rayleigh-Lamb frequency equations:

\[
u_s(f, d) \equiv \frac{\tan(qh)}{\tan(ph)} = \frac{-4k^2pq}{(q^2 - k^2)^2}
\]

(1.3)

\[
u_a(f, d) \equiv \frac{\tan(qh)}{\tan(ph)} = \frac{(q^2 - k^2)^2}{-4k^2pq}
\]

(1.4)

We can solve the above two equations numerically to obtain the dispersion curves. The dispersion curve can then be used along with the length of the path to estimate the phase response associated with the propagation of each mode. The magnitude response is typically estimated from the data. Once these quantities are known, we can use Eqns (1.1–1.2) to estimate the sensor signals. Since each mode has now been characterized we can also use this model to separately identify the different modes and reflections in the sensor signal as shown in the third panel in Figure 1.8. Thus, given an input signal, we use the Fourier transform to obtain the frequencies involved; these are then propagated according to the dispersion values, and the received signal can be determined. The analytical model for the sensor signal is then given by:

\[y(t) = F^{-1}[\sum_{m=1}^{N} X(\omega)H_m(\omega)]\]

where \(x(t)\) is the input signal, \(y(t)\) is the output signal, \(m\) is the mode number, \(X(\omega)\) the FFT of the input signal, and \(H_m(\omega)\) the phase response (obtained from the dispersion curves). Figure 1.10 shows our mobile robot taking data on the aluminum plate used for initial studies, as well as some readings from the sensors. The aluminum panel was 1.6 mm thick, the sensors were VS900-RIC Vallen transducers, and the excitation signal was a 200 KHz 5 cycle, Hann-windowed waveform.

1.3.2 SLAMBOT: Simultaneous Localization and Mapping using Lamb Waves

We are currently developing a mobile robot platform which can move around on a structure to take data (see Figure 1.10). Based on a modified Systronix Trackbot mobile platform, the SLAMBOT has two attached actuation systems which cause the robot to be lifted off the surface when the ultrasound sensors are used, thus, reducing the interference from the robot on the sensor signals. Our current work is on Simultaneous Localization and Mapping (SLAM) using Lamb waves (see [26] for a detailed account of the SLAM methodology). The damage (and boundary) locations are considered point landmarks since the reflected signal returned from the closest reflecting point determines the range value. The range calculation method described earlier (shown in principle in Figure 1.7) is used by finding the arrival time of the second Lamb wave signal received (the first being from the straight line path from the transducer). The total number of features is controlled by the data acquisition process, and both the range data and the robot motion are assumed to have been corrupted by additive Gaussian noise.

Because we only use positive landmark detection (landmarks that show up in the range data as opposed to those occluded by other objects), as well as the conditions given above, EKF SLAM works in this setting (see [26]). We therefore estimate the robot pose \(s_t = (x, y, \theta)\) as well as the landmark locations \((F_i = f_{i,x}f_{i,y}f_{i,z})\)
Figure 1.10: SLAMBOT for Dynamic Data Acquisition. The SLAMBOT is shown on the left; on the right, the structure is excited in three different locations, and the final column is the received signal for each; note that the reflected damage signal can be seen trailing the direct signal.

\[ i = 1 \ldots n, \text{ simultaneously using a combined state vector. Then given a motion mode for the robot:} \]

\[ p(s_t \mid u_t, s_{t-1}) \]

where \( u_t \) here indicates the robot control. The measurement model is:

\[ p(z_t \mid s_t, F, n_t) \]

The SLAM problem is to find all landmark locations and the robot’s pose using the measurements and control values; that is, the posterior:

\[ p(s^t, F \mid z^t, u^t) \]

We assume feature correspondence is known, and use Algorithm EKF SLAM known correspondences (see Table 10.1 [26], p. 314). The results of a simulation of the Lamb wave based range finder are shown in Figure 1.11 (left). In this example, a 2 m X 2 m aluminum plate is used with the origin at the center (thus range in \( x = [-1, 1] \) and range in \( y = [-1, 1] \) with one damage location at \((-0.4, -0.4)\). The robot places the actuator and receiver at six different locations around the damage, and each range value constrains the location of the reflecting point to be on an ellipse with the actuator and receiver locations as foci. Thus, by using an accumulator array and adding a 'vote' to each location on the ellipse, these six sensed range values allow the determination of the most likely location of the reflecting point (damage in this case). This ‘voting’ is done with a Guassian spread which leads to the smooth accumulator surface shown in the figure. Figure 1.11 (right) shows a 2-D visualization of the strength of damage location likelihood based on this data.
1.4 Conclusions and Future Work

We propose a Bayesian Computational Sensor Network approach as a formal basis for Dynamic Data Drive Application Systems. To date, we have shown that this can be effective in the 1D domain of heat flow, and we are currently working to develop a robust aircraft structural health monitoring framework based on the use of Lamb waves. A dynamic data acquisition method using a mobile robot has been described. Future work includes the experimental validation of the approach as well as a formal analysis of the uncertainty quantification. We are constructing several mobile robots and will perform experiments using single and multiple robots to map damage in plate structures. The experiments will first be performed with Aluminum plates, and then on composite structures.

The field of uncertainty quantification aims to find methods to provide bounds on the confidence of inferences about the behavior of physical systems based on computational models and sensor data. In order to estimate the error, one can resort to Monte Carlo sampling, or construct a response surface from sampling the system. We follow the procedure of Li and Xiu [19, 28] wherein a surrogate model is developed using polynomial chaos: “stochastic solutions are expressed as orthogonal polynomials of the input random parameters.” We follow the method described in Section 5.2 in [28].

The equations for symmetric (Equation 1.3) and anti-symmetric waves (Equation 1.4) form the basis of our damage analysis range sensor. That is, damage modifies the received signal according to the distance from the actuator to the damage and then on to the receiver. Taking a polynomial chaos approach, the generalized polynomial chaos (gPC) approximation to the solution is obtained by projecting $u$ (the function of interest – in this case, the wave speed) onto a polynomial basis, $P_k$:

$$u|_{P_n} = \sum \alpha_i P_i$$
where the Fourier coefficients are defined as:

$$\alpha_i \approx \int uP_i \rho dX = <u, P_i>$$

where $<u, w>$ is the inner product of $v$ and $w$. That is, the coefficients of the polynomial projection are approximated by the inner product of the function $u$ and the polynomial basis functions.

Figure 1.12 shows a plot of the error generated by sampling the function $fun(F, D)$ (wave speed) exactly and the approximated polynomial up to maximum total degree 6. In this case, we are considering the speed of the antisymmetric modes:

$$fun(F, D) = c_{p_{anti}}$$

We are currently working on characterizing the uncertainty properties of the range sensor function described earlier.
Bibliography


